E.M.U. - FACULTY OF ARTS AND SCIENCES
DEPARTMENT OF MATHEMATICS
Math106 -- Linear Algebra
First Midterm Examination
26.11.2015

Name-Surname		**Student Number**	
Group number		**Signature**	

Question 1	**Question 2**	**Question 3**	**Question 4**	**Question 5**	**Question 6**	**Total**
/20	**/20**	**/20**	**/15**	**/20**	**/15**	

Duration: 90 mins.

Q1) Determine for what values of $a \in \mathbb{R}$, the linear system

$$\begin{array}{rcrcrcl} x & + & y & + & az & = & 1 \\ x & + & ay & + & z & = & 1 \\ ax & + & y & + & z & = & 1 \end{array}$$

has

a) no solution
b) unique solution
c) infinitely many solutions.

Q2) Let $A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 3 \\ 3 & 8 & 7 \end{bmatrix}$. Express the matrix A^{-1} as a product of elementary matrices.

Q3) Consider the following linear system:

$$\begin{array}{rcrcrcr} x & - & 3y & + & z & = & 4 \\ 2x & - & y & & & = & -2 \\ 4x & & & - & 3z & = & 0 \end{array}$$

a) Find the inverse of the coefficient matrix A.
b) Solve the system, by using the inverse of A.

Q4) Decide whether the given matrix is invertible. If so, use **Adjoint method** to find its inverse.

$$A = \begin{pmatrix} 2 & 0 & 3 \\ 0 & 3 & 2 \\ -2 & 0 & -4 \end{pmatrix}$$

Q5)

a) Find the following determinant, by reducing the matrix to **Row-Echelon Form**:

$$\begin{pmatrix} 2 & 1 & 3 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 2 & 1 & 0 \\ 0 & 1 & 2 & 3 \end{pmatrix}$$

b) By using the properties of determinants, show that

$$\begin{vmatrix} b+c & c+a & b+a \\ a & b & c \\ 2 & 2 & 2 \end{vmatrix} = 0$$

Q6)

a) Prove that, if A is $n \times n$ matrix, then

$$\det\left(adj(A)\right) = \left(\det(A)\right)^{n-1}$$

b) Show that, if B is a square matrix, then

i. BB^T is symmetric.

ii. $B + B^T$ is symmetric.