MENG353 - FLUID MECHANICS

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CHAPTER 8 VISCOUS FLOW IN PIPES
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Learning Objectives

After completing this chapter, you should be able to:

- identify and understand various characteristics of the flow in pipes.
- discuss the main properties of laminar and turbulent pipe flow and appreciate their differences.
- calculate losses in straight portions of pipes as well as those in various pipe system components.
- apply appropriate equations and principles to analyze a variety of pipe flow situations.
- predict the flowrate in a pipe by use of common flowmeters.

Some of the basic components of a typical *pipe system* are shown in Fig. 8.1. They include the pipes themselves (perhaps of more than one diameter), the various fittings used to connect the individual pipes to form the desired system, the flowrate control devices (valves), and the pumps or turbines that add energy to or remove energy from the fluid. Even the most simple pipe systems are actually quite complex when they are viewed in terms of rigorous analytical considerations.

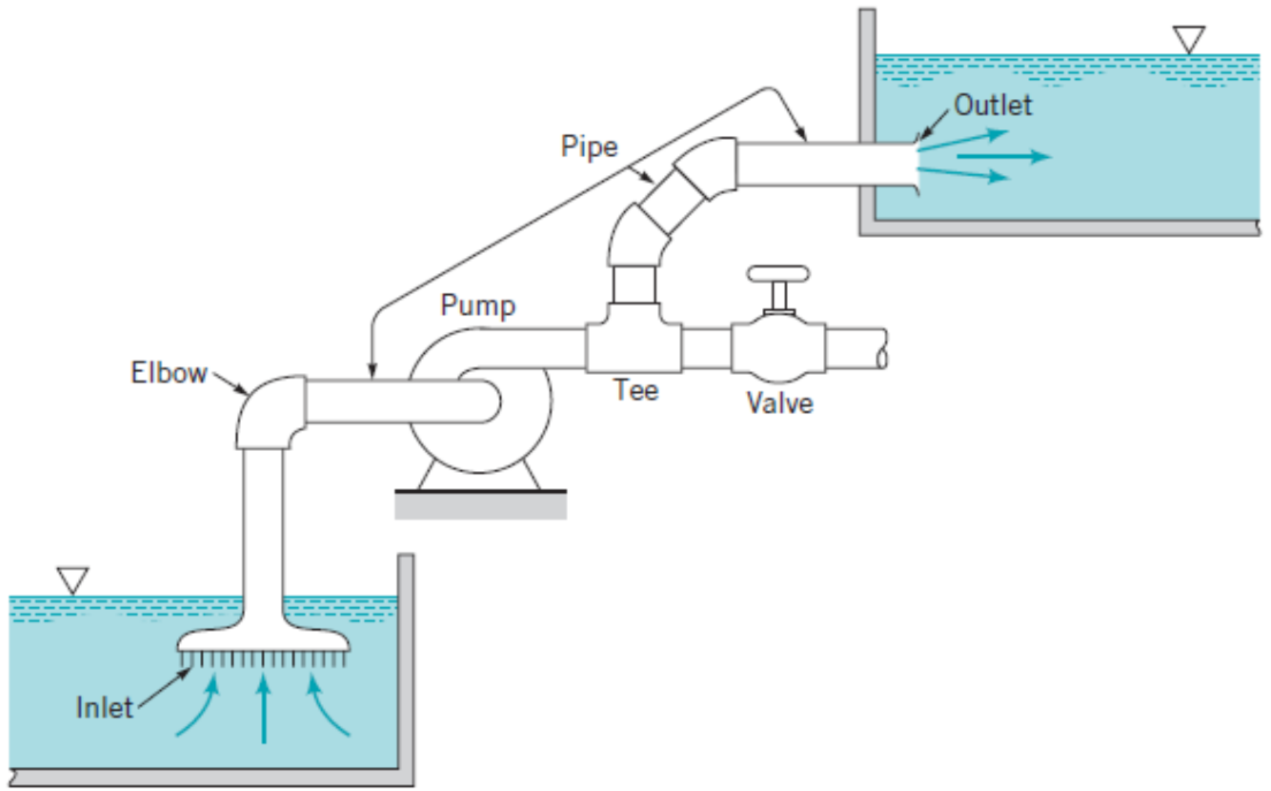


FIGURE 8.1 Typical pipe system components.

8.1 General Characteristics of Pipe Flow

Before we apply the various governing equations to pipe flow examples, we will discuss some of the basic concepts of pipe flow. With these ground rules established we can then proceed to formulate and solve various important flow problems.

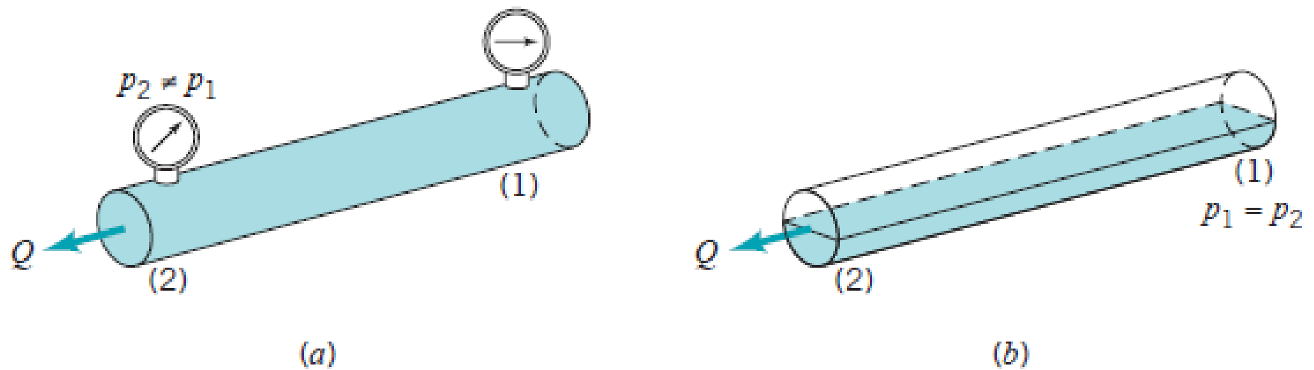


FIGURE 8.2 (*a*) Pipe flow. (*b*) Open-channel flow.

8.1.1 Laminar or Turbulent Flow

The flow of a fluid in a pipe may be laminar flow or it may be turbulent flow.

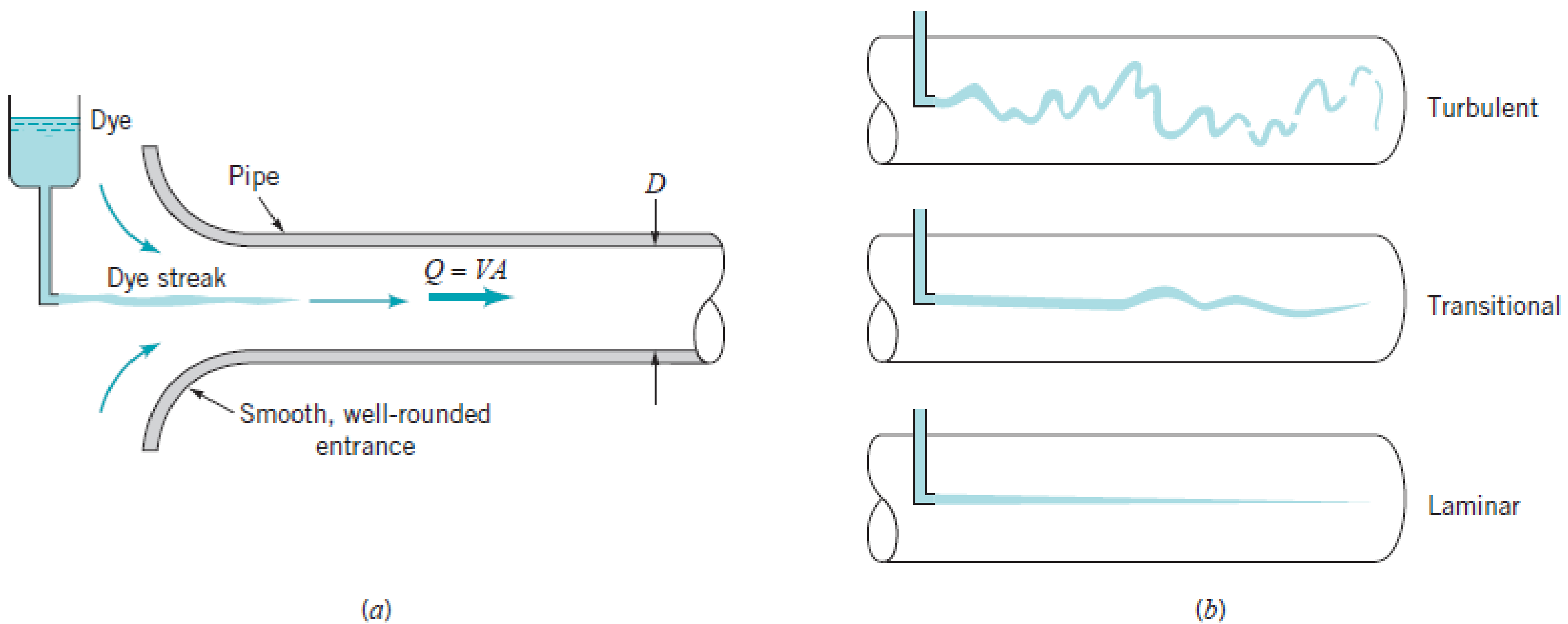
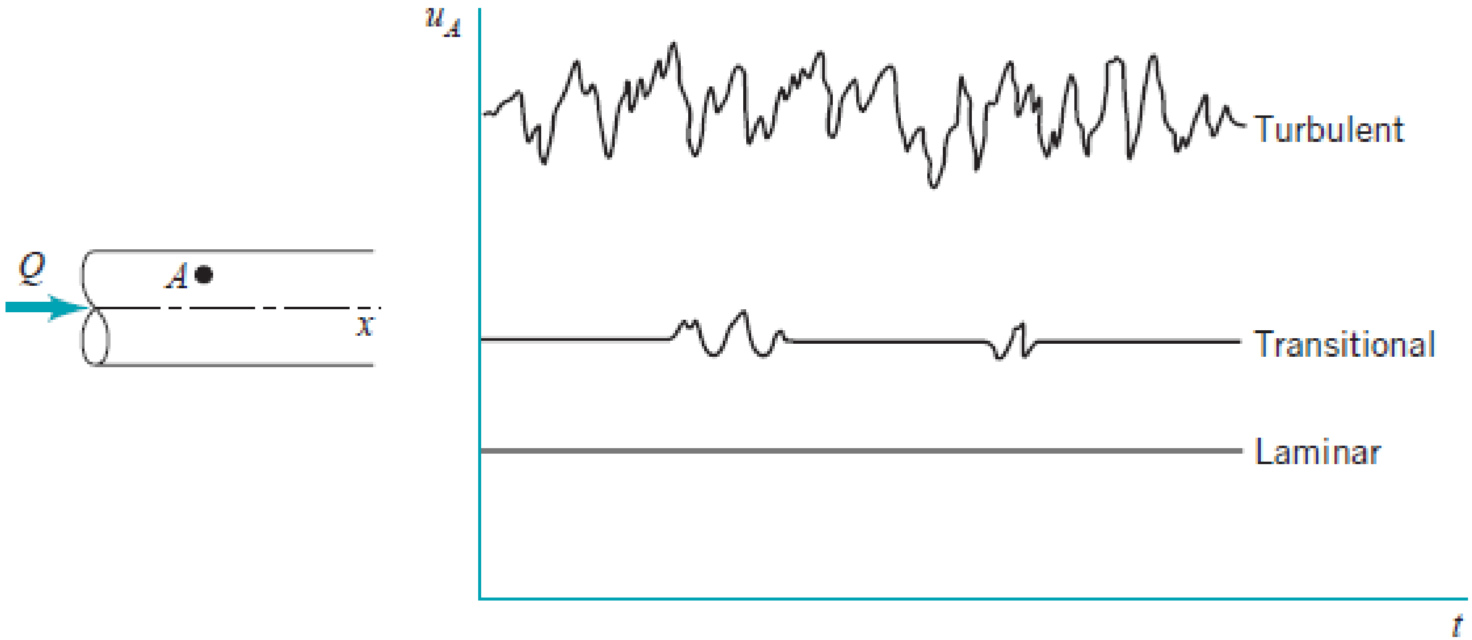


FIGURE 8.3 (*a*) Experiment to illustrate type of flow. (*b*) Typical dye streaks.



■ FIGURE 8.4 Time dependence of fluid velocity at a point.

