MENG203

EXPERIMENTAL METHODS FOR ENGINEERS

PRESSURE, FLOW AND TEMPERATURE MEASUREMENT

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WHAT IS PRESSURE?

- PRESSURE IS THE FORCE PER UNIT AREA.

1) ABSOLUTE PRESSURE: IS THE ABSOLUTE FORCE PER UNIT AREA EXERTED BY FLUID ON THE CONTAINING WALL BY A FLUID.
2) GAGE PRESSURE: REPRESENTS THE DIFFERENCE BETWEEN ABSOLUTE PRESSURE AN LOCAL ATYMOSPHERIC PRESSURE.
3) VACUM PRESSURE: REPRESENTS THE AMOUNT BY WHICH THE ATMOSPHERIC PRESSURE EXCEEDS THE ABSOLUTE PRESSURE.

SOME COMMON PRESSURE UNITS

$$
\begin{aligned}
1 \text{ atmosphere (atm)} &= 14.696 \text{ pounds per square inch absolute} \\
&= 1.01325 \times 10^5 \text{ newtons per square meter (Pa)} \\
&= 2116 \text{ pounds-force per square foot (lbf/ft}^2\text{)} \\
1 \text{ N/m}^2 &\equiv 1 \text{ pascal (Pa)} \\
1 \text{ atmosphere (atm)} &= 760 \text{ millimeters of mercury (mmHg)} \\
1 \text{ bar} &= 10^5 \text{ newtons per square meter (100 kPa)}
\end{aligned}
$$

GRAPHICAL REPRESENTATION OF THREE PRESSURES

$$p\ (\text{absolute}) = p\ (\text{gage}) + p\ (\text{atm})$$

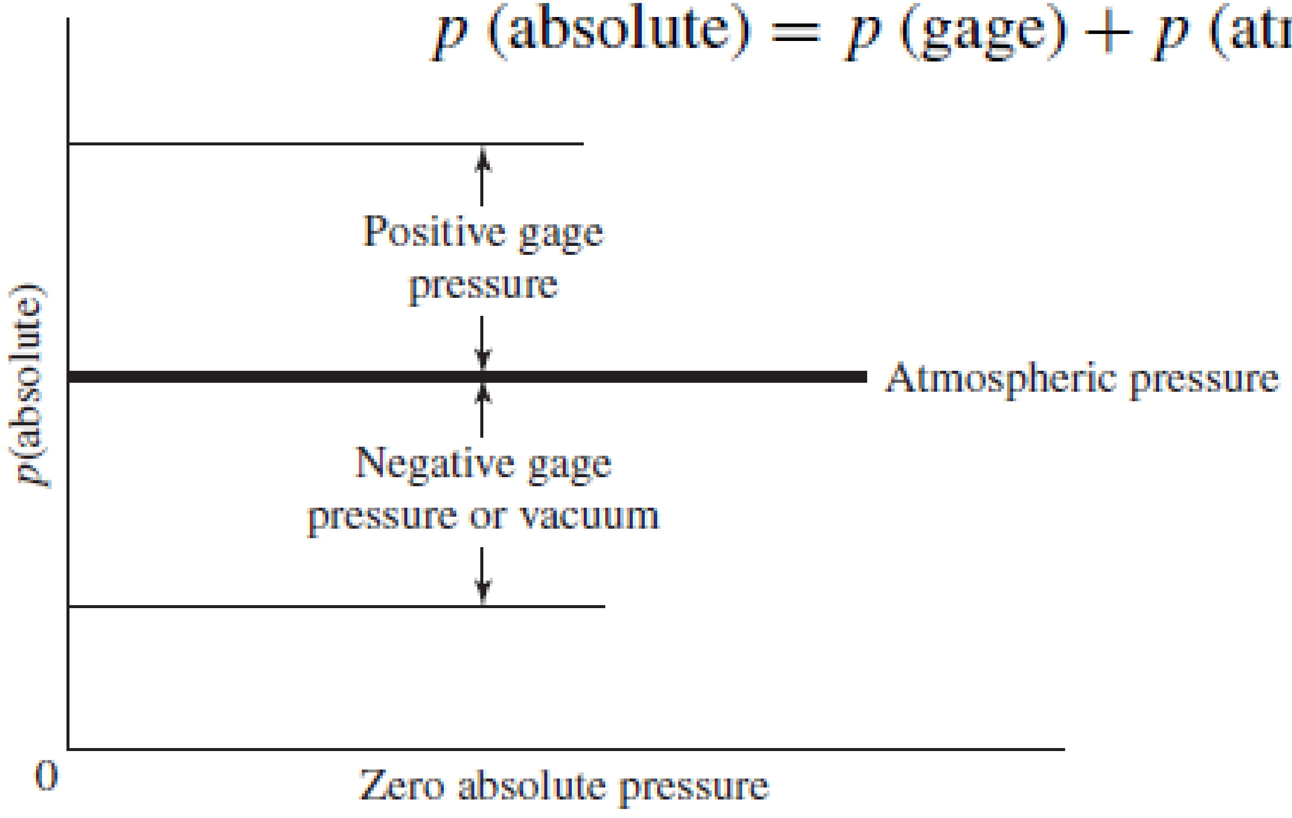


Figure 1 Relationship between pressure terms.

MECHANICAL PRESSURE MEASUREMENT DEVICES

Mechanical devices offer the simplest means for pressure measurement. In this section we shall examine the principles of some of the more important arrangements.

1) MANOMETER
2) BOURDON TUBE GAUGE
3) DIAPHRAGM AND BELLOW S GAUGE

1) U - TUBE MANOMETER

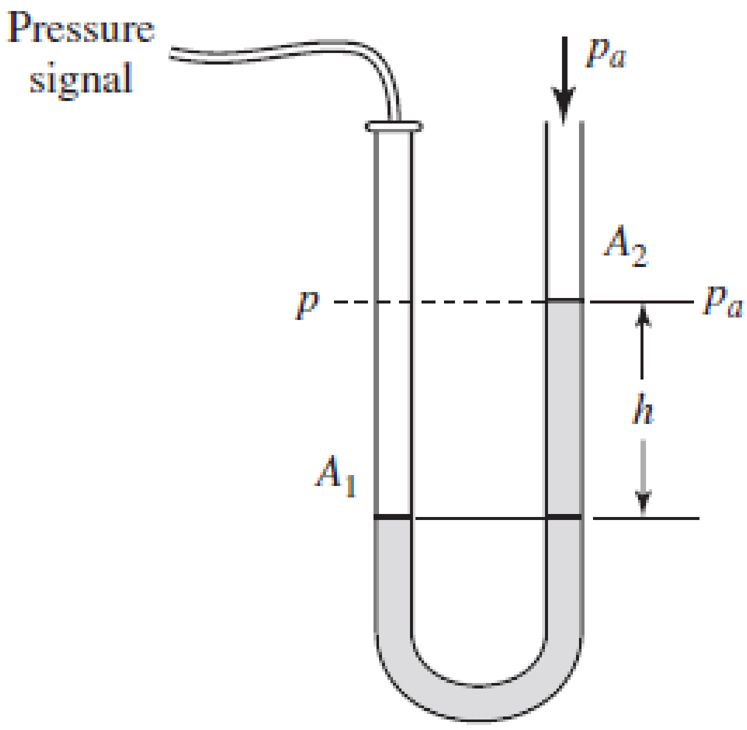


Figure 2 U-tube manometer.

A pressure balance of the two columns dictates that

$$p_a + \frac{g}{g_c} h \rho_m = p + \frac{g}{g_c} h \rho_f$$

$$p - p_a = \frac{g}{g_c} h (\rho_m - \rho_f)$$

SENSITIVITY OF U TUBE MANOMETER

The *sensitivity* of the U-tube manometer may be defined as

$$\text{Sensitivity} = h/(p - p_a) = h/\Delta p = 1/(g/g_c)(\rho_m - \rho_f)$$

or for a manometer with $\rho_m \gg \rho_f$,

$$\text{Sensitivity} = 1/\rho_m(g/g_c)$$

EXAMPLE: 1

U-TUBE MANOMETER. A U-tube manometer employs a special oil having a specific gravity of 0.82 for the manometer fluid. One side of the manometer is open to local atmospheric pressure of 29.3 inHg and the difference in column heights is measured as 20 cm $\pm$ 1.0 mm when exposed to an air source at 25°C. Standard acceleration of gravity is present. Calculate the pressure of the air source in pascals and its uncertainty.

Solution

The manometer fluid has a density of 82 percent of that of water at 25°C; so,

$$\rho_m = 0.82\rho_w = (0.82)(996\ \text{kg/m}^3) = 816.7\ \text{kg/m}^3$$

The local atmospheric pressure is

$$p_a = 29.3\ \text{inHg} = 9.922 \times 10^4\ \text{Pa}$$

The "fluid" in this problem is the air which has a density at the above pressure and 25°C (298 K) of

$$\rho_f = \rho_a = \frac{p}{RT} = \frac{9.922 \times 10^4}{(287)(298)} = 1.16\ \text{kg/m}^3$$

For this problem the density is negligible compared to that of the manometer fluid, but we shall include it anyway. From Eq. (6.11)

$$\begin{aligned} p - p_a &= \frac{g}{g_c} h(\rho_m - \rho_f) \\ &= \frac{9.807}{1.0}(0.2)(816.7 - 1.16) \\ &= 1600\ \text{Pa} \end{aligned}$$

or $$p = 1600 + 9.922 \times 10^4 = 1.0082 \times 10^5\ \text{Pa}$$

For altitudes between 0 and 36,000 ft the standard atmosphere is expressed by

$$p = p_0 \left(1 - \frac{BZ}{T_0} \right)^{5.26}$$

where p_0 = standard atmospheric pressure at sea level

Z = altitude, m or ft

$T_0 = 518.69°\text{R} = 288.16\ \text{K} = 15°\text{C}$

$B = 0.003566°\text{R/ft} = 0.00650\ \text{K/m}$

EXAMPLE 2:

INFLUENCE OF BAROMETER READING ON VACUUM MEASUREMENT. A pressure measurement is made in Denver, Colorado (elevation 5000 ft), indicating a *vacuum* of 75 kPa. The weather bureau reports a barometer reading of 29.92 inHg. The absolute pressure is to be calculated from this information. What percent error would result if the above barometric pressure were taken at face value?

