

**MATH 104 TUTORIAL**  
**Elasticity and Lagrange**

**Question1.** Locate and identify all **criticals** on

a)  $z = f(x, y) = x^2 + y^2 - 5x + 4y$

b)  $z = f(x, y) = \frac{1}{3}(x^3 + 8y^3) - 2(x^2 + y^2) + 1$

**Question2.** Given  $f(x, y) = \ln(x + y)(1 + e^{3x})^2 - 5(x + y)^2 + x^2y^3$ . Find  $f_{xy}$  and  $f_{yx}$ .

**Question3. Pricing Model**

A manufacturer sells two related products, the demands for which are estimated by the following two demand functions:

$$q_1(p_1, p_2) = 150 - 2p_1 - p_2$$

$$q_2(p_1, p_2) = 200 - p_1 - 3p_2$$

Where  $p_j$ : the price (in dollars) of product j,  $q_j$ : the demand (in thousand of units) for product j.

- The firm wants to determine the price it should charge for each product in order to maximize total revenue from the sale of the two products.
- Determine the Marginal changes at  $p_1 = 30\$, p_2 = 10\$$  demands and classify these products, interpret.
- Find  $\eta_{11}, \eta_{21}$  at revenue maximizing prices, interpret.

**Question4.** Find the critical points and their natures for  $z = f(x, y) = 3x - y + 6$  subject to the constraint  $x^2 + y^2 = 4$ . (LAGRANGE)

**Question5.** Find the critical points and their natures for  $z = f(x, y) = 5x^2y$  subject to the constraint  $x + 2y = 24$ . (LAGRANGE)

**Question6.** Given

Minimize  $f(x, y) = 2x^2 + 4y^2 - 3xy - 2x - 23y + 3$

subject to  $x + y = 15$

What will be the change in function value at optimum if the constraint changes to  $x + y = 16$ .