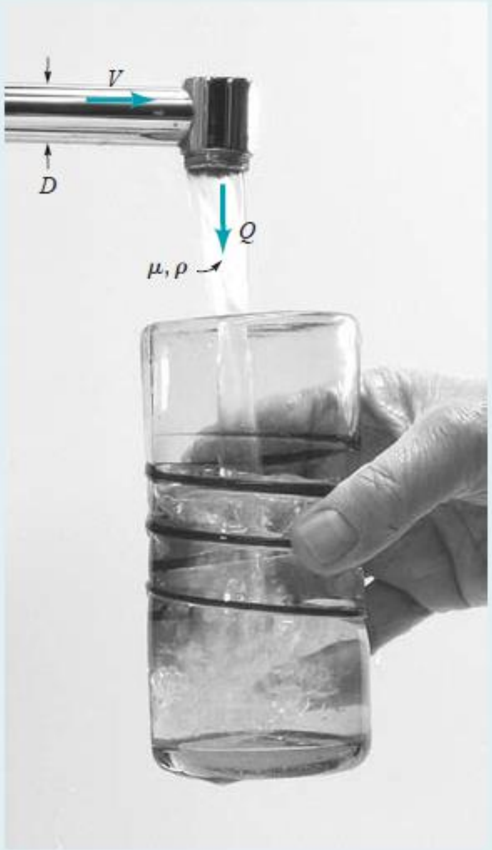
EXAMPLE 8.1 Laminar or Turbulent Flow

GIVEN Water at a temperature of 50 °F flows through a pipe of diameter $D = 0.73$ in. and into a glass as shown in Fig. E8.1*a*.

FIND Determine

(a) the minimum time taken to fill a 12-oz glass (volume = 0.0125 ft^3) with water if the flow in the pipe is to be laminar. Repeat the calculations if the water temperature is 140 °F.

(b) the maximum time taken to fill the glass if the flow is to be turbulent. Repeat the calculations if the water temperature is 140 °F.



■ FIGURE E8.1*a*

SOLUTION

(a) If the flow in the pipe is to remain laminar, the minimum time to fill the glass will occur if the Reynolds number is the maximum allowed for laminar flow, typically $\mathrm{Re} = \rho V D/\mu = 2100$. Thus, $V = 2100\,\mu/\rho D$, where from Table B.1, $\rho = 1.94$ slugs/ft^3 and $\mu = 2.73 \times 10^{-5}$ lb · s/ft^2 at 50 °F, while $\rho = 1.91$ slugs/ft^3 and $\mu = 0.974 \times 10^{-5}$ lb · s/ft^2 at 140 °F. Thus, the maximum average velocity for laminar flow in the pipe is

$$V = \frac{2100\mu}{\rho D} = \frac{2100(2.73 \times 10^{-5}\ \mathrm{lb \cdot s/ft^2})}{(1.94\ \mathrm{slugs/ft^3})(0.73/12\ \mathrm{ft})}$$
$$= 0.486\ \mathrm{lb \cdot s/slug} = 0.486\ \mathrm{ft/s}$$

Similarly, $V = 0.176$ ft/s at 140 °F. With $\mathcal{V}$ = volume of glass and $\mathcal{V} = Qt$ we obtain

$$t = \frac{\forall}{Q} = \frac{\forall}{(\pi/4)D^2V} = \frac{4(0.0125\ \text{ft}^3)}{(\pi[0.73/12]^2\text{ft}^2)(0.486\ \text{ft/s})}$$

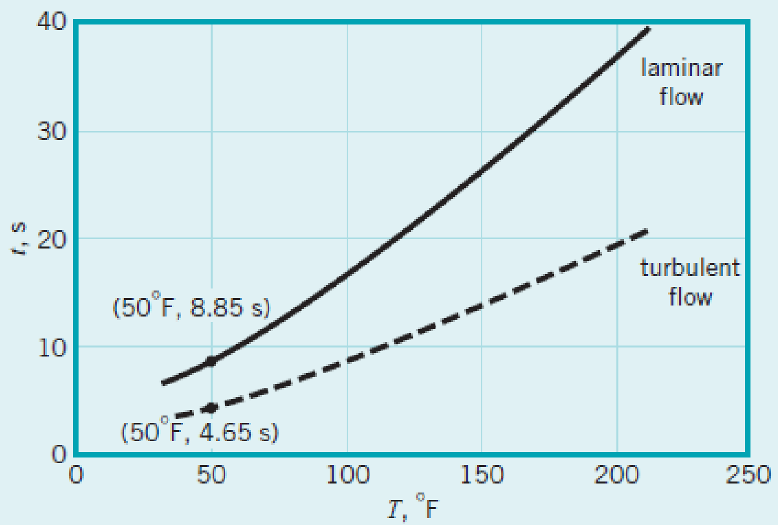
$$= 8.85\ \text{s at } T = 50\ °\text{F} \qquad \textbf{(Ans)}$$

Similarly, $t = 24.4$ s at 140 °F. To maintain laminar flow, the less viscous hot water requires a lower flowrate than the cold water.

(b) If the flow in the pipe is to be turbulent, the maximum time to fill the glass will occur if the Reynolds number is the minimum allowed for turbulent flow, Re = 4000. Thus, $V = 4000\mu/\rho D = 0.925$ ft/s and

$$t = 4.65\ \text{s at } 50\ °\text{F} \qquad \textbf{(Ans)}$$

Similarly, $V = 0.335$ ft/s and $t = 12.8$ s at 140 °F.



■ FIGURE E8.1b

8.1.2 Entrance Region and Fully Developed Flow

Any fluid flowing in a pipe had to enter the pipe at some location. The region of flow near where the fluid enters the pipe is termed the *entrance region* and is illustrated in Fig. 8.5. It may be the first few feet of a pipe connected to a tank or the initial portion of a long run of a hot air duct coming from a furnace.

Viscous effects are of considerable importance within the boundary layer. For fluid outside the boundary layer [within the *inviscid core* surrounding the centerline from (1) to (2)], viscous effects are negligible.

The shape of the velocity profile in the pipe depends on whether the flow is laminar or turbulent, as does the length of the entrance region, ℓ_e. As with many other properties of pipe flow, the dimensionless **entrance length,** ℓ_e/D, correlates quite well with the Reynolds number. Typical entrance lengths are given by

$$\frac{\ell_e}{D} = 0.06 \text{ Re for laminar flow} \qquad \textbf{(8.1)}$$

and

$$\frac{\ell_e}{D} = 4.4 (\text{Re})^{1/6} \text{ for turbulent flow} \qquad \textbf{(8.2)}$$

For very low Reynolds number flows the entrance length can be quite short ($\ell_e = 0.6D$ if $\text{Re} = 10$), whereas for large Reynolds number flows it may take a length equal to many pipe diameters before the end of the entrance region is reached ($\ell_e = 120D$ for $\text{Re} = 2000$). For many practical engineering problems, $10^4 < \text{Re} < 10^5$ so that as shown by the figure in the margin, $20D < \ell_e < 30D$.

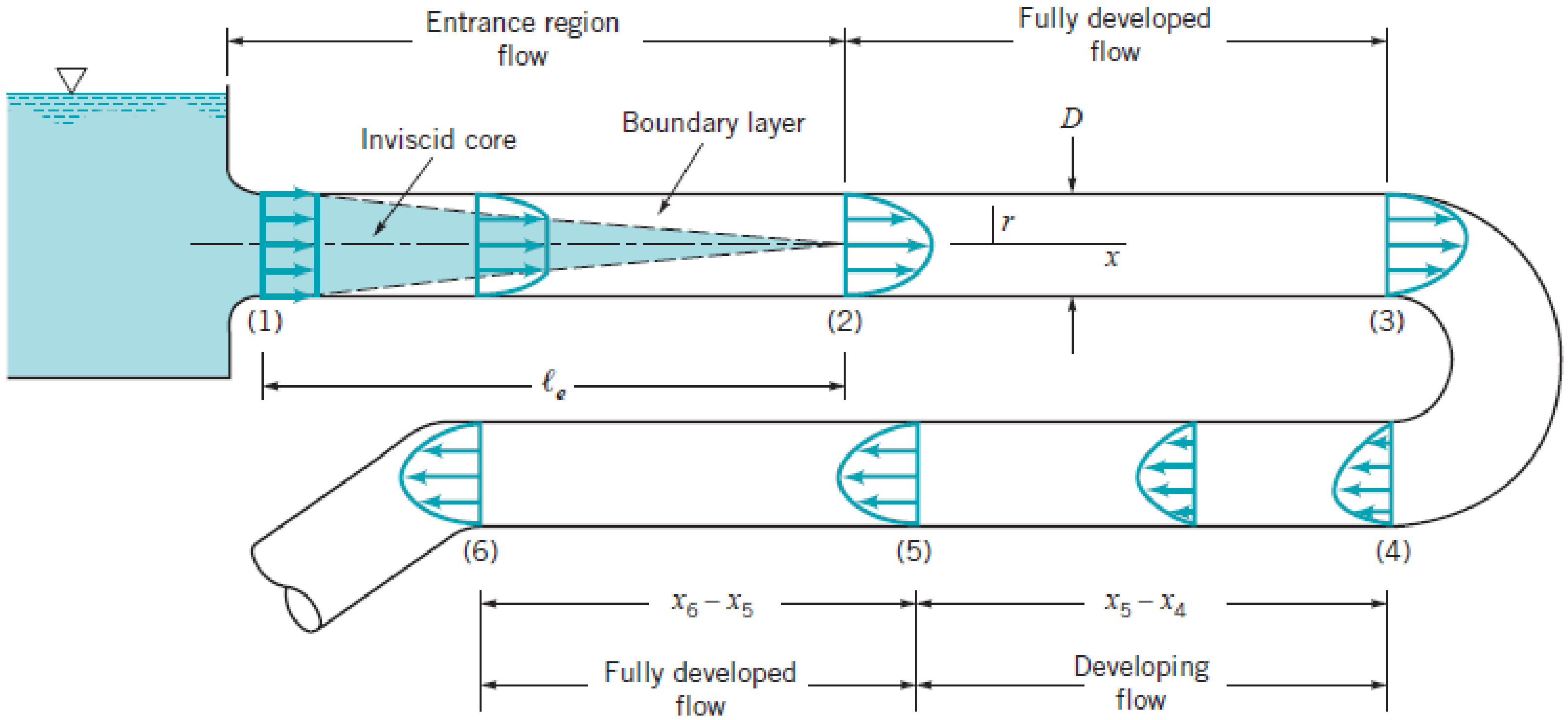


FIGURE 8.5 **Entrance region, developing flow, and fully developed flow in a pipe system.**

8.1.3 Pressure and Shear Stress

Fully developed steady flow in a constant diameter pipe may be driven by gravity and/or pressure forces. For horizontal pipe flow, gravity has no effect except for a hydrostatic pressure variation across the pipe, γD, that is usually negligible. It is the pressure difference, $\Delta p = p_1 - p_2$, between one section of the horizontal pipe and another which forces the fluid through the pipe. Viscous effects provide the restraining force that exactly balances the pressure force, thereby allowing the fluid to flow through the pipe with no acceleration. If viscous effects were absent in such flows, the pressure would be constant throughout the pipe, except for the hydrostatic variation.

The fact that there is a nonzero pressure gradient along the horizontal pipe is a result of viscous effects. As is discussed in Chapter 3, if the viscosity were zero, the pressure would not vary with x. The need for the pressure drop can be viewed from two different standpoints. In terms of a force balance, the pressure force is needed to overcome the viscous forces generated. In terms of an energy balance, the work done by the pressure force is needed to overcome the viscous dissipation of energy throughout the fluid. If the pipe is not horizontal, the pressure gradient along it is due in part to the component of weight in that direction. As is discussed in Section 8.2.1, this contribution due to the weight either enhances or retards the flow, depending on whether the flow is downhill or uphill.