Solution

The absolute pressure is given by

$$p_{\text{absolute}} = p_{\text{atm}} - p_{\text{vacuum}} \tag{a}$$

If the barometer report is taken at face value,

$$p_{\text{atm}} = (29.92)(25.4) = 760 \text{ mmHg} = 101.32 \text{ kPa}$$

and the absolute pressure is

$$p_{\text{absolute}} = 101.32 - 75 = 26.32 \text{ kPa} \tag{b}$$

Assuming the correction for altitude is given by Eq. (6.14), the true atmospheric pressure at the weather bureau is

$$p_{\text{atm}} = (760)[1 - (0.003566)(5000)/518.69]^{5.26} = 632.3 \text{ mmHg} = 84.29 \text{ kPa}$$

Assuming the local atmospheric pressure where the measurement is taken has this same value, the true absolute pressure is therefore

$$P_{\text{absolute}} = 84.29 - 75 = 9.29 \text{ kPa} \tag{c}$$

The percent error between the values in Eqs. (b) and (c) is

$$\% \text{ error} = \frac{26.32 - 9.29}{9.29} \times 100 = +183 \text{ percent}$$

Obviously, the *local* barometric pressure must be used instead of the value reported by the weather bureau.

Bourdon-Tube Pressure Gage

The construction of a bourdon-tube gage is shown in Figure . The bourdon tube itself is usually an elliptical cross-sectional tube having a C-shape configuration. When the pressure is applied to the inside of the tube, an elastic deformation results, which, ideally, is proportional to the pressure. The degree of linearity depends on the quality of the gage. The end of the gage is connected to a spring-loaded linkage, which amplifies the displacement and transforms it to an angular rotation of the pointer.

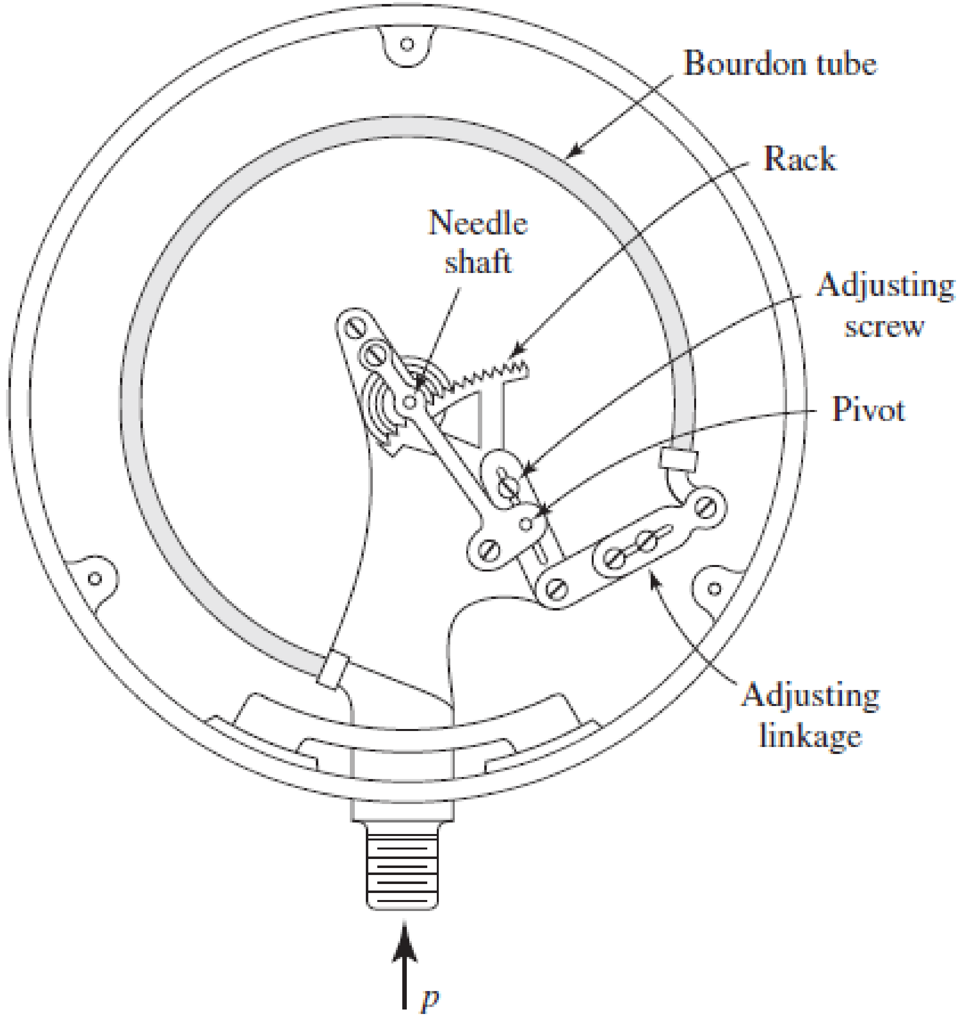


Figure Schematic of a bourdon-tube pressure gage.

3) Diaphragm and Bellows Gages

Diaphragm and bellows gages represent similar types of elastic deformation devices useful for many pressure-measurement applications. Consider first the flat diaphragm subjected to the differential pressure $p_1 - p_2$, as shown in Figure . The diaphragm will be deflected in accordance with this pressure differential and the deflection sensed by an appropriate displacement transducer.

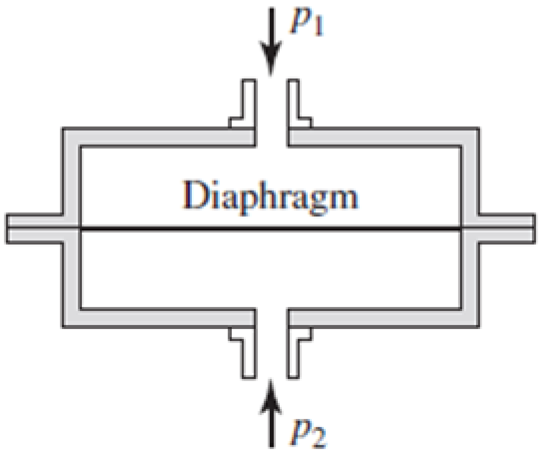


Figure 4 Schematic of a diaphragm gage.

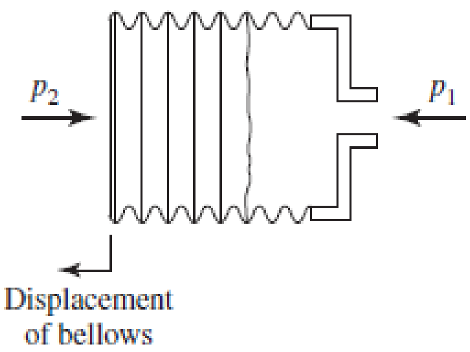


Figure 5 Schematic of a bellows pressure gage.

EXAMPLE:

NATURAL FREQUENCY OF A DIAPHRAGM GAGE. A diaphragm pressure gage is to be constructed of spring steel ($E = 200$ GN/m^2, $\mu = 0.3$) 5.0 cm in diameter and is to be designed to measure a maximum pressure of 1.4 MPa. Calculate the thickness of the gage required so that the maximum deflection is one-third this thickness. Calculate the natural frequency of this diaphragm.

Solution

Using the relation from Fig. 6.10, we have

$$\frac{1}{3}t = \frac{3\,\Delta p}{16Et^3}a^4(1-\mu^2)$$

$$t^4 = \frac{(0)(1.4\times 10^6)(0.025)^4[1-(0.3)^2]}{(16)(2\times 10^{11})}$$

$$t = 1.09 \text{ mm}$$

We may calculate the natural frequency from Eq. (6.16)

$$f = \frac{10.21}{(0.025)^2}\left[\frac{(1.0)(2\times 10^{11})(0.00109)^2}{(12)[1-(0.3)^2](7800)}\right]^{1/2}$$

$$= 27{,}285 \text{ Hz}$$

FLOW MEASUREMENT

The measurement of fluid flow is important in applications ranging from measurements of blood-flow rates in a human artery to the measurement of the flow of liquid oxygen in a rocket. Many research projects and industrial processes depend on a measurement of fluid flow to furnish important data for analysis. In some cases extreme precision is called for in the flow measurement, while in other instances only crude measurements are necessary. The selection of the proper instrument for a particular application is governed by many variables, including cost.

