MATH 106 Linear Algebra Quiz 1

Name-Surname

Student No

Group No

Signature

-Solution Manual-

Q1	Q2	Total

March 19, 2012. Duration is 45 minutes.

Question 1) Find a 3×1 matrix X with entries not all zero such that $AX=3X$, where

$$A=\begin{bmatrix}1 & 2 & -1\\ 1 & 0 & 1\\ 4 & -4 & 5\end{bmatrix}.$$

Let $X=\begin{pmatrix}a\\ b\\ c\end{pmatrix}$

Hint: Please, use the elementary row operations to solve the system.

$$\begin{pmatrix}1 & 2 & -1\\ 1 & 0 & 1\\ 4 & -4 & 5\end{pmatrix}\begin{pmatrix}a\\ b\\ c\end{pmatrix}=3\begin{pmatrix}a\\ b\\ c\end{pmatrix} \Rightarrow \begin{aligned} a+2b-c&=3a\\ a+c&=3b\\ 4a-4b+5c&=3c.\end{aligned}$$

$$\begin{aligned}-2a+2b-c&=0\\ a-3b+c&=0\\ 4a-4b+2c&=0.\end{aligned} \iff \begin{pmatrix}-2 & 2 & -1\\ 1 & -3 & 1\\ 4 & -4 & 2\end{pmatrix} \sim \begin{pmatrix}1 & -1 & 1/2\\ 1 & -3 & 1\\ 4 & -4 & 2\end{pmatrix}$$

$R_2 \to -R_1+R_2$
$R_3 \to -4R_1+R_3$

$$\sim \begin{pmatrix}1 & -1 & 1/2\\ 0 & -2 & 1/2\\ 0 & 0 & 0\end{pmatrix} \to \begin{pmatrix}1 & -1 & 1/2\\ 0 & 1 & -1/4\\ 0 & 0 & 0\end{pmatrix}$$

$a-b+\frac{1}{2}c=0$.

$b-1/4c=0$. $\boxed{\text{If } c=t}$

$\boxed{b=1/4t}$ $a=1/4t-\frac{1}{2}t$

$\boxed{a=-1/4t}$ inf. many solutions.

So, $X=\begin{pmatrix}-1\\ 1\\ 4\end{pmatrix}$ is a solution.

2nd way;

From $A\underline{X}=3\underline{X}$

$(A-3I)\underline{X}=\underline{0}$

& solve this by Gaussian Elim.

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Question 2) In the following linear system

$$x+y-z=2$$
$$x+2y+z=3$$
$$x+y+(a^2-5)z=a$$

i. determine all values of a for which the resulting linear system has
   a) no solution
   b) a unique solution
   c) infinitely many solution.

ii. Find the unique solution of the given system.

$$\left(\begin{array}{ccc|c} 1 & 1 & -1 & 2 \\ 1 & 2 & 1 & 3 \\ 1 & 1 & a^2-5 & a \end{array}\right) \quad \begin{array}{c} R_2 \to -R_1+R_2 \\ \sim \\ R_3 \to -R_1+R_3 \end{array} \quad \left(\begin{array}{ccc|c} 1 & 1 & -1 & 2 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & a^2-4 & a-2 \end{array}\right)$$

If $a=2$ $\quad 0 \quad 0 \quad 0 \mid 0$ $\quad$ infinitely many solutions

If $a=-2$ $\quad 0 \quad 0 \quad 0 \mid -4$ $\quad$ no solution.

If $a \neq 2,-2$ continue Gauss elimination (for unique solution).

If $a \neq 2,-2$ $\quad R_3 \to R_3/a^2-4$.

$$\left(\begin{array}{ccc|c} 1 & 1 & -1 & 2 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 1 & 1/a+2 \end{array}\right) \quad \begin{array}{c} R_2 \to -2R_3+R_2 \\ \overline{} \\ R_1 \to R_3+R_1 \end{array} \quad \left(\begin{array}{ccc|c} 1 & 1 & 0 & 2+1/a+2 \\ 0 & 1 & 0 & -2/a+2 + 1 \\ 0 & 0 & 1 & 1/a+2 \end{array}\right)$$

$R_1 \to -R_2+R_1$

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 3/a+2 + 1 \\ 0 & 1 & 0 & -2/a+2 + 1 \\ 0 & 0 & 1 & 1/a+2 \end{array}\right)$$

unique sol^n for $a \neq \pm 2$.