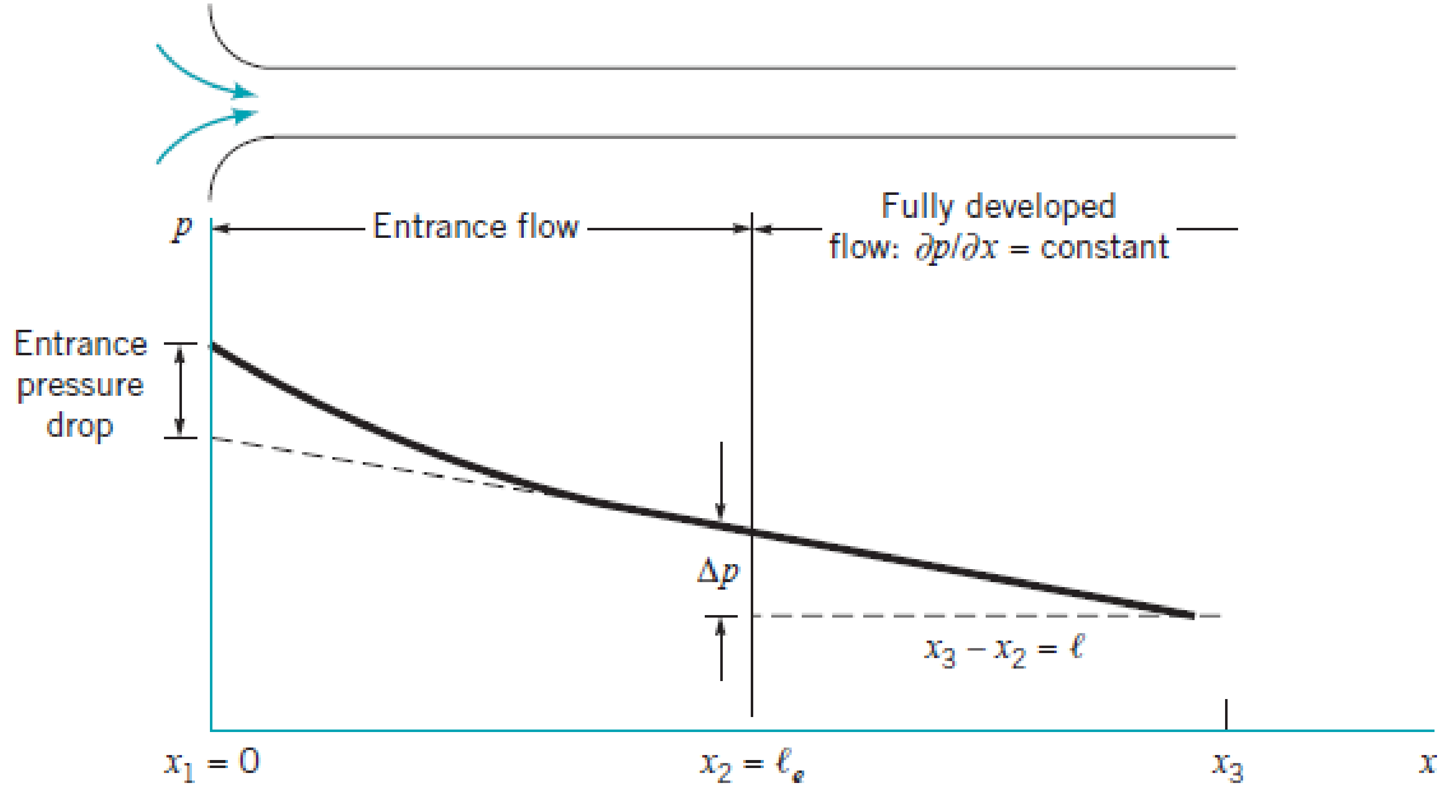


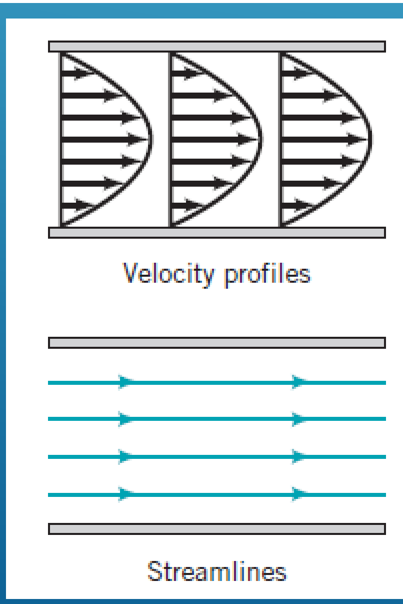
FIGURE 8.6 **Pressure distribution along a horizontal pipe.**

There are numerous ways to derive important results pertaining to fully developed laminar flow. Three alternatives include: (1) from $\mathbf{F} = m\mathbf{a}$ applied directly to a fluid element, (2) from the Navier–Stokes equations of motion, and (3) from dimensional analysis methods.

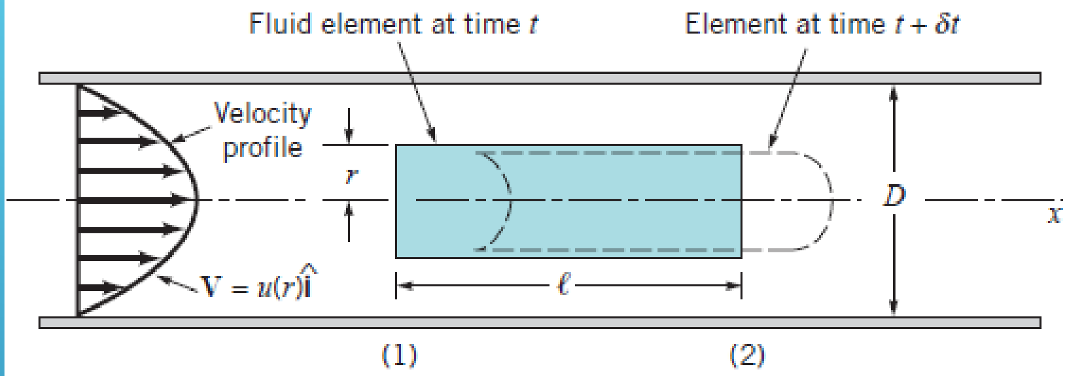
8.2.1 From F = *m*a Applied Directly to a Fluid Element

The local acceleration is zero ($\partial \mathbf{V}/\partial t = 0$) because the flow is steady, and the convective acceleration is zero ($\mathbf{V} \cdot \nabla \mathbf{V} = u\, \partial u/\partial x\, \hat{\mathbf{i}} = 0$) because the flow is fully devel oped. Thus, every part of the fluid merely flows along its streamline parallel to the pipe walls with constant velocity.

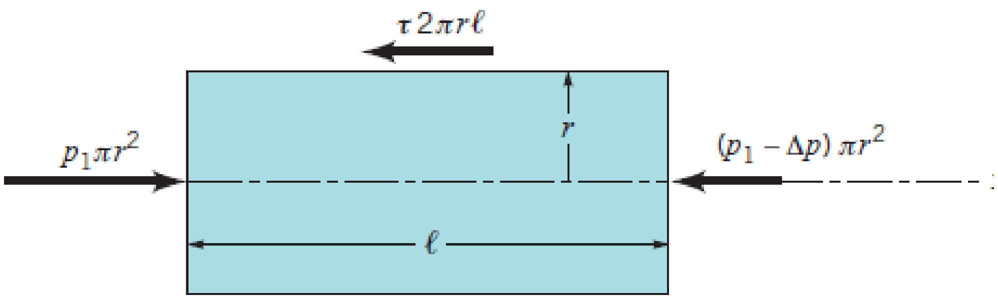
If gravitational effects are neglected, the pressure is constant across any vertical cross section of the pipe, although it varies along the pipe from one section to the next. Thus, if the pressure is $p = p_1$ at section (1), it is $p_2 = p_1 - \Delta p$ at section (2) where Δp is the pressure drop between sections (1) and (2). We anticipate the fact that the pressure decreases in the direction of flow so that $\Delta p > 0$. A shear stress, τ, acts on the surface of the cylinder of fluid. This viscous stress is a function of the radius of the cylinder, $\tau = \tau(r)$.



Steady, fully developed pipe flow experiences no acceleration.



■ FIGURE 8.7 Motion of a cylindrical fluid element within a pipe.



■ FIGURE 8.8 Free-body diagram of a cylinder of fluid.

This force balance can be written as

$$(p_1)\pi r^2 - (p_1 - \Delta p)\pi r^2 - (\tau)2\pi r\ell = 0$$

which can be simplified to give

$$\frac{\Delta p}{\ell} = \frac{2\tau}{r} \tag{8.3}$$

$$\tau = \frac{2\tau_w r}{D} \quad \text{(8.4)}$$

$$\Delta p = \frac{4\ell\tau_w}{D} \quad \text{(8.5)}$$

Basic horizontal pipe flow is governed by a balance between viscous and pressure forces.

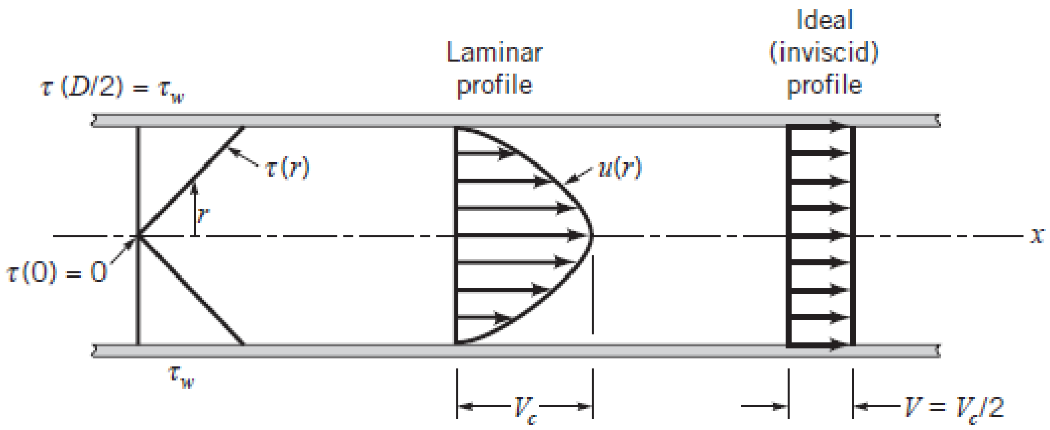


FIGURE 8.9 Shear stress distribution within the fluid in a pipe (laminar or turbulent flow) and typical velocity profiles.

$$\tau = -\mu \frac{du}{dr} \quad \text{(8.6)}$$

Equations 8.3 and 8.6 represent the two governing laws for fully developed laminar flow of a Newtonian fluid within a horizontal pipe. The one is Newton's second law of motion and the other is the definition of a Newtonian fluid. By combining these two equations we obtain

$$\frac{du}{dr} = -\left(\frac{\Delta p}{2\mu\ell}\right)r$$

which can be integrated to give the velocity profile as follows:

$$\int du = -\frac{\Delta p}{2\mu\ell}\int r\,dr$$

or

$$u = -\left(\frac{\Delta p}{4\mu\ell}\right)r^2 + C_1$$

where C_1 is a constant. Because the fluid is viscous it sticks to the pipe wall so that $u = 0$ at $r = D/2$. Thus, $C_1 = (\Delta p/16\mu\ell)D^2$. Hence, the velocity profile can be written as

$$u(r) = \left(\frac{\Delta p D^2}{16\mu\ell}\right)\left[1 - \left(\frac{2r}{D}\right)^2\right] = V_c\left[1 - \left(\frac{2r}{D}\right)^2\right] \qquad \textbf{(8.7)}$$

where $V_c = \Delta p D^2/(16\mu\ell)$ is the centerline velocity. An alternative expression can be written by using the relationship between the wall shear stress and the pressure gradient (Eqs. 8.5 and 8.7) to give

$$u(r) = \frac{\tau_w D}{4\mu}\left[1 - \left(\frac{r}{R}\right)^2\right]$$

where $R = D/2$ is the pipe radius.

Since the flow is axisymmetric about the centerline, the velocity is constant on small area elements consisting of rings of radius r and thickness dr as shown in the figure in the margin. Thus,

$$Q = \int u\, dA = \int_{r=0}^{r=R} u(r) 2\pi r\, dr = 2\pi\, V_c \int_0^R \left[1 - \left(\frac{r}{R}\right)^2\right] r\, dr$$

or

$$Q = \frac{\pi R^2 V_c}{2}$$

By definition, the average velocity is the flowrate divided by the cross-sectional area, $V = Q/A = Q/\pi R^2$, so that for this flow

$$V = \frac{\pi R^2 V_c}{2\pi R^2} = \frac{V_c}{2} = \frac{\Delta p D^2}{32\mu\ell} \qquad \textbf{(8.8)}$$

and

$$\boxed{Q = \frac{\pi D^4\, \Delta p}{128\mu\ell}} \qquad \textbf{(8.9)}$$

The above results confirm the following properties of laminar pipe flow. For a horizontal pipe the flowrate is (a) directly proportional to the pressure drop, (b) inversely proportional to the viscosity, (c) inversely proportional to the pipe length, and (d) proportional to the pipe diameter to the fourth power. With all other parameters fixed, an increase in diameter by a factor of 2 will increase the flowrate by a factor of $2^4 = 16$—the flowrate is very strongly dependent on pipe size.