Positive-displacement flowmeters are generally used for those applications where consistently high accuracy is desired under steady-flow conditions. A typical positive-displacement device is the home water meter shown schematically

Flow rate is expressed in both volume and mass units of varying sizes. Some commonly used terms are

1 gallon per minute (gpm)
$= 231$ cubic inches per minute (in^3/min)
$= 63.09$ cubic centimeters per second (cm^3/s)

1 liter
$= 0.26417$ gallon $= 1000$ cubic centimeters

1 cubic foot per minute (cfm, or ft^3/min)
$= 0.028317$ cubic meter per minute
$= 471.95$ cubic centimeters per second

1 standard cubic foot per minute of air at 20°C, 1 atm
$= 0.07513$ pound-mass per minute
$= 0.54579$ gram per second

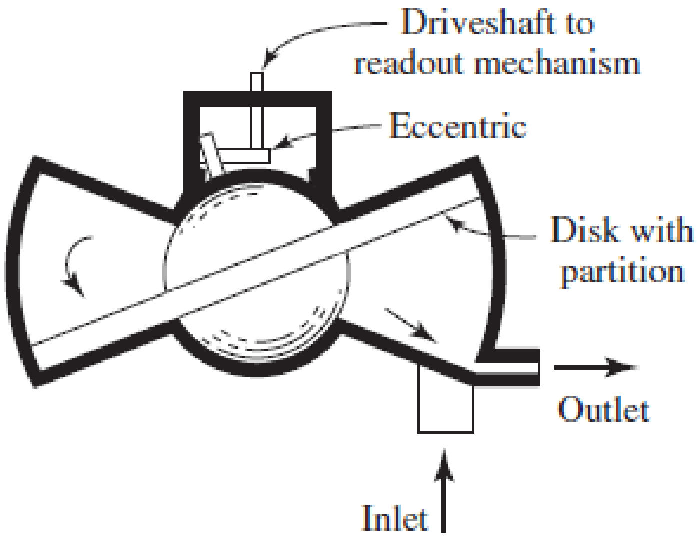


Figure 6 Schematic of a nutating-disk meter.

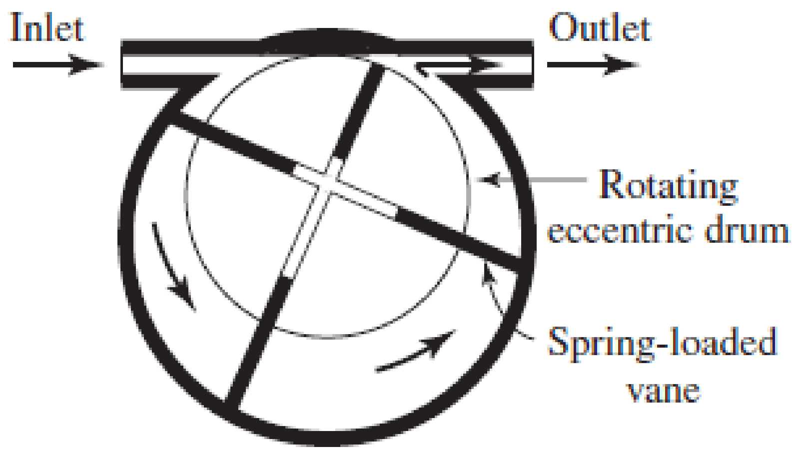


Figure 7 Schematic of rotary-vane flowmeter.

The nutating-disk meter may give reliable flow measurements within 1 percent, over an extended period of time. The uncertainties of rotary-vane meters are of the order of 0.5 percent,

EXAMPLE

UNCERTAINTY IN FLOW CAUSED BY UNCERTAINTIES IN TEMPERATURE AND PRESSURE. A lobed-impeller flowmeter is used for measurement of the flow of nitrogen at 20 psia and 100°F. The meter has been calibrated so that it indicates the volumetric flow with an accuracy of ± one-half of 1 percent from 1000 to 3000 cfm. The uncertainties in the gas pressure and temperature measurements are ±0.025 psi and ±1.0°F, respectively. Calculate the uncertainty in a mass flow measurement at the given pressure and temperature conditions.

Solution

The mass flow is given by

$$\dot{m} = \rho Q$$

where the density of nitrogen is given by

$$\rho = \frac{p}{R_{N_2} T}$$

Using Eq. (3.2), we obtain the following equation for the uncertainty in the mass flow:

$$\frac{w_{\dot{m}}}{\dot{m}} = \left[\left(\frac{w_Q}{Q} \right) + \left(\frac{w_p}{p} \right)^2 + \left(\frac{w_T}{T} \right)^2 \right]^{1/2}$$

Using the given data, we obtain

$$\frac{w_{\dot{m}}}{\dot{m}} = \left[(0.005)^2 + \left(\frac{0.025}{20}\right)^2 + \left(\frac{1}{560}\right)^2\right]^{1/2} = 5.05 \times 10^{-3}$$

or 0.505 percent. Thus, the uncertainties in the pressure and temperature measurements do not appreciably influence the overall uncertainty in the mass flow measurements.

FLOW OBSTRUCTION METHODS

Several types of flowmeters fall under the category of obstruction devices. Such devices are sometimes called *head meters* because a head-loss or pressure-drop measurement is taken as an indication of the flow rate. They are also called *differential pressure meters*. Let us first consider some of the general relations for obstruction meters. We shall then examine the applicability of these relations to specific devices.

Consider the one-dimensional flow system shown in Fig. 7.4. The continuity relation for this situation is

$$\dot{m} = \rho_1 A_1 u_1 = \rho_2 A_2 u_2 \qquad [\;1]$$

where u is the velocity. If the flow is adiabatic and frictionless and the fluid is incompressible, the familiar Bernoulli equation may be written

$$\frac{p_1}{\rho_1} + \frac{u_1^2}{2g_c} = \frac{p_2}{\rho_2} + \frac{u_2^2}{2g_c} \qquad [\;2]$$

where now $\rho_1 = \rho_2$. Solving Eqs. (7.1) and (7.2) simultaneously gives for the pressure drop

$$p_1 - p_2 = \frac{u_2^2 \rho}{2g_c}\left[1 - \left(\frac{A_2}{A_1}\right)^2\right] \qquad [\;3]$$

and the volumetric flow rate may be written

$$Q = A_2 u_2 = \frac{A_2}{\sqrt{1 - (A_2/A_1)^2}} \sqrt{\frac{2g_c}{\rho}(p_1 - p_2)} \qquad [\;4]$$

where $Q = \text{ft}^3/\text{s or m}^3/\text{s}$

$A = \text{ft}^2 \text{ or m}^2$

$\rho = \text{lbm/ft}^3 \text{ or kg/m}^3$

$p = \text{lbf/ft}^2 \text{ or N/m}^2$

$g_c = 32.17 \text{ lbm} \cdot \text{ft/lbf} \cdot \text{s or } 1.0 \text{ kg} \cdot \text{m/N} \cdot \text{s}^2$

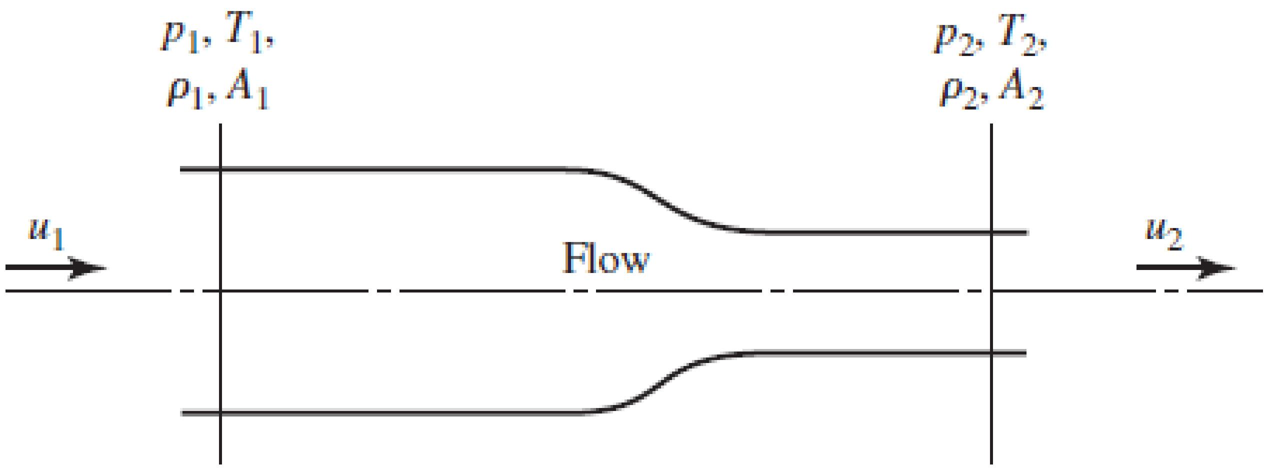


Figure 8 General one-dimensional flow system.

discharge coefficient C by the following relation:

$$\frac{Q_{\text{actual}}}{Q_{\text{ideal}}} = C \qquad [5]$$

The discharge coefficient is not a constant and may depend strongly on the flow Reynolds number and the channel geometry.