This flow, the properties of which were first established experimentally by two independent workers, G. Hagen (1797–1884) in 1839 and J. Poiseuille (1799–1869) in 1840, is termed *Hagen–Poiseuille flow*. Equation 8.9 is commonly referred to as ***Poiseuille's law.***

$$\frac{\Delta p - \gamma \ell \sin \theta}{\ell} = \frac{2\tau}{r} \tag{8.10}$$

Thus, all of the results for the horizontal pipe are valid provided the pressure gradient is adjusted for the elevation term, that is, Δp is replaced by $\Delta p - \gamma \ell \sin \theta$ so that

$$V = \frac{(\Delta p - \gamma \ell \sin \theta) D^2}{32 \mu \ell} \tag{8.11}$$

and

$$Q = \frac{\pi (\Delta p - \gamma \ell \sin \theta) D^4}{128 \mu \ell} \tag{8.12}$$

It is seen that the driving force for pipe flow can be either a pressure drop in the flow direction, Δp, or the component of weight in the flow direction, $-\gamma \ell \sin \theta$. If the flow is downhill, gravity helps the flow (a smaller pressure drop is required; $\sin \theta < 0$). If the flow is uphill, gravity works against the flow (a larger pressure drop is required; $\sin \theta > 0$). Note that $\gamma \ell \sin \theta = \gamma \Delta z$ (where

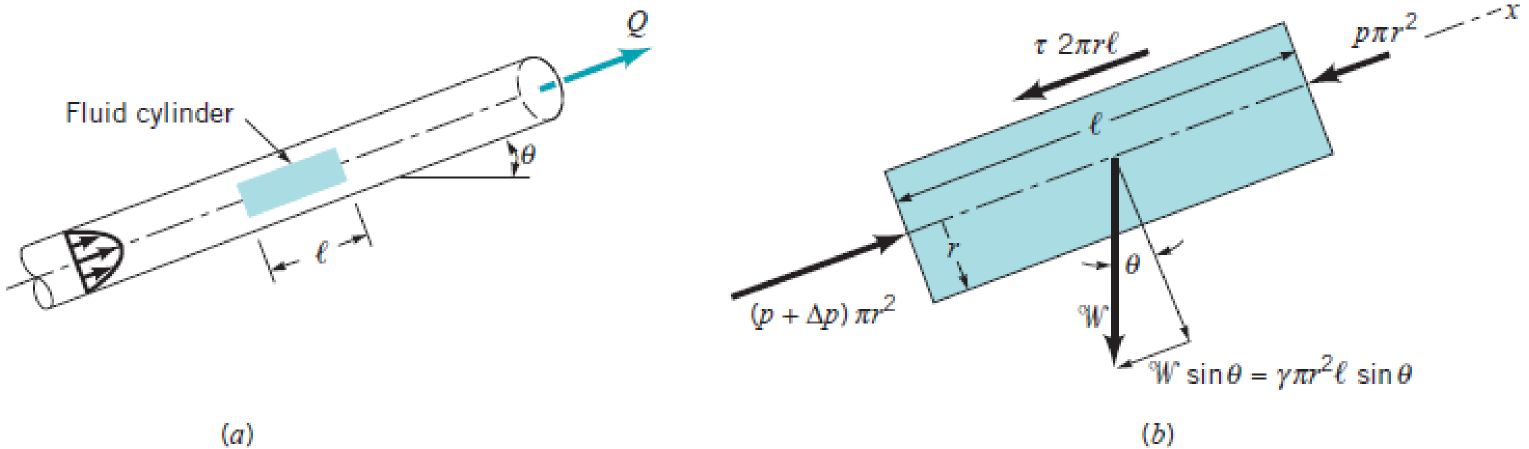


FIGURE 8.10 Free-body diagram of a fluid cylinder for flow in a nonhorizontal pipe.

EXAMPLE 8.2 Laminar Pipe Flow

GIVEN An oil with a viscosity of $\mu = 0.40\ \text{N}\cdot\text{s/m}^2$ and density $\rho = 900\ \text{kg/m}^3$ flows in a pipe of diameter $D = 0.020$ m.

FIND **(a)** What pressure drop, $p_1 - p_2$, is needed to produce a flowrate of $Q = 2.0 \times 10^{-5}\ \text{m}^3/\text{s}$ if the pipe is horizontal with $x_1 = 0$ and $x_2 = 10$ m?

(b) How steep a hill, θ, must the pipe be on if the oil is to flow through the pipe at the same rate as in part (a), but with $p_1 = p_2$?

(c) For the conditions of part (b), if $p_1 = 200$ kPa, what is the pressure at section $x_3 = 5$ m, where x is measured along the pipe?

SOLUTION

(a) If the Reynolds number is less than 2100 the flow is laminar and the equations derived in this section are valid. Since the average velocity is $V = Q/A = (2.0 \times 10^{-5}\ \text{m}^3/\text{s})/[\pi(0.020)^2\text{m}^2/4] = 0.0637$ m/s, the Reynolds number is $\text{Re} = \rho VD/\mu = 2.87 < 2100$. Hence, the flow is laminar and from Eq. 8.9 with $\ell = x_2 - x_1 = 10$ m, the pressure drop is

$$\Delta p = p_1 - p_2 = \frac{128\mu\ell Q}{\pi D^4}$$

$$= \frac{128(0.40\ \text{N}\cdot\text{s/m}^2)(10.0\ \text{m})(2.0 \times 10^{-5}\ \text{m}^3/\text{s})}{\pi(0.020\ \text{m})^4}$$

or

$$\Delta p = 20{,}400\ \text{N/m}^2 = 20.4\ \text{kPa} \qquad \textbf{(Ans)}$$

(b) If the pipe is on a hill of angle θ such that $\Delta p = p_1 - p_2 = 0$, Eq. 8.12 gives

$$\sin\theta = -\frac{128\mu Q}{\pi \rho g D^4} \quad \textbf{(1)}$$

or

$$\sin\theta = \frac{-128(0.40 \text{ N} \cdot \text{s/m}^2)(2.0 \times 10^{-5} \text{ m}^3\text{/s})}{\pi(900 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.020 \text{ m})^4} \quad \textbf{(Ans)}$$

Thus, $\theta = -13.34°$.

(c) With $p_1 = p_2$ the length of the pipe, ℓ, does not appear in the flowrate equation (Eq. 1). This is a statement of the fact that for such cases the pressure is constant all along the pipe (provided the pipe lies on a hill of constant slope). This can be seen by substituting the values of Q and θ from case (b) into Eq. 8.12 and noting that $\Delta p = 0$ for any ℓ. For example, $\Delta p = p_1 - p_3 = 0$ if $\ell = x_3 - x_1 = 5$ m. Thus, $p_1 = p_2 = p_3$ so that

$$p_3 = 200 \text{ kPa} \quad \textbf{(Ans)}$$

8.2.2 From the Navier–Stokes Equations

General motion of an incompressible Newtonian fluid is governed by the continuity equation (conservation of mass, Eq. 6.31) and the momentum equation (Eq. 6.127), which are rewritten here for convenience:

$$\nabla \cdot \mathbf{V} = 0 \tag{8.13}$$

$$\frac{\partial \mathbf{V}}{\partial t} + \mathbf{V} \cdot \nabla \mathbf{V} = -\frac{\nabla p}{\rho} + \mathbf{g} + \nu \nabla^2 \mathbf{V} \tag{8.14}$$

For steady, fully developed flow in a pipe, the velocity contains only an axial component, which is a function of only the radial coordinate $[\mathbf{V} = u(r)\hat{\mathbf{i}}]$. For such conditions, the left-hand side of the Eq. 8.14 is zero. This is equivalent to saying that the fluid experiences no acceleration as it flows along. The same constraint was used in the previous section when considering $\mathbf{F} = m\mathbf{a}$ for the fluid cylinder. Thus, with $\mathbf{g} = -g\hat{\mathbf{k}}$ the Navier–Stokes equations become

$$\nabla \cdot \mathbf{V} = 0$$

$$\nabla p + \rho g \hat{\mathbf{k}} = \mu \nabla^2 \mathbf{V} \tag{8.15}$$

The flow is governed by a balance of pressure, weight, and viscous forces in the flow direction,

Poiseuille's law can be obtained from the Navier–Stokes equations.

When it is written in terms of polar coordinates (as was done in Section 6.9.3), the component of Eq. 8.15 along the pipe becomes

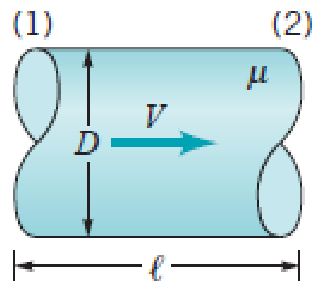
$$\frac{\partial p}{\partial x} + \rho g \sin\theta = \mu \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) \tag{8.16}$$

8.2.3 From Dimensional Analysis

$$\Delta p = F(V, \ell, D, \mu)$$

There are five variables that can be described in terms of three reference dimensions (M, L, T). According to the results of dimensional analysis (Chapter 7), this flow can be described in terms of $k - r = 5 - 3 = 2$ dimensionless groups. One such representation is

$$\frac{D\,\Delta p}{\mu V} = \phi\left(\frac{\ell}{D}\right) \tag{8.17}$$



$\Delta p = p_1 - p_2 = F(V, \ell, D, \mu)$

$$\frac{D\,\Delta p}{\mu V} = \frac{C\ell}{D}$$

which can be rewritten as

$$\frac{\Delta p}{\ell} = \frac{C\mu\,V}{D^2}$$

or

$$Q = AV = \frac{(\pi/4C)\,\Delta p D^4}{\mu\ell} \qquad \textbf{(8.18)}$$

It is usually advantageous to describe a process in terms of dimensionless quantities. To this end we rewrite the pressure drop equation for laminar horizontal pipe flow, Eq. 8.8, as $\Delta p = 32\mu\ell V/D^2$ and divide both sides by the dynamic pressure, $\rho V^2/2$, to obtain the dimensionless form as

$$\frac{\Delta p}{\frac{1}{2}\rho V^2} = \frac{(32\mu\ell V/D^2)}{\frac{1}{2}\rho V^2} = 64\left(\frac{\mu}{\rho VD}\right)\left(\frac{\ell}{D}\right) = \frac{64}{\text{Re}}\left(\frac{\ell}{D}\right)$$

This is often written as

$$\Delta p = f\frac{\ell}{D}\frac{\rho V^2}{2}$$

where the dimensionless quantity

$$f = \Delta p(D/\ell)/(\rho V^2/2)$$

is termed the ***friction factor,*** or sometimes the *Darcy friction factor*

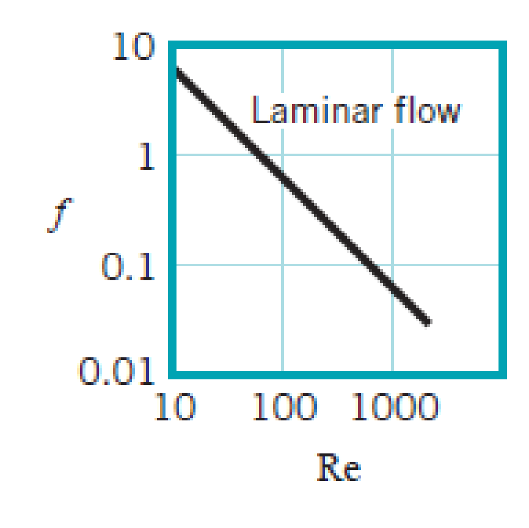
factor, which is defined to be $f/4$. In this text we will use only the Darcy friction factor.) Thus, the friction factor for laminar fully developed pipe flow is simply

$$f = \frac{64}{\text{Re}} \tag{8.19}$$

as shown by the figure in the margin.

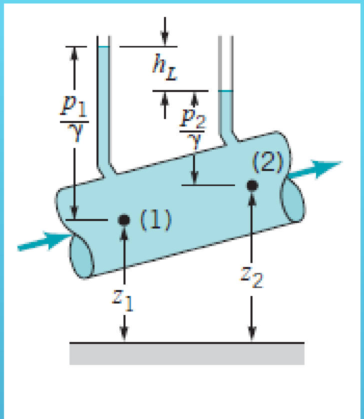
By substituting the pressure drop in terms of the wall shear stress (Eq. 8.5), we obtain an alternate expression for the friction factor as a dimensionless wall shear stress

$$f = \frac{8\tau_w}{\rho V^2} \tag{8.20}$$



8.2.4 Energy Considerations

$$\frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_L \qquad \textbf{(8.21)}$$



For the ideal (inviscid) cases discussed in previous chapters, $\alpha_1 = \alpha_2 = 1$, $h_L = 0$.

Even though the velocity profile in viscous pipe flow is not uniform, for fully developed flow it does not change from section (1) to section (2) so that $\alpha_1 = \alpha_2$. Thus, the kinetic energy is the same at any section ($\alpha_1 V_1^2/2 = \alpha_2 V_2^2/2$) and the energy equation becomes

$$\left(\frac{p_1}{\gamma} + z_1\right) - \left(\frac{p_2}{\gamma} + z_2\right) = h_L \qquad \textbf{(8.22)}$$

The energy dissipated by the viscous forces within the fluid is supplied by the excess work done by the pressure and gravity forces as shown by the figure in the margin.

A comparison of Eqs. 8.22 and 8.10 shows that the head loss is given by

$$h_L = \frac{2\tau \ell}{\gamma r}$$

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A comparison of Eqs. 8.22 and 8.10 shows that the head loss is given by

$$h_L = \frac{2\tau\ell}{\gamma r}$$

(recall $p_1 = p_2 + \Delta p$ and $z_2 - z_1 = \ell \sin\theta$), which, by use of Eq. 8.4, can be rewritten in the form

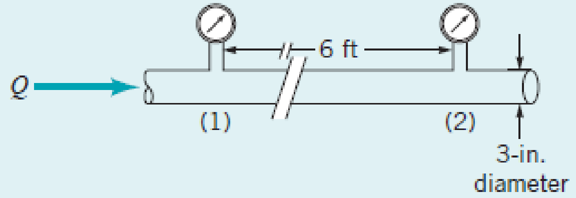
$$h_L = \frac{4\ell\tau_w}{\gamma D} \qquad \textbf{(8.23)}$$

The head loss in a pipe is a result of the viscous shear stress on the wall.

EXAMPLE 8.3 Laminar Pipe Flow Properties

GIVEN The flowrate, Q, of corn syrup through the horizontal pipe shown in Fig. E8.3*a* is to be monitored by measuring the pressure difference between sections (1) and (2). It is proposed that $Q = K\,\Delta p$, where the calibration constant, K, is a function of temperature, T, because of the variation of the syrup's viscosity and density with temperature. These variations are given in Table E8.3.