When the flow of an ideal gas is considered, the following equation of state applies:

$$p = \rho R T \qquad [6]$$

where T is the absolute temperature and R is the gas constant for the particular gas, which can be expressed in terms of the universal gas constant $\Re$ and the molecular weight by

$$R = \frac{\Re}{M}$$

The value of $\Re$ is 8314 kJ/kg · mol · K or 1545 ft · lbf/lbm · mol · °R. For reversible adiabatic flow the steady-flow energy equation for an ideal gas is

$$c_p T_1 + \frac{u_1^2}{2g_c} = c_p T_2 + \frac{u_2^2}{2g_c} \qquad [7]$$

where c_p is the specific heat at constant pressure and is assumed constant for an ideal gas. When Eqs. (7.1), (7.6), and (7.7) are combined, there results

$$\dot{m}^2 = 2g_c A_2^2 \frac{\gamma}{\gamma - 1} \frac{p_1^2}{RT_1} \left[\left(\frac{p_2}{p_1} \right)^{2/\gamma} - \left(\frac{p_2}{p_1} \right)^{(\gamma+1)/\gamma} \right] \quad [8]$$

where the velocity of approach, that is, the velocity at section 1 of Fig. 7.4, is assumed to be very small. This relationship may be simplified to

$$\dot{m} = \sqrt{\frac{2g_c}{RT_1}} A_2 \left[p_2 \Delta p - \left(\frac{1.5}{\gamma} - 1 \right) (\Delta p)^2 + \cdots \right]^{1/2} \quad [9]$$

with $\Delta p = p_1 - p_2$ and $\gamma = c_p/c_v$ is the ratio of specific heats for the gas. Equation (7.9) is valid for $\Delta p < p_1/4$. When $\Delta p < p_1/10$, a further simplification may be made to give

$$\dot{m} = A_2 \sqrt{\frac{2g_c p_2}{RT_1} (p_1 - p_2)} \quad [10]$$

where $\dot{m}$ = mass flow rate, lbm/s or kg/s

A = area, ft^2 or m^2

g_c = 32.17 lbm · ft/lbf · s^2 or 1.0 kg · m/N · s^2

p = pressure, lbf/ft^2 or N/m^2(Pa)

R = gas constant, lbf · ft/lbm · °R or N · m/kg · K

T = absolute temperature, °R or K

Note that Eq. (10) reduces to Eq. (4) when the relation for density from Eq. (6) is substituted. Thus, for small values of Δp compared with p_1 the flow of a compressible fluid may be approximated by the flow of an incompressible fluid.

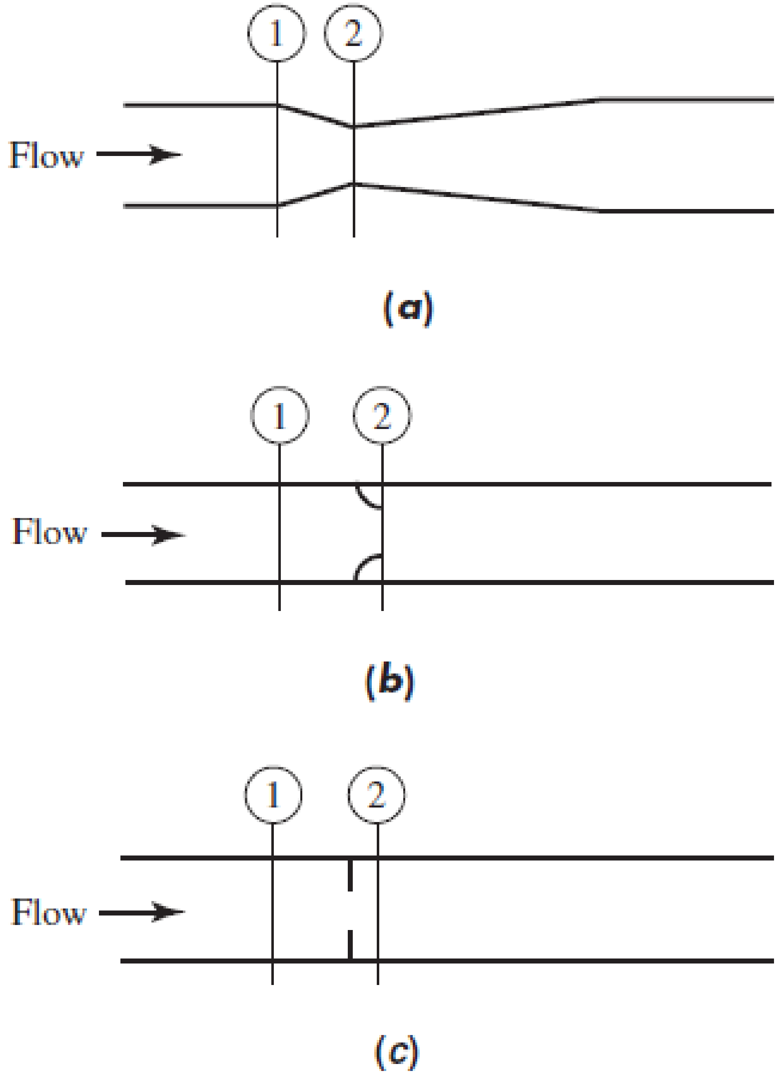


Figure 9 Schematic of three typical obstruction meters. (*a*) Venturi; (*b*) flow nozzle; (*c*) orifice.

Three typical obstruction meters are shown in Fig. **9** . The venturi offers the advantages of high accuracy and small pressure drop, while the orifice is considerably lower in cost. Both the flow nozzle and the orifice have a relatively high permanent pressure drop. Flow-rate calculations for all three devices are made on the basis of Eq. (4) with appropriate empirical constants defined as follows:

$$M = \text{velocity of approach factor} = \frac{1}{\sqrt{1-(A_2/A_1)^2}} \qquad [\ 11]$$

$$K = \text{flow coefficient} = CM \qquad [\ 12]$$

$$\beta = \text{diameter ratio} = \frac{d}{D} = \sqrt{\frac{A_2}{A_1}} \qquad [\ 13]$$

When flow measurements of a compressible fluid are made, an additional parameter, the *expansion factor* Y, is used. For venturis and nozzles this factor is given by

$$Y_a = \left[\left(\frac{p_2}{p_1}\right)^{2/\gamma} \frac{\gamma}{\gamma-1} \frac{1-(p_2/p_1)^{(\gamma-1)/\gamma}}{1-(p_2/p_1)} \frac{1-(A_2/A_1)^2}{1-(A_2/A_1)^2(p_2/p_1)^{2/\gamma}}\right]^{1/2} \qquad [\ 14]$$

while for orifices an empirical expression for Y is given as

$$Y_1 = 1 - \left[0.41 + 0.35\left(\frac{A_2}{A_1}\right)^2\right]\frac{p_1 - p_2}{\gamma p_1} \qquad [15]$$

when either flange taps or vena contracta taps are used. For orifices with pipe taps the following relation applies:

$$Y_2 = 1 - [0.333 + 1.145(\beta^2 + 0.7\beta^5 + 12\beta^{13})]\frac{p_1 - p_2}{\gamma p_1} \qquad [16]$$

The empirical expansion factors given by Eqs. (7.15) and (7.16) are accurate within ± 0.5 percent for $0.8 < p_2/p_1 < 1.0$. Plots of the expansion factors Y_a and Y_1 are given in Figs. 7.14 and 7.15, respectively.

We thus have the following semiempirical equations, which are conventionally applied to venturis, nozzles, or orifices:

VENTURIS, INCOMPRESSIBLE FLOW:

$$Q_{\text{actual}} = CMA_2\sqrt{\frac{2g_c}{\rho}}\sqrt{p_1 - p_2} \qquad [17]$$

NOZZLES AND ORIFICES, INCOMPRESSIBLE FLOW:

$$Q_{\text{actual}} = KA_2\sqrt{\frac{2g_c}{\rho}}\sqrt{p_1 - p_2} \qquad [18]$$

The use of the flow coefficient instead of the product CM is merely a matter of convention. When compressible fluids are used, the above equations are modified by the factor Y and the fluid density is evaluated at inlet conditions. We then have

VENTURIS, COMPRESSIBLE FLOW:

$$\dot{m}_{\text{actual}} = YCMA_2\sqrt{2g_c\rho_1(p_1 - p_2)} \qquad [\ 19]$$

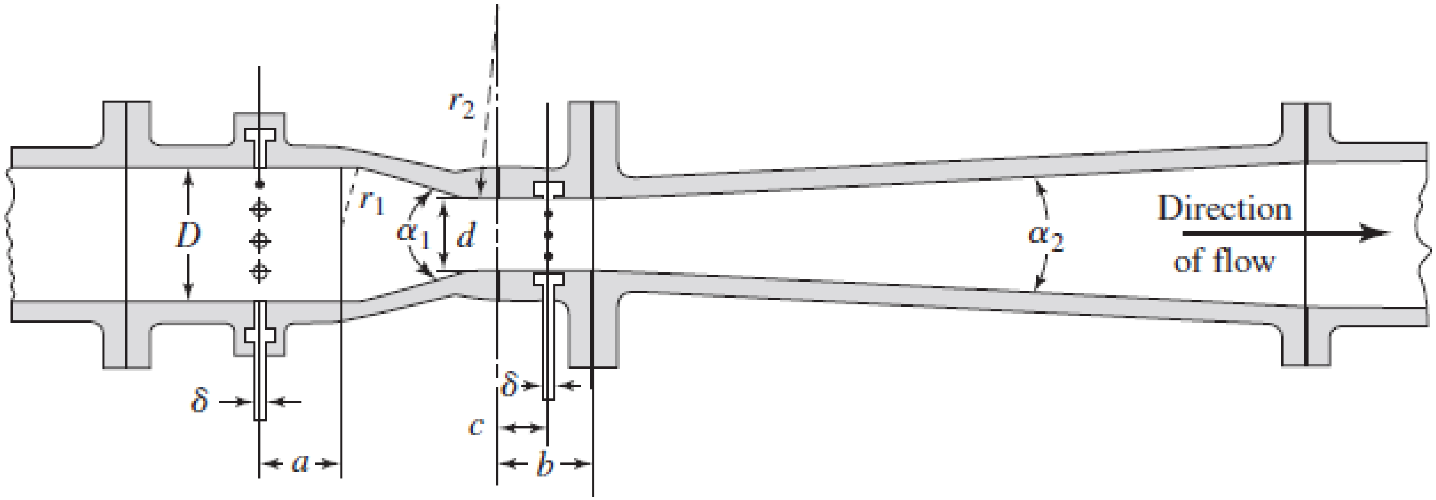
NOZZLES AND ORIFICES, COMPRESSIBLE FLOW:

$$\dot{m}_{\text{actual}} = YKA_2\sqrt{2g_c\rho_1(p_1 - p_2)} \qquad [\ 20]$$

In Eqs. (17) to (20) the appropriate units are

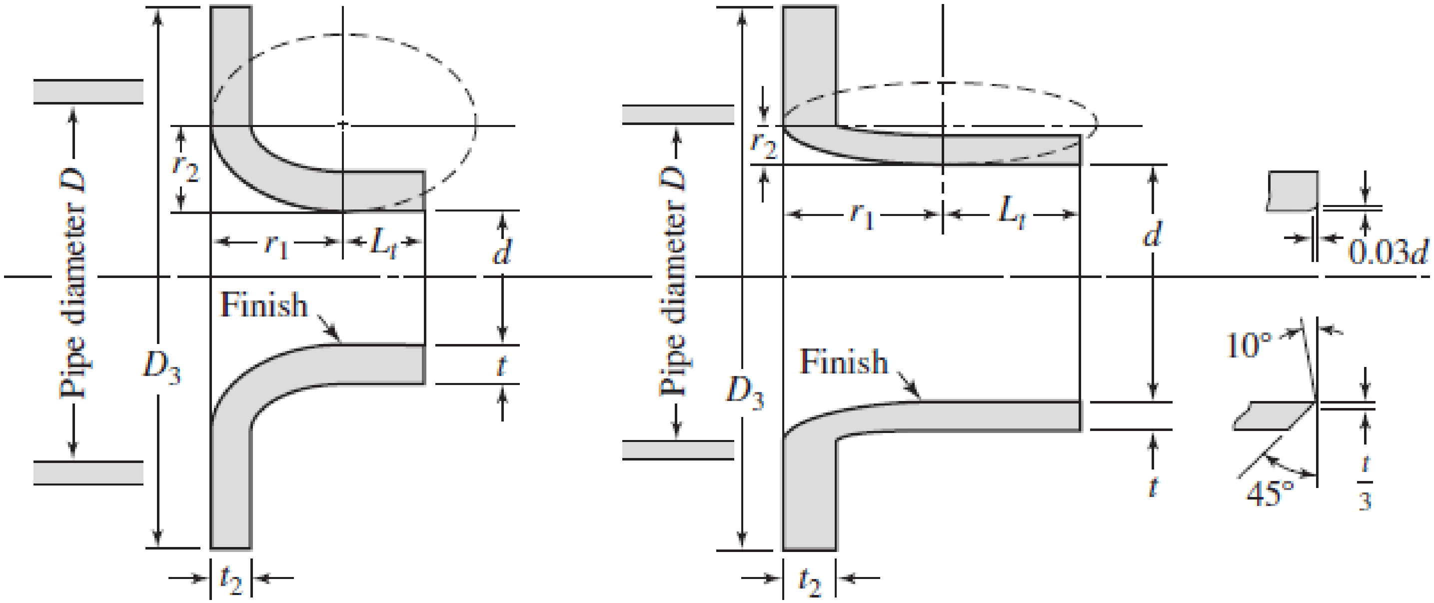
Q = volume flow rate, ft^3/s or m^3/s
A = area, ft^2 or m^2
g_c = 32.17 $lbm \cdot ft/lbf \cdot s^2$ or 1.0 $kg \cdot m/N \cdot s^2$
ρ = density, lbm/ft^3 or kg/m^3
p = pressure, lbf/ft^2 or N/m^2

Detailed tabulations of the various coefficients have been made in Ref. [1], some of which are presented in Figs. **10** through **16** . Examples 2 and 3 illustrate the use of these charts for practical calculations.



D = Pipe diameter inlet and outlet
d = Throat diameter as required
$a = 0.25D$ to $0.75D$ for $4'' \leqq D \leqq 6''$, $0.25D$ to $0.50D$ for $6'' \leqq D \leqq 32''$
$b = d$
$c = d/2$
δ = 3/16 in to 1/2 in according to D. Annular pressure chamber with at least four piezometer vents
$r_2 = 3.5d$ to $3.75d$
$r_1 = 0$ to $1.375D$
$\alpha_1 = 21\infty \pm 2\infty$
$\alpha_2 = 5\infty$ to 15∞

Figure 10 Recommended proportions of venturi tubes, according to Ref. [1].



Low β series: $\beta < 0.5$

$r_1 = d$

$r_2 = \frac{2}{3}d$

$L_t = 0.6d$

$1/8'' \lesseqgtr t \lesseqgtr 1/2''$

$1/8'' \lesseqgtr t_2 \lesseqgtr 0.15D$

High β series: $\beta > 0.25$

$r_1 = \frac{1}{2}D$

$\frac{1}{2}$

$L_t \lesseqgtr 0.6d$ or $L_t \lesseqgtr \frac{1}{3}D$

$2t \lesseqgtr D - (d + 1/8'')$

$1/8'' \lesseqgtr t_2 \lesseqgtr 0.15D$

Optional designs of nozzle outlet

$r_2 = \quad (D - d)$

Figure 11 Recommended proportions of the ASME long-radius flow nozzle, according to Ref. [1].

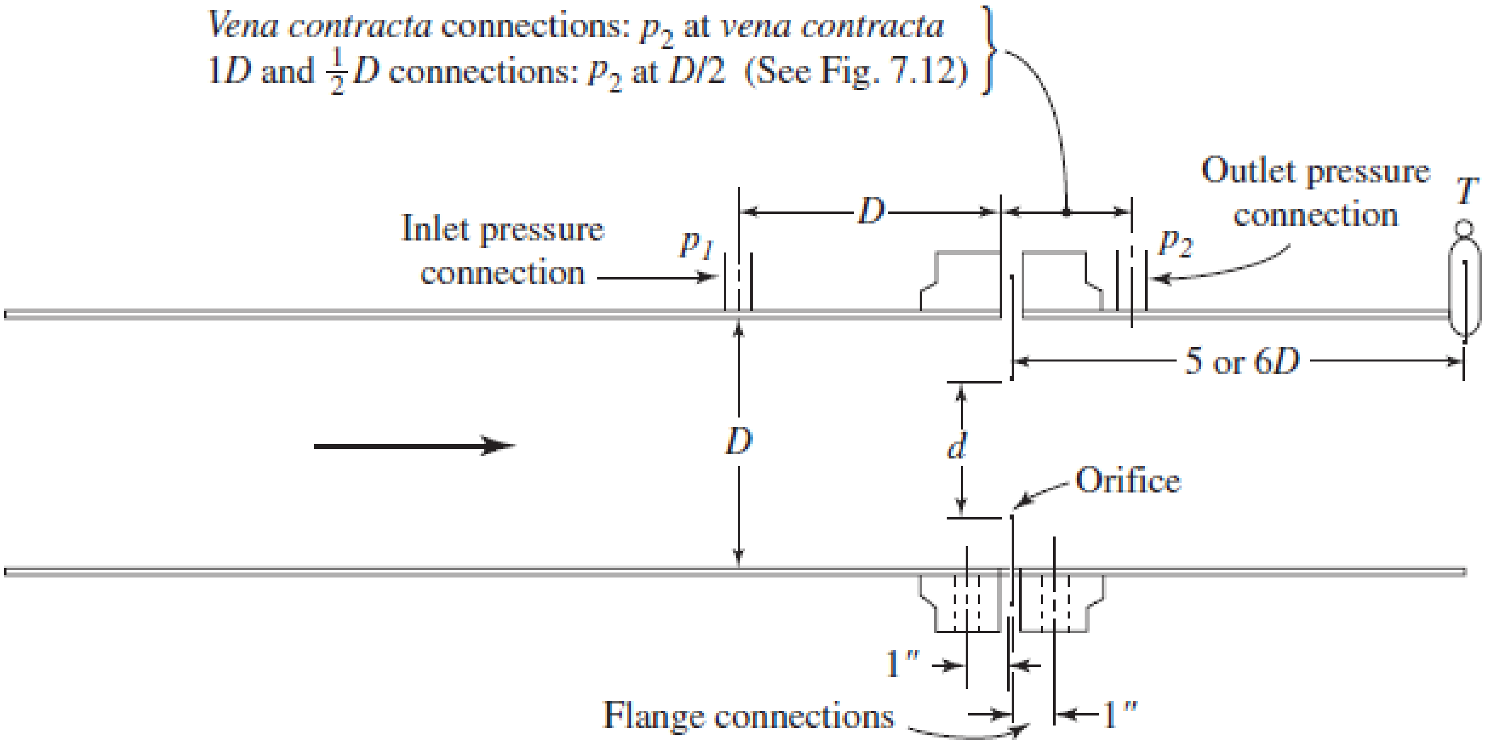


Figure 12 Recommended location of pressure taps for use with concentric, thin-plate, square-edged orifices, according to Ref. [1].