FIND **(a)** Plot $K(T)$ versus T for 60 °F ≤ T ≤ 160 °F. **(b)** Determine the wall shear stress and the pressure drop, $\Delta p = p_1 - p_2$, for $Q = 0.5\ \text{ft}^3/\text{s}$ and $T = 100$ °F. **(c)** For the conditions of part (b), determine the net pressure force, $(\pi D^2/4)\,\Delta p$, and the net shear force, $\pi D \ell \tau_w$, on the fluid within the pipe between the sections (1) and (2).



(*a*)

■ **TABLE E8.3**

T (°F)	ρ (slugs/ft^3)	μ (lb · s/ft^2)
60	2.07	4.0×10^{-2}
80	2.06	1.9×10^{-2}
100	2.05	3.8×10^{-3}
120	2.04	4.4×10^{-4}
140	2.03	9.2×10^{-5}
160	2.02	2.3×10^{-5}

SOLUTION

(a) If the flow is laminar it follows from Eq. 8.9 that

$$Q = \frac{\pi D^4\,\Delta p}{128\mu\ell} = \frac{\pi(\frac{3}{12}\ \text{ft})^4\,\Delta p}{128\mu(6\ \text{ft})}$$

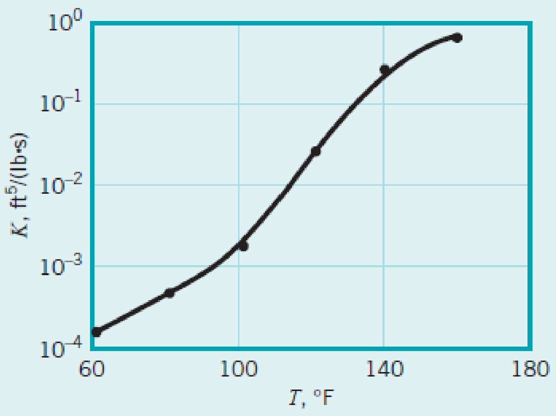
or

$$Q = K\,\Delta p = \frac{1.60 \times 10^{-5}}{\mu}\Delta p \qquad \textbf{(1)}$$

where the units on Q, Δp, and μ are ft^3/s, lb/ft^2, and lb · s/ft^2, respectively. Thus

$$K = \frac{1.60 \times 10^{-5}}{\mu} \qquad \textbf{(Ans)}$$

where the units of K are ft^5/lb · s. By using values of the viscosity from Table E8.3, the calibration curve shown in Fig. E8.3*b* is obtained. This result is valid only if the flow is laminar.



(*b*)

(b) For $T = 100\ °\text{F}$, the viscosity is $\mu = 3.8 \times 10^{-3}\ \text{lb} \cdot \text{s/ft}^2$ so that with a flowrate of $Q = 0.5\ \text{ft}^3\text{/s}$ the pressure drop (according to Eq. 8.9) is

$$\Delta p = \frac{128\mu\ell Q}{\pi D^4}$$

$$= \frac{128(3.8 \times 10^{-3}\ \text{lb} \cdot \text{s/ft}^2)(6\ \text{ft})(0.5\ \text{ft}^3\text{/s})}{\pi(\frac{3}{12}\ \text{ft})^4}$$

$$= 119\ \text{lb/ft}^2 \qquad \textbf{(Ans)}$$

provided the flow is laminar. For this case

$$V = \frac{Q}{A} = \frac{0.5\ \text{ft}^3\text{/s}}{\frac{\pi}{4}(\frac{3}{12}\ \text{ft})^2} = 10.2\ \text{ft/s}$$

so that

$$\text{Re} = \frac{\rho V D}{\mu} = \frac{(2.05\ \text{slugs/ft}^3)(10.2\ \text{ft/s})(\frac{3}{12}\ \text{ft})}{(3.8 \times 10^{-3}\ \text{lb} \cdot \text{s/ft}^2)}$$

$$= 1380 < 2100$$

Hence, the flow is laminar. From Eq. 8.5 the wall shear stress is

$$\tau_w = \frac{\Delta p D}{4\ell} = \frac{(119\ \text{lb/ft}^2)(\frac{3}{12}\ \text{ft})}{4(6\ \text{ft})} = 1.24\ \text{lb/ft}^2 \qquad \textbf{(Ans)}$$

(c) For the conditions of part (b), the net pressure force, F_p, on the fluid within the pipe between sections (1) and (2) is

$$F_p = \frac{\pi}{4} D^2 \, \Delta p = \frac{\pi}{4} \left(\frac{3}{12} \text{ ft} \right)^2 (119 \text{ lb/ft}^2) = 5.84 \text{ lb} \qquad \textbf{(Ans)}$$

Similarly, the net viscous force, F_v, on that portion of the fluid is

$$F_v = 2\pi \left(\frac{D}{2} \right) \ell \tau_w$$

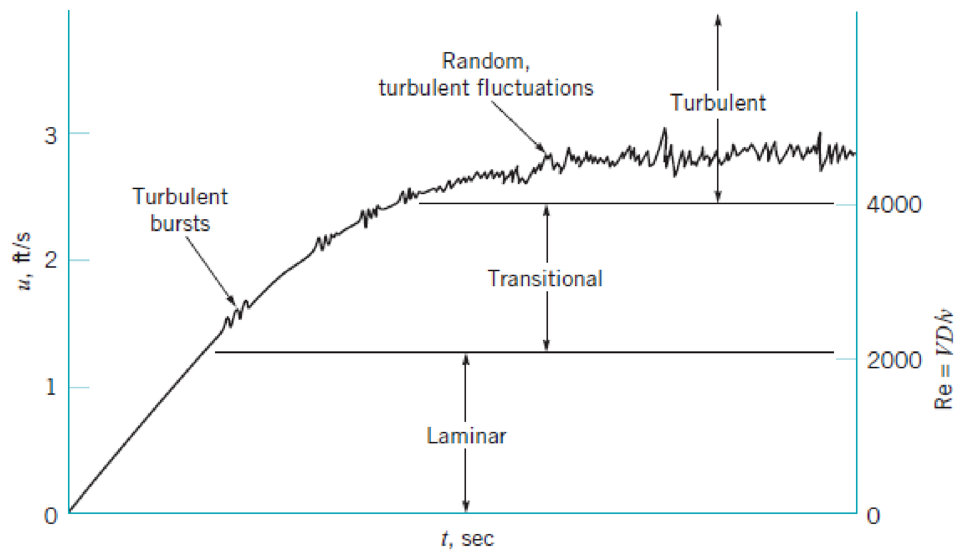
$$= 2\pi \left[\frac{3}{2(12)} \text{ ft} \right] (6 \text{ ft})(1.24 \text{ lb/ft}^2) = 5.84 \text{ lb} \qquad \textbf{(Ans)}$$

8.3 Fully Developed Turbulent Flow

8.3.1 Transition from Laminar to Turbulent Flow

Flows are classified as laminar or turbulent. For any flow geometry, there is one (or more) dimensionless parameter such that with this parameter value below a particular value the flow is laminar, whereas with the parameter value larger than a certain value the flow is turbulent. The important parameters involved (i.e., Reynolds number, Mach number) and their critical values depend on the specific flow situation involved.

As a general rule for pipe flow, the value of the Reynolds number must be less than approximately 2100 for laminar flow and greater than approximately 4000 for turbulent flow.



■ FIGURE 8.11 Transition from laminar to turbulent flow in a pipe.

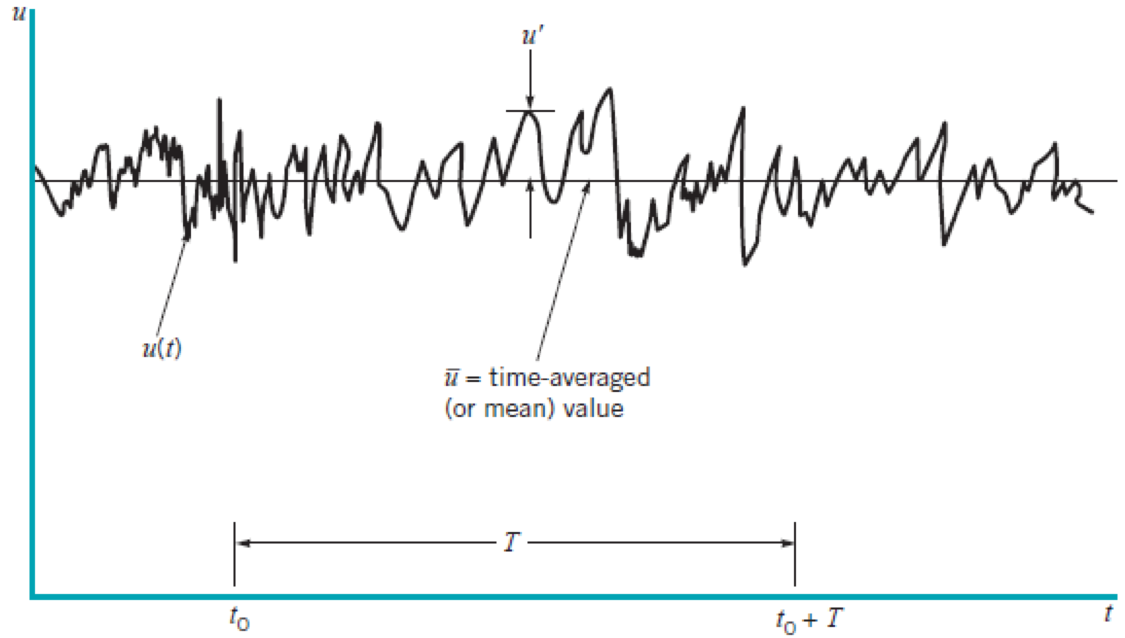


FIGURE 8.12 The time-averaged, $\overline{u}$, and fluctuating, u', description of a parameter for turbulent flow.

8.3.2 Turbulent Shear Stress

The fundamental difference between laminar and turbulent flow lies in the chaotic, random behavior of the various fluid parameters. Such variations occur in the three components of velocity, the pressure, the shear stress, the temperature, and any other variable that has a field description.

$$\overline{u} = \frac{1}{T}\int_{t_0}^{t_0+T} u(x, y, z, t)\, dt \tag{8.24}$$

where the time interval, T, is considerably longer than the period of the longest fluctuations, but considerably shorter than any unsteadiness of the average velocity. This is illustrated in Fig. 8.12.

The *fluctuating part* of the velocity, u', is that time-varying portion that differs from the average value

$$u = \overline{u} + u' \quad \text{or} \quad u' = u - \overline{u} \tag{8.25}$$

Clearly, the time average of the fluctuations is zero, since

$$\overline{u'} = \frac{1}{T}\int_{t_0}^{t_0+T} (u - \overline{u})\, dt = \frac{1}{T}\left(\int_{t_0}^{t_0+T} u\, dt - \overline{u}\int_{t_0}^{t_0+T} dt\right)$$

$$= \frac{1}{T}(T\overline{u} - T\overline{u}) = 0$$

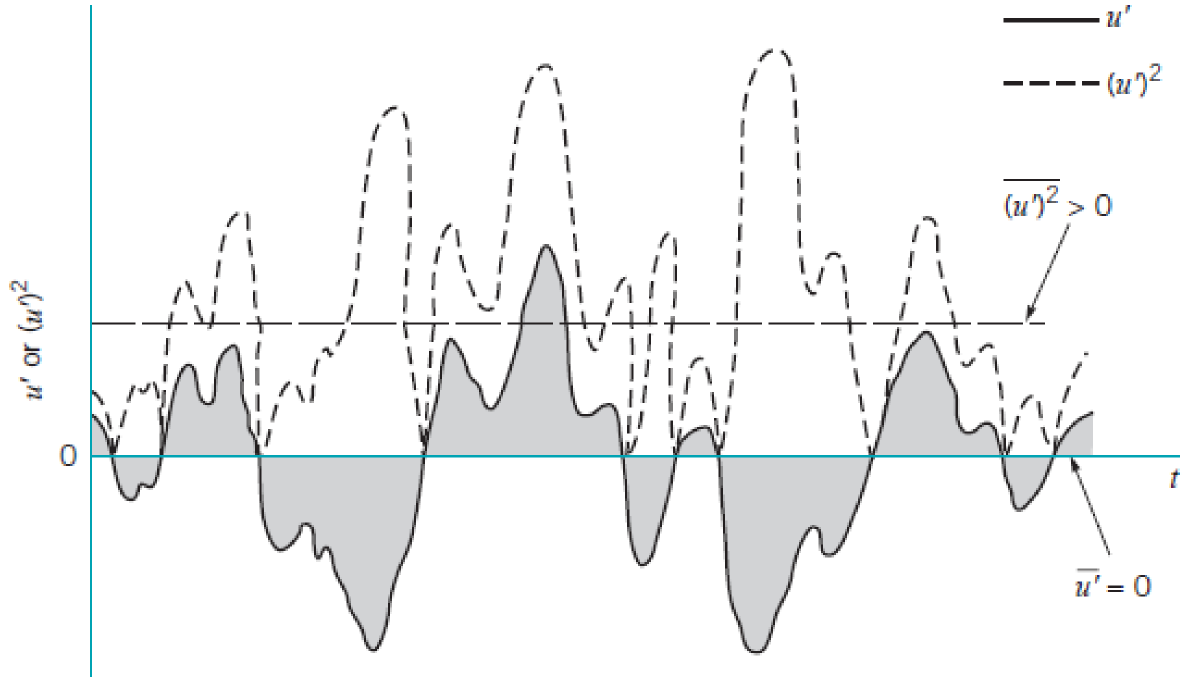


FIGURE 8.13 Average of the fluctuations and average of the square of the fluctuations.