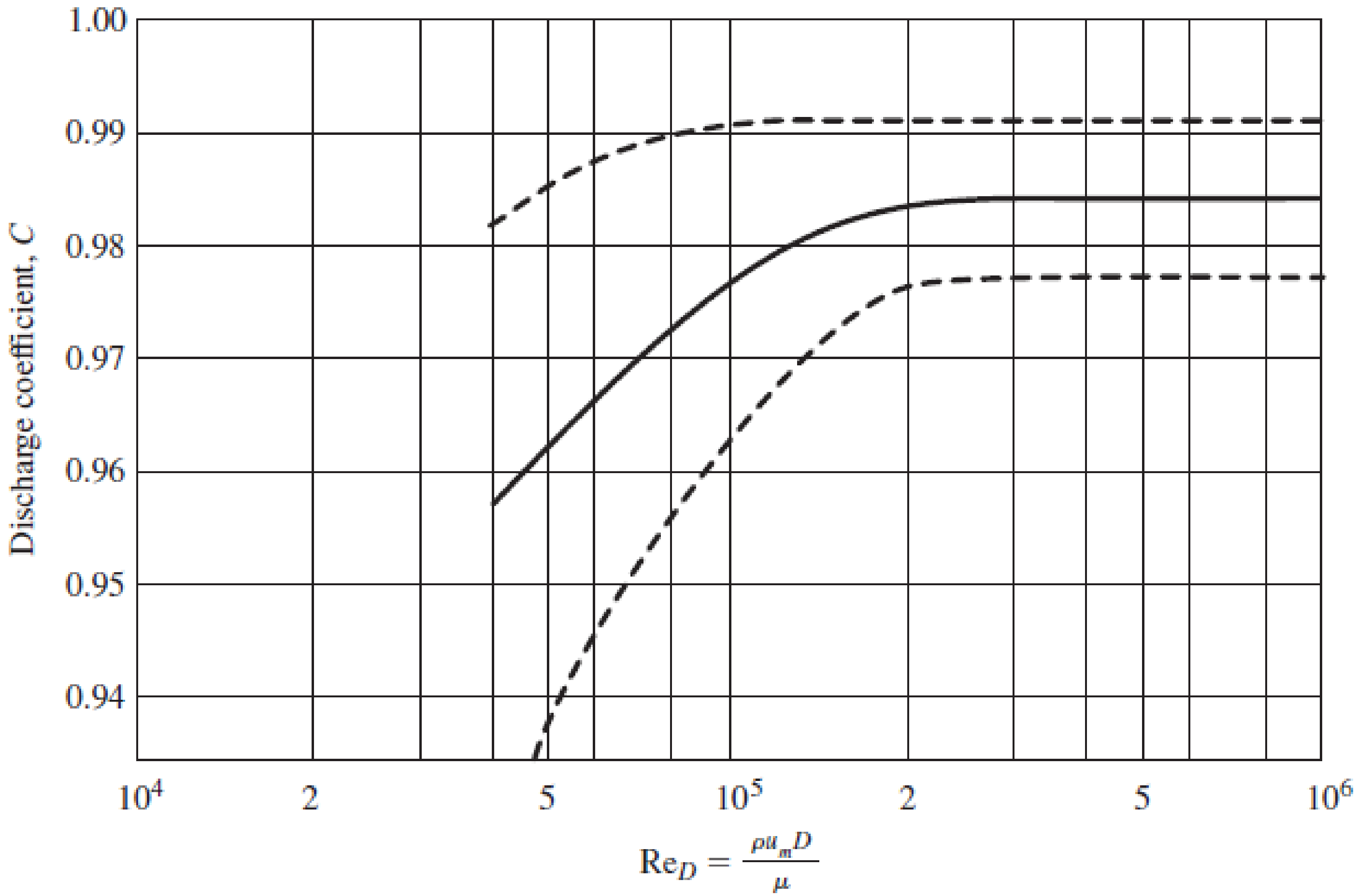


Figure 13 Discharge coefficients for the venturi tube shown in Fig. 7.6, according to Ref. [1]. Values are applicable for $0.25 < \beta < 0.75$ and $D > 2$ in.

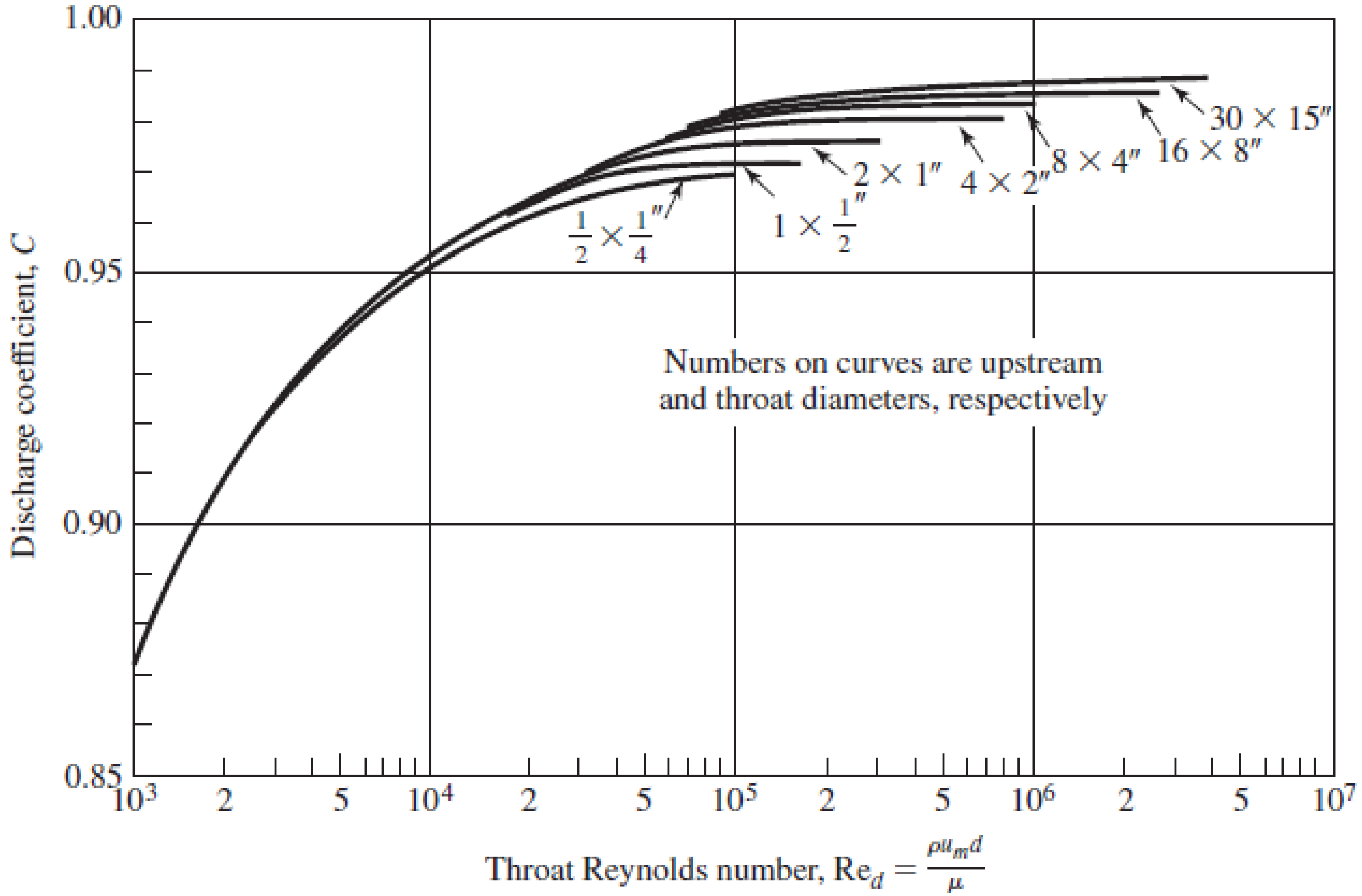


Figure 14 Approximate venturi coefficients for various throat diameters, according to Ref. [15].

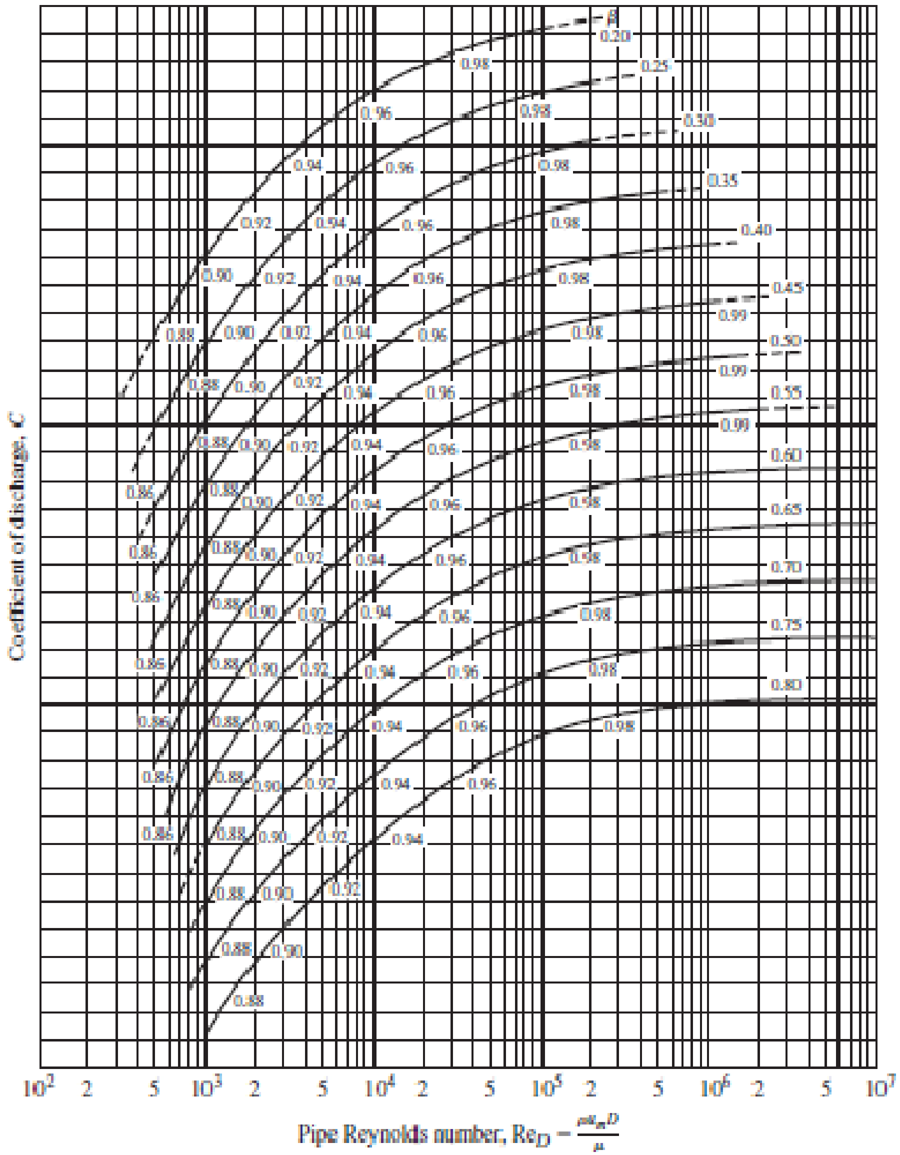


Figure 15 Discharge coefficients for ASME long-radius nozzles shown in Fig. 7.7, according to Ref. [1].