The fluctuations are equally distributed on either side of the average. It is also clear, as is indicated in Fig. 8.13, that since the square of a fluctuation quantity cannot be negative $[(u')^2 \geq 0]$, its average value is positive. Thus,

$$\overline{(u')^2} = \frac{1}{T}\int_{t_0}^{t_0+T} (u')^2 \, dt > 0$$

The structure and characteristics of turbulence may vary from one flow situation to another.

The turbulence intensity, $\mathcal{I}$,

$$\mathcal{I} = \frac{\sqrt{\overline{(u')^2}}}{\overline{u}} = \frac{\left[\frac{1}{T}\int_{t_0}^{t_0+T} (u')^2\, dt\right]^{1/2}}{\overline{u}}$$

The relationship between fluid motion and shear stress is very complex for turbulent flow.

Turbulent flow shear stress is larger than laminar flow shear stress because of the irregular, random motion.

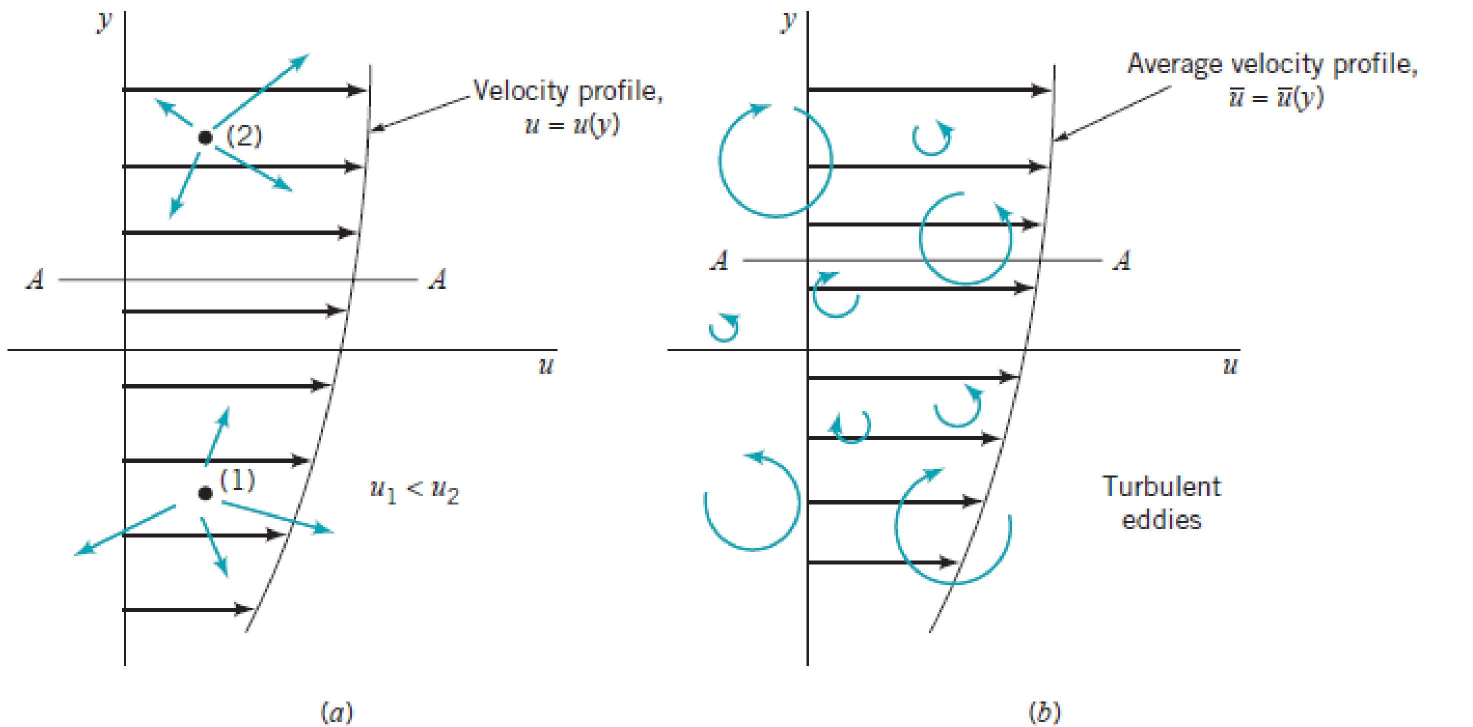


FIGURE 8.14 (*a*) Laminar flow shear stress caused by random motion of molecules. (*b*) Turbulent flow as a series of random, three-dimensional eddies.

The random velocity components that account for this momentum transfer (hence, the shear force) are u' (for the x component of velocity) and v' (for the rate of mass transfer crossing the plane). A more detailed consideration of the processes involved will show that the apparent shear stress on plane $A-A$ is given by the following (Ref. 2):

$$\tau = \mu \frac{d\overline{u}}{dy} - \rho\overline{u'v'} = \tau_{\text{lam}} + \tau_{\text{turb}} \tag{8.26}$$

Note that if the flow is laminar, $u' = v' = 0$, so that $\overline{u'v'} = 0$ and Eq. 8.26 reduces to the customary random molecule-motion-induced *laminar shear stress*, $\tau_{\text{lam}} = \mu\, d\overline{u}/dy$. For turbulent flow it is found that the ***turbulent shear stress***, $\tau_{\text{turb}} = -\rho\overline{u'v'}$, is positive.

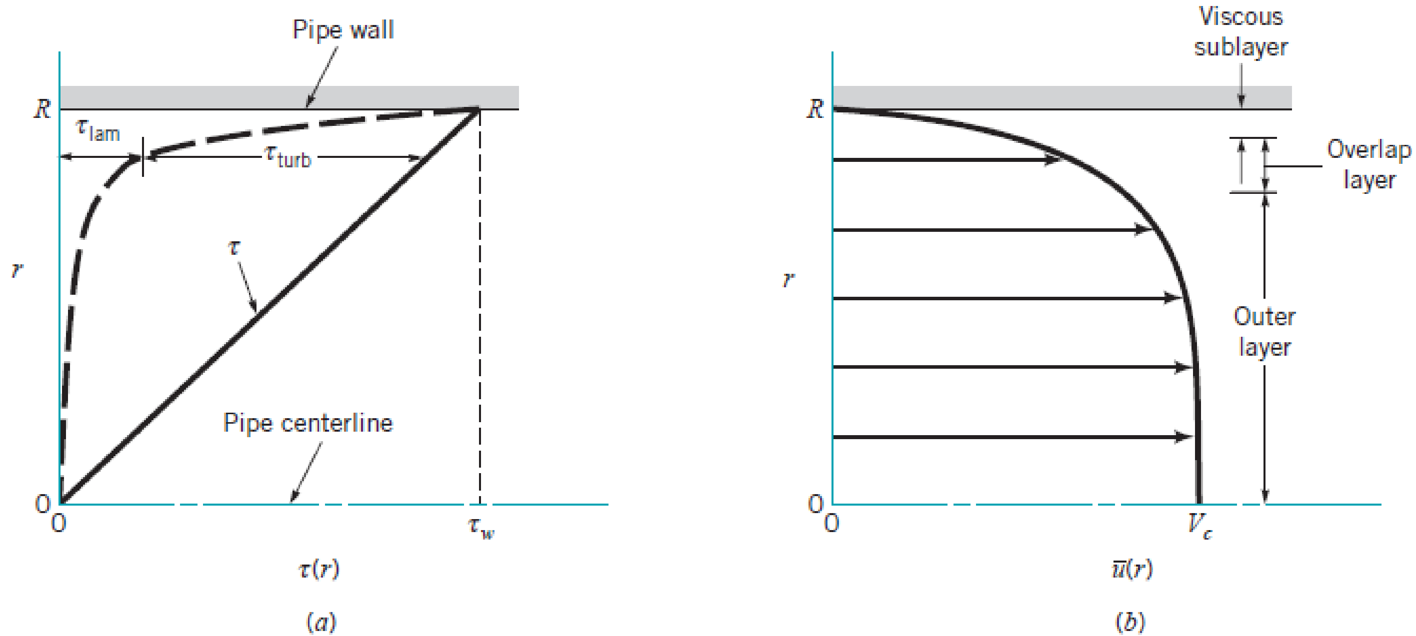


FIGURE 8.15 Structure of turbulent flow in a pipe. (*a*) Shear stress. (*b*) Average velocity.

An alternate form for the shear stress for turbulent flow is given in terms of the *eddy viscosity*, η, where

$$\tau_{\text{turb}} = \eta \frac{d\overline{u}}{dy} \tag{8.27}$$

This extension of laminar flow terminology was introduced by J. Boussinesq, a French scientist, in 1877. Although the concept of an eddy viscosity is intriguing, in practice it is not an easy parameter to use. Unlike the absolute viscosity, μ, which is a known value for a given fluid, the eddy viscosity is a function of both the fluid and the flow conditions. That is, the eddy viscosity of water cannot be looked up in handbooks—its value changes from one turbulent flow condition to another and from one point in a turbulent flow to another.

The inability to accurately determine the Reynolds stress, $\rho\overline{u'v'}$, is equivalent to not knowing the eddy viscosity. Several semiempirical theories have been proposed (Ref. 3) to determine approximate values of η. L. Prandtl (1875–1953), a German physicist and aerodynamicist, proposed that the turbulent process could be viewed as the random transport of bundles of fluid particles over a certain distance, ℓ_m, the *mixing length*, from a region of one velocity to another region of a different velocity. By the use of some ad hoc assumptions and physical reasoning, it was concluded that the eddy viscosity was given by

$$\eta = \rho \ell_m^2 \left| \frac{d\overline{u}}{dy} \right|$$

Thus, the turbulent shear stress is

$$\tau_{\text{turb}} = \rho \ell_m^2 \left(\frac{d\overline{u}}{dy} \right)^2 \tag{8.28}$$

In the viscous sublayer the velocity profile can be written in dimensionless form as

$$\frac{\overline{u}}{u^*} = \frac{yu^*}{\nu} \qquad (8.29)$$

commonly called the *law of the wall* is valid very near the smooth wall, for $0 \le yu^*/\nu \lesssim 5$.

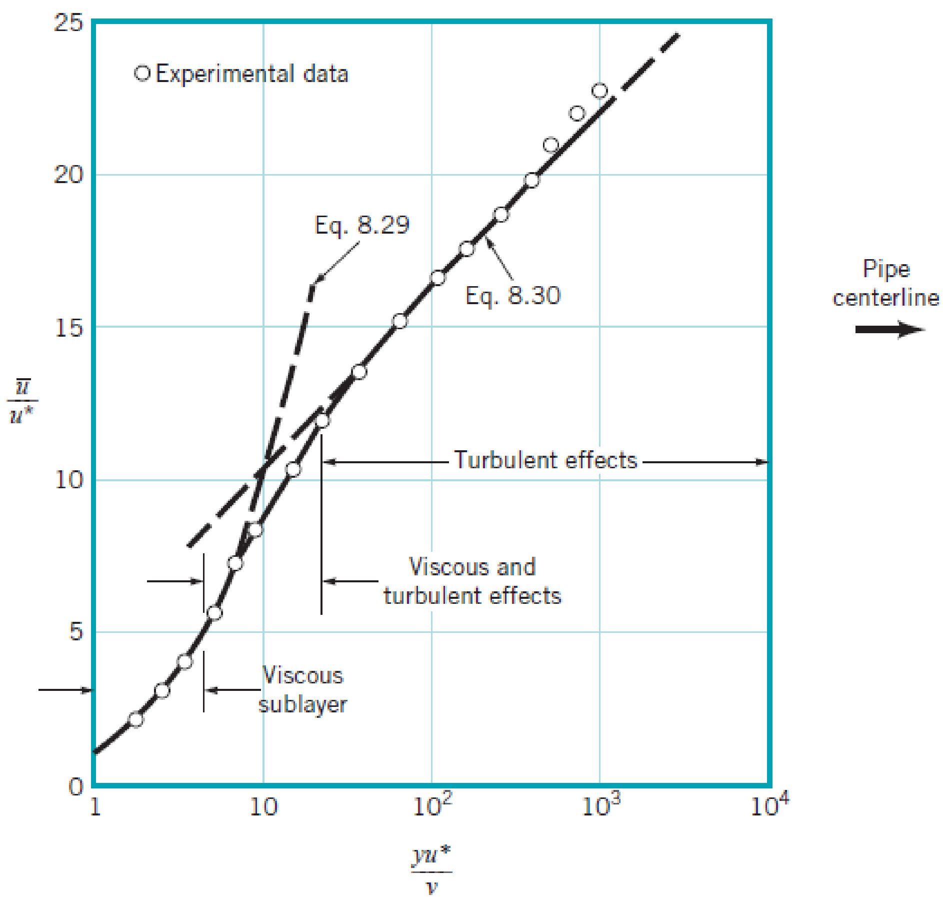


FIGURE 8.16 Typical structure of the turbulent velocity profile in a pipe.