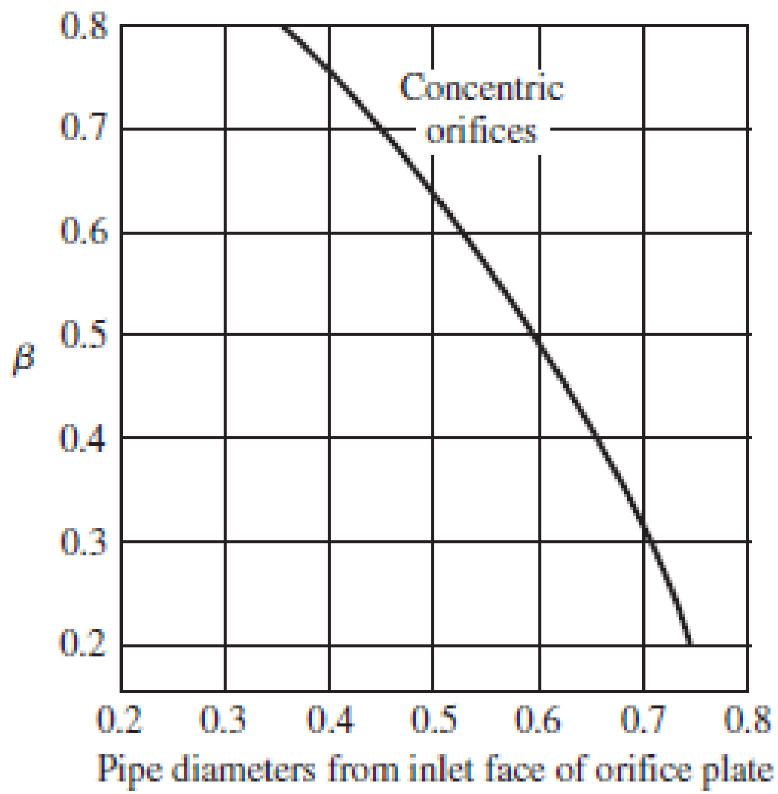


Figure 16 Location of outlet pressure connections for orifices with vena contracta taps, according to Ref. [1].

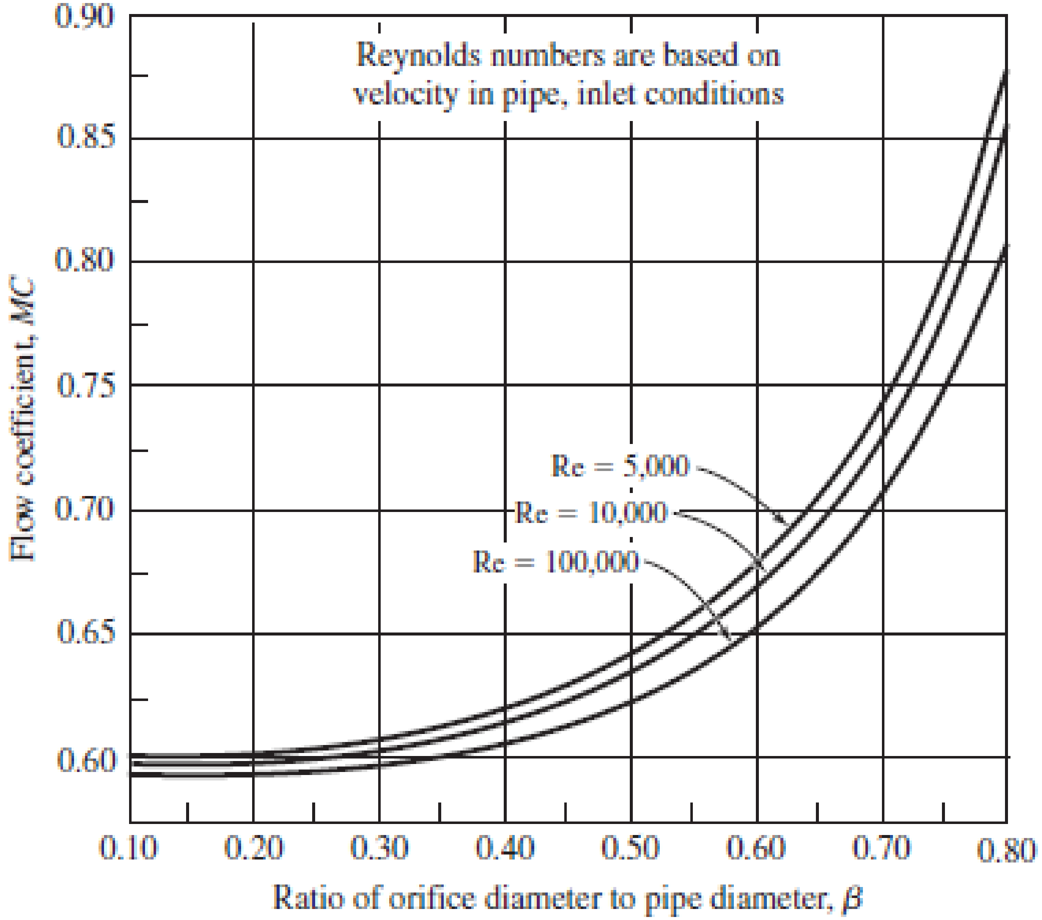


Figure 17 Flow coefficients for concentric orifices in pipes. Pressure taps one diameter upstream and one-half diameter downstream. Applicable for $1.25 < D < 3.00$ in. (*From Ref.* [15].)

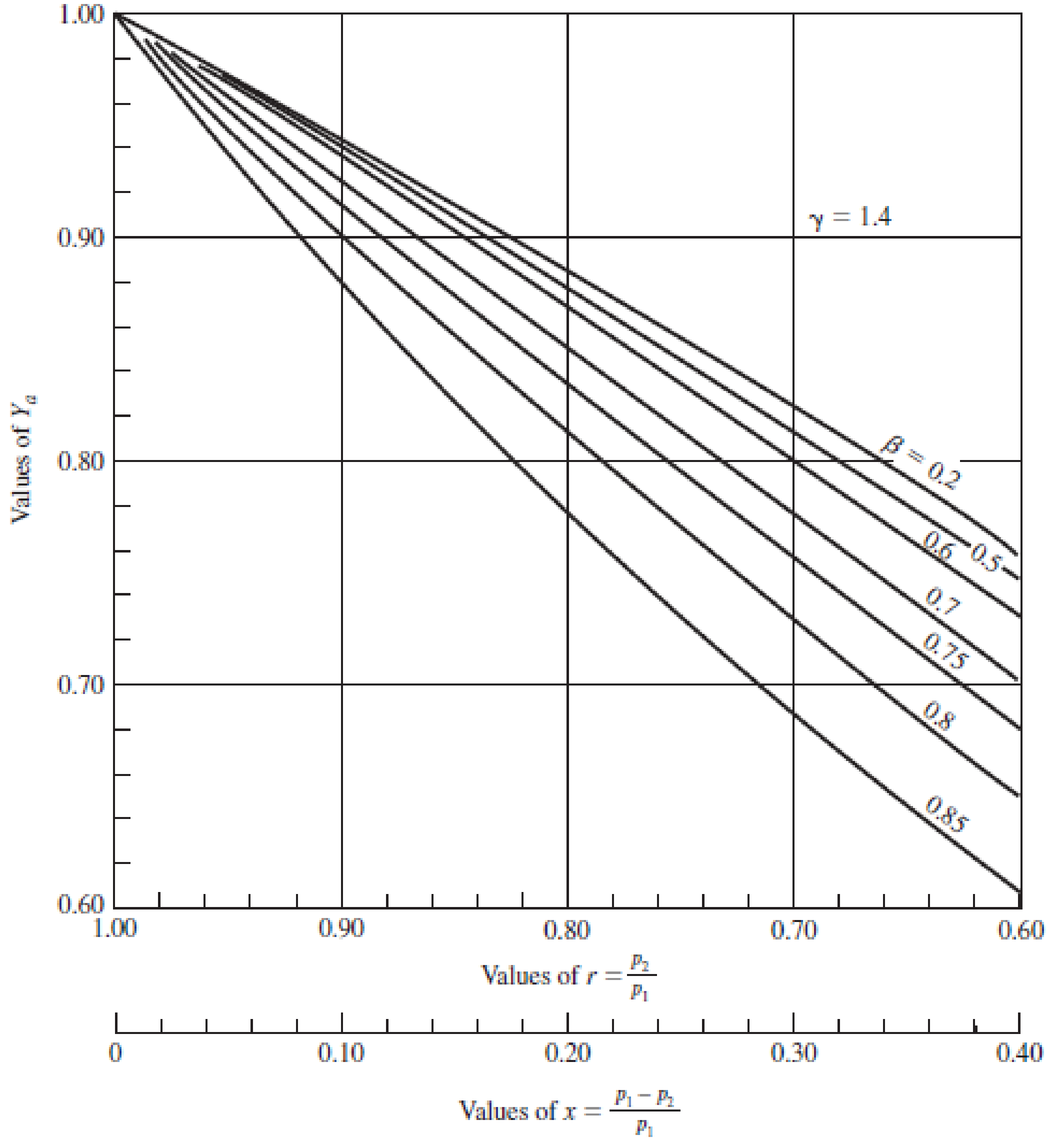


Figure 18 Adiabatic expansion factors for use with venturis and flow nozzles as calculated from Eq. (7.14). (*From Ref. [2].*)