Dimensional analysis arguments indicate that in the overlap region the velocity should vary as the logarithm of y. Thus, the following expression has been proposed:

$$\frac{\overline{u}}{u^*} = 2.5 \ln\left(\frac{yu^*}{\nu}\right) + 5.0 \qquad \textbf{(8.30)}$$

$u^* = (\tau_w/\rho)^{1/2}$ is termed the *friction velocity*.

$\overline{u}$ is the time-averaged x component of velocity

A number of other correlations exist for the velocity profile in turbulent pipe flow. In the central region (the outer turbulent layer) the expression $(V_c - \overline{u})/u^* = 2.5 \ln(R/y)$, where V_c is the centerline velocity, is often suggested as a good correlation with experimental data. Another often-used (and relatively easy to use) correlation is the empirical *power-law velocity profile*

$$\frac{\overline{u}}{V_c} = \left(1 - \frac{r}{R}\right)^{1/n} \qquad \textbf{(8.31)}$$

A closer examination of Eq. 8.31 shows that the power-law profile cannot be valid near the wall, since according to this equation the velocity gradient is infinite there. In addition, Eq. 8.31 cannot be precisely valid near the centerline because it does not give $d\overline{u}/dr = 0$ at $r = 0$. However, it does provide a reasonable approximation to the measured velocity profiles across most of the pipe.

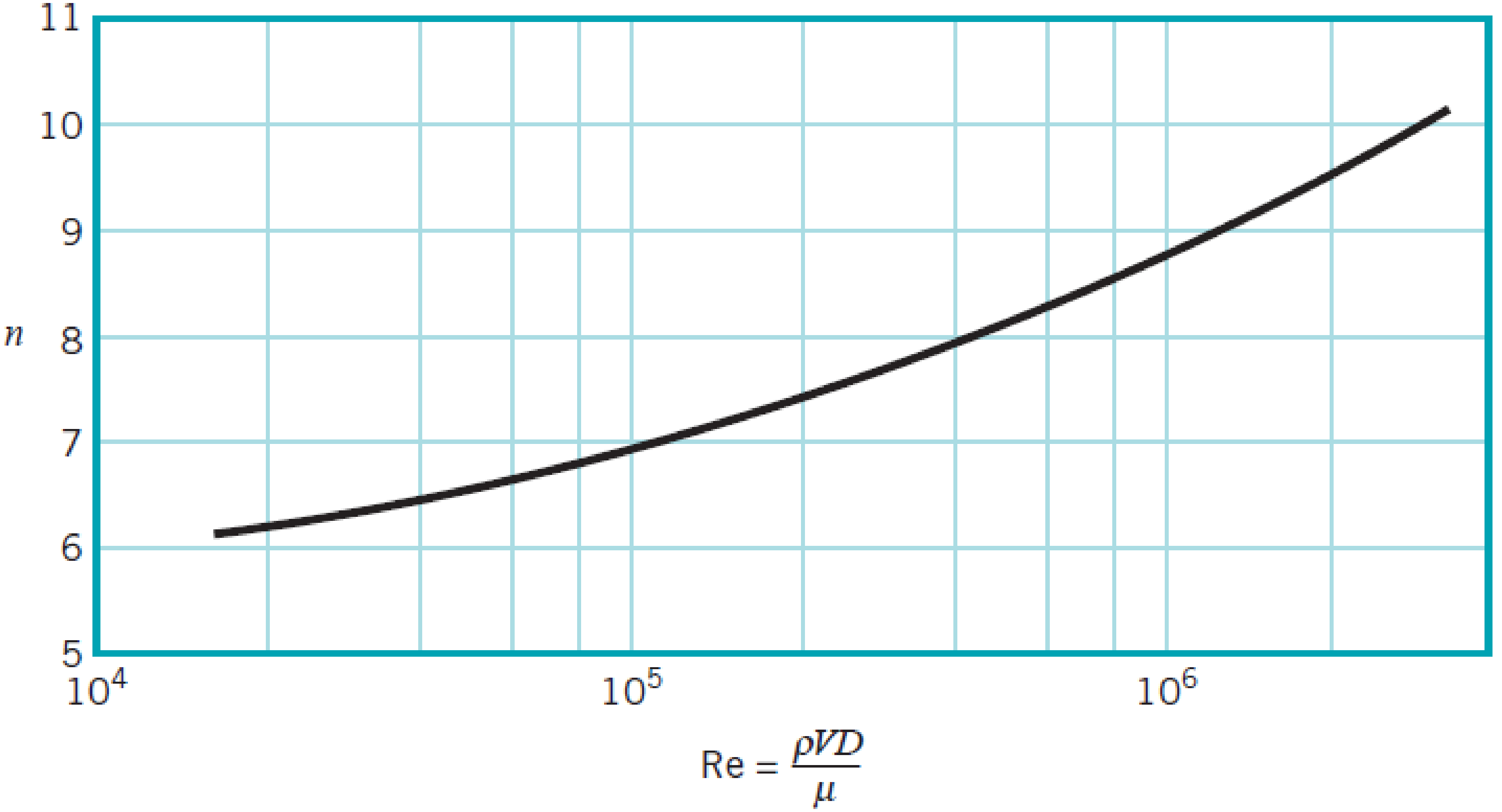


FIGURE 8.17 Exponent, n, for power-law velocity profiles. (Adapted from Ref. 1.)

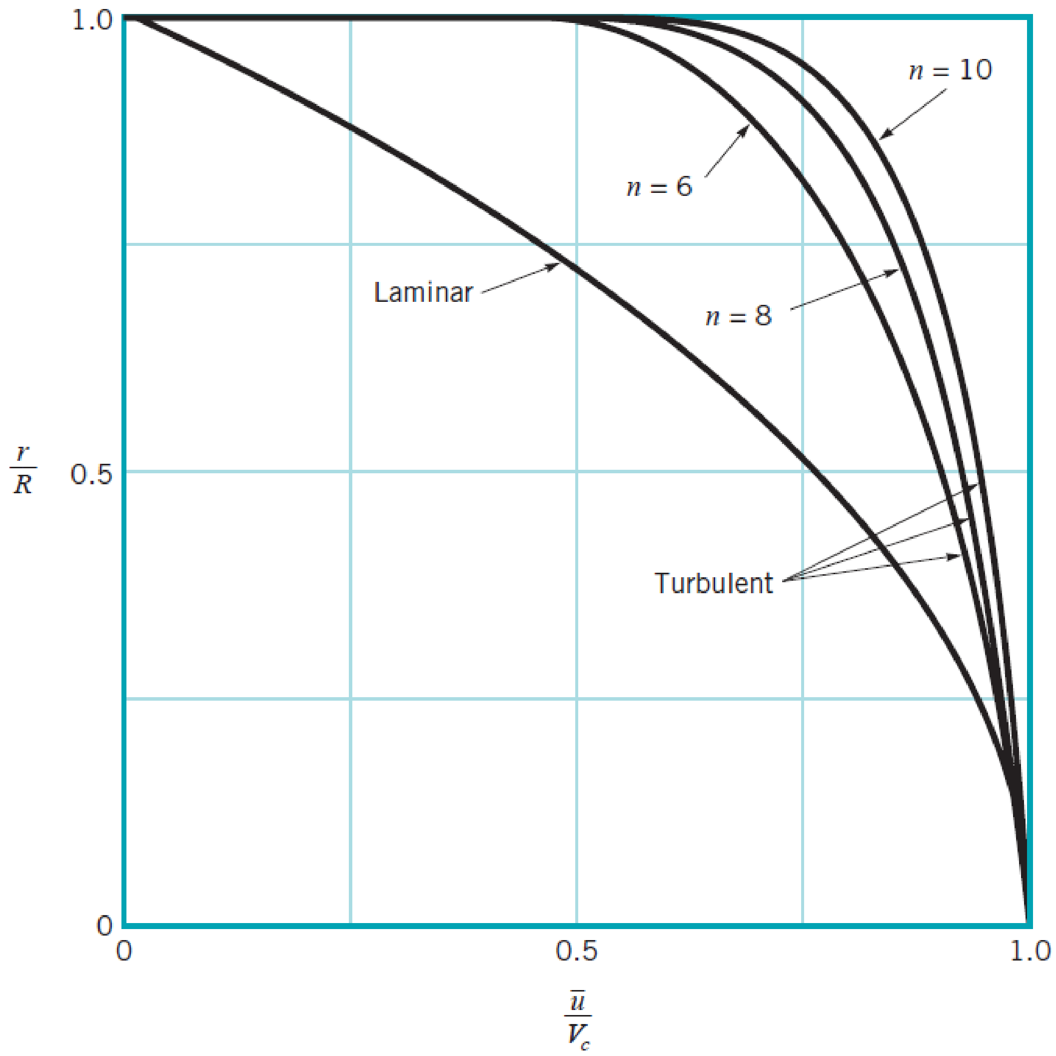


FIGURE 8.18 Typical laminar flow and turbulent flow velocity profiles.

Example 8.4 Turbulent Pipe Flow Properties

GIVEN Water at 20 °C ($\rho = 998 \text{ kg/m}^3$ and $\nu = 1.004 \times 10^{-6} \text{ m}^2\text{/s}$) flows through a horizontal pipe of 0.1-m diameter with a flowrate of $Q = 4 \times 10^{-2} \text{ m}^3\text{/s}$ and a pressure gradient of 2.59 kPa/m.

FIND **(a)** Determine the approximate thickness of the viscous sublayer.

(b) Determine the approximate centerline velocity, V_c.

(c) Determine the ratio of the turbulent to laminar shear stress, $\tau_{\text{turb}}/\tau_{\text{lam}}$, at a point midway between the centerline and the pipe wall (i.e., at $r = 0.025$ m).

Solution

(a) According to Fig. 8.16, the thickness of the viscous sublayer, δ_s, is approximately

$$\frac{\delta_s u^*}{\nu} = 5$$

Therefore,

$$\delta_s = 5\,\frac{\nu}{u^*}$$

where

$$u^* = \left(\frac{\tau_w}{\rho}\right)^{1/2} \qquad \textbf{(1)}$$

The wall shear stress can be obtained from the pressure drop data and Eq. 8.5, which is valid for either laminar or turbulent flow. Thus,

$$\tau_w = \frac{D\,\Delta p}{4\ell} = \frac{(0.1 \text{ m})(2.59 \times 10^3 \text{ N/m}^2)}{4(1 \text{ m})} = 64.8 \text{ N/m}^2$$

Hence, from Eq. 1 we obtain

$$u^* = \left(\frac{64.8 \text{ N/m}^2}{998 \text{ kg/m}^3}\right)^{1/2} = 0.255 \text{ m/s}$$

so that

$$\delta_s = \frac{5(1.004 \times 10^{-6} \text{ m}^2\text{/s})}{0.255 \text{ m/s}}$$

$$= 1.97 \times 10^{-5} \text{ m} \simeq 0.02 \text{ mm} \qquad \textbf{(Ans)}$$

(b) The centerline velocity can be obtained from the average velocity and the assumption of a power-law velocity profile as follows. For this flow with

$$V = \frac{Q}{A} = \frac{0.04 \text{ m}^3/\text{s}}{\pi(0.1 \text{ m})^2/4} = 5.09 \text{ m/s}$$

the Reynolds number is

$$\text{Re} = \frac{VD}{\nu} = \frac{(5.09 \text{ m/s})(0.1 \text{ m})}{(1.004 \times 10^{-6} \text{ m}^2/\text{s})} = 5.07 \times 10^5$$

Thus, from Fig. 8.17, $n = 8.4$ so that

$$\frac{\overline{u}}{V_c} \approx \left(1 - \frac{r}{R}\right)^{1/8.4}$$

To determine the centerline velocity, V_c, we must know the relationship between V (the average velocity) and V_c. This can be obtained by integration of the power-law velocity profile as follows. Since the flow is axisymmetric,

$$Q = AV = \int \overline{u}\, dA = V_c \int_{r=0}^{r=R} \left(1 - \frac{r}{R}\right)^{1/n} (2\pi r)\, dr$$

which can be integrated to give

$$Q = 2\pi R^2 V_c \frac{n^2}{(n + 1)(2n + 1)}$$

Thus, since $Q = \pi R^2 V$, we obtain

$$\frac{V}{V_c} = \frac{2n^2}{(n + 1)(2n + 1)}$$

With $n = 8.4$ in the present case, this gives

$$V_c = \frac{(n + 1)(2n + 1)}{2n^2} V = 1.186V = 1.186\,(5.09 \text{ m/s})$$

$$= 6.04 \text{ m/s}$$ **(Ans)**

Recall that $V_c = 2V$ for laminar pipe flow.