The various flow coefficients are plotted as a function of Reynolds number, defined by

$$\mathrm{Re} = \frac{\rho u_m d}{\mu} \qquad [\;21]$$

where ρ = fluid density

μ = dynamic viscosity

u_m = mean flow velocity

d = diameter *at the particular section for which the Reynolds number is specified*

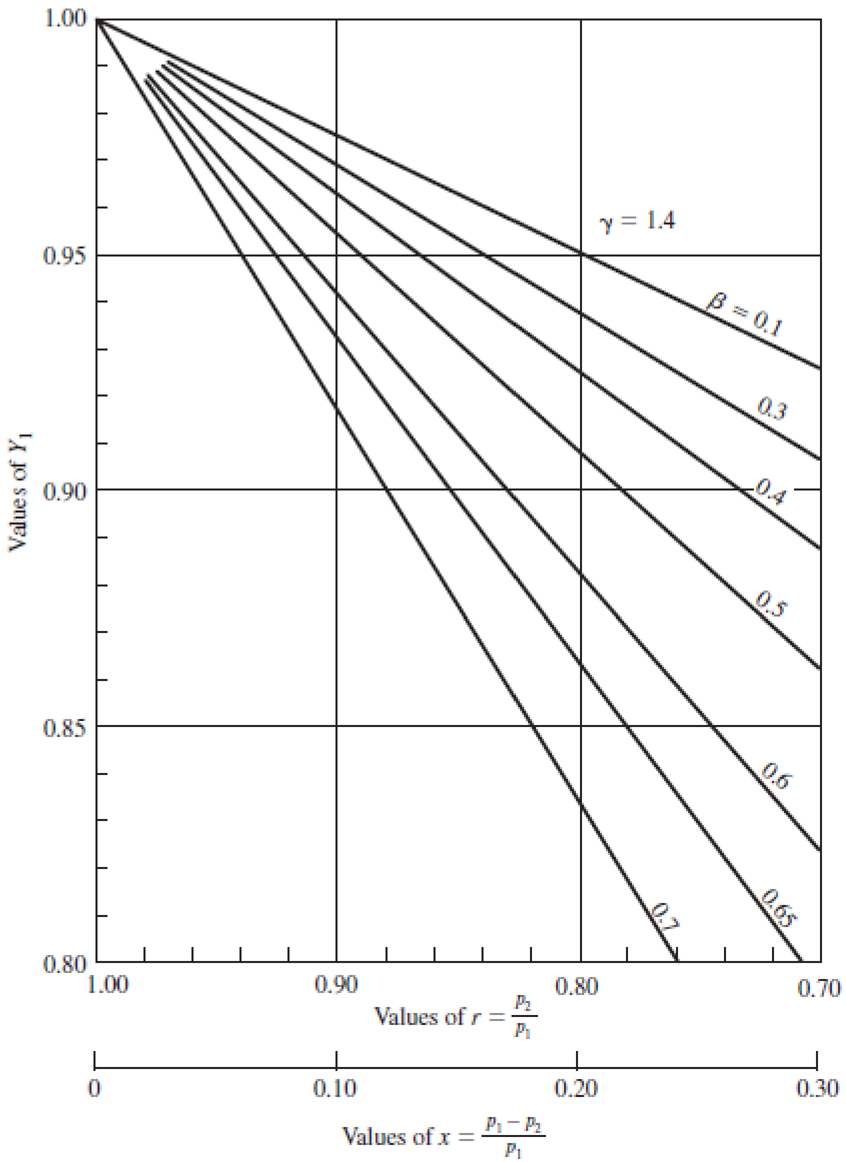


Figure 19 Expansion factors for square-edged orifices with pipe taps as calculated from Eq. (7.16). (*From Ref. [2].*)

$$\dot{m} = \rho u_m A_c \qquad [22]$$

where A_c is the cross-sectional area for the flow where u_m is measured. For a circular cross section $A_c = \pi d^2/4$. Further information on orifice and venturi meters is contained in Refs. [33] to [37].

EXAMPLE

DESIGN OF VENTURI METER. A venturi tube is to be used to measure a maximum flow rate of water of 50 gpm (gallons per minute) at 70°F. The throat Reynolds number is to be at least 10^5 at these flow conditions. A differential pressure gage is selected which has an accuracy of 0.25 percent of full scale, and the upper scale limit is to be selected to correspond to the maximum flow rate. Determine the size of the venturi and the maximum range of the differential pressure gage and estimate the uncertainty in the mass flow measurement at nominal flow rates of 50 and 25 gpm. Use either Fig. 7.9 or Fig. 7.10 to determine the discharge coefficient.

Solution

The properties of water are

$$\rho = 62.4\ \text{lbm/ft}^3 = 8.33\ \text{lbm/gal} \qquad \mu = 2.36\ \text{lbm/h}\cdot\text{ft}$$

From the given maximum flow rate and throat Reynolds number we may calculate the maximum allowable throat diameter:

$$\text{Re}_d = \frac{\rho u_m d}{\mu} = \frac{\dot{m} d}{(\pi d^2/4)\mu} = \frac{4\dot{m}}{\pi d \mu} = 10^5$$

The maximum flow rate is

$$\dot{m} = (50)(8.33)(60) = 2.5 \times 10^4 \text{ lbm/h } (3.027 \text{ kg/s})$$

so that $$d_{\max} = \frac{(4)(2.5 \times 10^4)}{\pi(10^5)(2.36)} = 0.135 \text{ ft} = 1.62 \text{ in } (4.11 \text{ cm})$$

We shall select a venturi with a 1.0-in throat diameter since we have a discharge coefficient curve for this size in Fig. 7.10. The upstream pipe diameter is taken as 2.0 in. From Fig. 7.10 we estimate the discharge coefficient for this size venturi as 0.976 for $8 \times 10^4 < \mathrm{Re}_d < 3 \times 10^5$. The uncertainty in this coefficient will be taken as ± 0.002 since Fig. 7.10 is a general set of curves. With this selection of venturi size, the maximum throat Reynolds number becomes

$$(\mathrm{Re}_d)_{\max} = (10^5)\left(\frac{1.62}{1.0}\right) = 1.62 \times 10^5$$

The minimum Reynolds number is thus one-half this value, or 8.1×10^4. The maximum pressure differential may be calculated with Eq. (7.17).

$$Q_{\text{actual}} = CMA_2\sqrt{\frac{2g_c}{\rho}}\sqrt{\Delta p} \qquad [\quad 17]$$

or

$$\frac{(50)(231)}{(60)(1728)} = \frac{(0.976)\pi(1.0)^2}{(4)(144)\sqrt{1-\left(\frac{1}{2}\right)^2}}\sqrt{\frac{(2)(32.2)}{62.4}}\sqrt{\Delta p}$$

This yields

$$\Delta p = 948 \text{ psf} = 6.58 \text{ psi } (45.4 \text{ kPa})$$

Let us assume that a differential pressure gage with a maximum range of 1000 psf is at our disposal. In accordance with the problem statement the uncertainty in the pressure reading would be

$$w_{\Delta p} = \pm 2.5 \text{ psf } (119.7 \text{ Pa})$$

When the flow is reduced to 25 gpm, the pressure differential will be *one-fourth* of that at 50 gpm. To estimate the uncertainty in the flow measurement, we shall assume that the dimensions of the venturi are known exactly, as well as the density of the water. For the calculation we utilize Eq. (3.2). The quantities of interest are

$$\frac{\partial Q}{\partial C} = MA_2\sqrt{\frac{2g_c}{\rho}}\sqrt{\Delta p}$$

$$\frac{\partial Q}{\partial \Delta p} = \frac{CMA_2}{2\sqrt{\Delta p}}\sqrt{\frac{2g_c}{\rho}}$$

$$w_c = \pm 0.002$$

Thus,

$$\frac{w_Q}{Q} = \left[\left(\frac{w_c}{C}\right)^2 + \frac{1}{4}\left(\frac{w_{\Delta p}}{\Delta p}\right)^2\right]^{1/2}$$

For $Q = 50$ gpm

$$\frac{w_Q}{Q} = \left[\left(\frac{0.002}{0.976}\right)^2 + \frac{1}{4}\left(\frac{2.5}{948}\right)^2\right]^{1/2}$$

$$= 0.002435 \qquad \text{or } 0.2435\%$$

For $Q = 25$ gpm

$$\frac{w_Q}{Q} = \left[\left(\frac{0.002}{0.976}\right)^2 + \frac{1}{4}\left(\frac{2.5}{984/4}\right)^2\right]^{1/2}$$

$$= 0.00566 \qquad \text{or } 0.566\%$$