(c) From Eq. 8.4, which is valid for laminar or turbulent flow, the shear stress at $r = 0.025$ m is

$$\tau = \frac{2\tau_w r}{D} = \frac{2(64.8\ \text{N/m}^2)(0.025\ \text{m})}{(0.1\ \text{m})}$$

or

$$\tau = \tau_{\text{lam}} + \tau_{\text{turb}} = 32.4\ \text{N/m}^2$$

where $\tau_{\text{lam}} = -\mu\, d\bar{u}/dr$. From the power-law velocity profile (Eq. 8.31) we obtain the gradient of the average velocity as

$$\frac{d\bar{u}}{dr} = -\frac{V_c}{nR}\left(1 - \frac{r}{R}\right)^{(1-n)/n}$$

which gives

$$\frac{d\bar{u}}{dr} = -\frac{(6.04\ \text{m/s})}{8.4(0.05\ \text{m})}\left(1 - \frac{0.025\ \text{m}}{0.05\ \text{m}}\right)^{(1-8.4)/8.4}$$
$$= -26.5/\text{s}$$

Thus,

$$\tau_{\text{lam}} = -\mu\frac{d\bar{u}}{dr} = -(\nu\rho)\frac{d\bar{u}}{dr}$$
$$= -(1.004 \times 10^{-6}\ \text{m}^2/\text{s})(998\ \text{kg/m}^3)(-26.5/\text{s})$$
$$= 0.0266\ \text{N/m}^2$$

Thus, the ratio of turbulent to laminar shear stress is given by

$$\frac{\tau_{\text{turb}}}{\tau_{\text{lam}}} = \frac{\tau - \tau_{\text{lam}}}{\tau_{\text{lam}}} = \frac{32.4 - 0.0266}{0.0266} = 1220 \qquad \textbf{(Ans)}$$

8.3.4 Turbulence Modeling

Although it is not yet possible to theoretically predict the random, irregular details of turbulent flows, it would be useful to be able to predict the time-averaged flow fields (pressure, velocity, etc.) directly from the basic governing equations. To this end one can time average the governing Navier–Stokes equations (Eqs. 6.31 and 6.127) to obtain equations for the average velocity and pressure. However, because the Navier–Stokes equations are nonlinear, the resulting time-averaged differential equations contain not only the desired average pressure and velocity as variables, but also averages of products of the fluctuations—terms of the type that one tried to eliminate by averaging the equations! For example, the Reynolds stress $-\rho \overline{u'v'}$ (see Eq. 8.26) occurs in the time-averaged momentum equation.

8.3.5 Chaos and Turbulence

Chaos theory is a relatively new branch of mathematical physics that may provide insight into the complex nature of turbulence. This method combines mathematics and numerical (computer) techniques to provide a new way to analyze certain problems. Chaos theory, which is quite complex and is currently under development, involves the behavior of nonlinear dynamical systems and their response to initial and boundary conditions. The flow of a viscous fluid, which is governed by the nonlinear Navier–Stokes equations (Eq. 6.127), may be such a system.

8.4 Dimensional Analysis of Pipe Flow

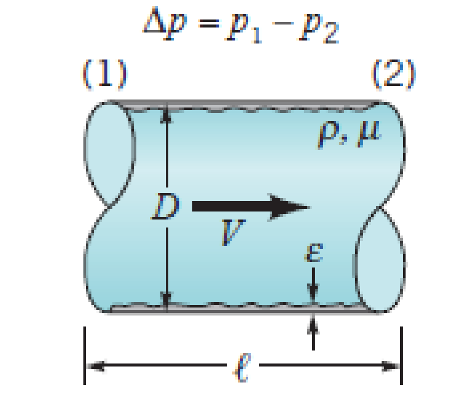
As noted previously, turbulent flow can be a very complex, difficult topic—one that as yet has defied a rigorous theoretical treatment. Thus, most turbulent pipe flow analyses are based on experimental data and semi-empirical formulas. These data are expressed conveniently in dimensionless form.

$$h_L = h_{L\,\text{major}} + h_{L\,\text{minor}}$$

8.4.1 Major Losses

Thus, as indicated by the figure in the margin, the pressure drop, Δp, for steady, incompressible turbulent flow in a horizontal round pipe of diameter D can be written in functional form as

$$\Delta p = F(V, D, \ell, \varepsilon, \mu, \rho) \qquad \textbf{(8.32)}$$



$$\frac{\Delta p}{\frac{1}{2}\rho V^2} = \tilde{\phi}\left(\frac{\rho V D}{\mu}, \frac{\ell}{D}, \frac{\varepsilon}{D}\right)$$

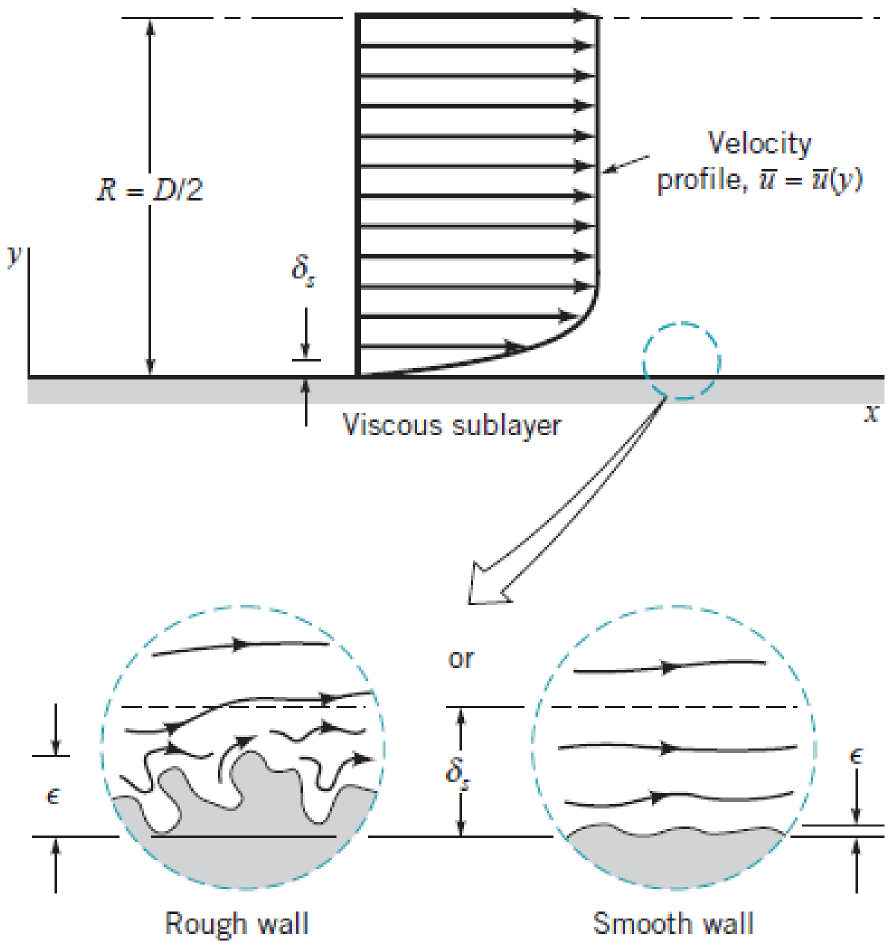


FIGURE 8.19 Flow in the viscous sublayer near rough and smooth walls.

$$\Delta p = f \frac{\ell}{D} \frac{\rho V^2}{2} \qquad (8.33)$$

From Eq. 5.89 the energy equation for steady incompressible flow is

$$\frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_L$$

where h_L is the head loss between sections (1) and (2). With the assumption of a constant diameter ($D_1 = D_2$ so that $V_1 = V_2$), horizontal ($z_1 = z_2$) pipe with fully developed flow ($\alpha_1 = \alpha_2$), this becomes $\Delta p = p_1 - p_2 = \gamma h_L$, which can be combined with Eq. 8.33 to give

$$h_{L\ \text{major}} = f \frac{\ell}{D} \frac{V^2}{2g} \quad \textbf{(8.34)}$$

Equation 8.34, called the *Darcy–Weisbach equation*, is valid for any fully developed, steady, incompressible pipe flow—whether the pipe is horizontal or on a hill. On the other hand, Eq. 8.33 is valid only for horizontal pipes. In general, with $V_1 = V_2$ the energy equation gives

$$p_1 - p_2 = \gamma(z_2 - z_1) + \gamma h_L = \gamma(z_2 - z_1) + f \frac{\ell}{D} \frac{\rho V^2}{2}$$

TABLE 8.1

Equivalent Roughness for New Pipes [From Moody (Ref. 7) and Colebrook (Ref. 8)]

	Equivalent Roughness, ε	
Pipe	**Feet**	**Millimeters**
Riveted steel	0.003–0.03	0.9–9.0
Concrete	0.001–0.01	0.3–3.0
Wood stave	0.0006–0.003	0.18–0.9
Cast iron	0.00085	0.26
Galvanized iron	0.0005	0.15
Commercial steel or wrought iron	0.00015	0.045
Drawn tubing	0.000005	0.0015
Plastic, glass	0.0 (smooth)	0.0 (smooth)

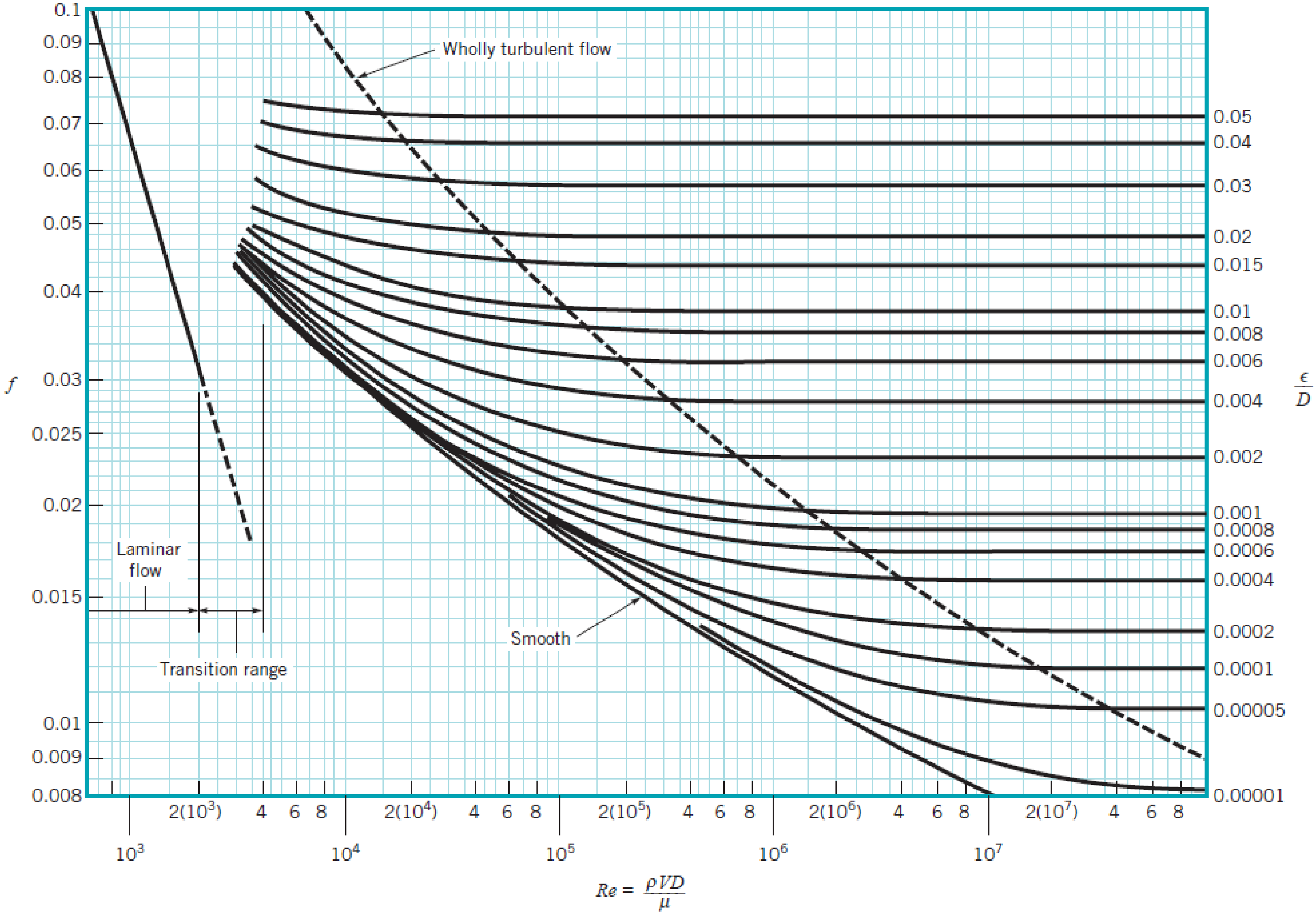


FIGURE 8.20 Friction factor as a function of Reynolds number and relative roughness for round pipes—the Moody chart. (Data from Ref. 7 with permission.)

The Moody chart, on the other hand, is universally valid for all steady, fully developed, incompressible pipe flows.

The following equation from Colebrook is valid for the entire nonlaminar range of the Moody chart

$$\frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\mathrm{Re}\sqrt{f}}\right) \qquad \textbf{(8.35a)}$$

$$\frac{1}{\sqrt{f}} = -1.8 \log\left[\left(\frac{\varepsilon/D}{3.7}\right)^{1.11} + \frac{6.9}{\mathrm{Re}}\right] \qquad \textbf{(8.35b)}$$

EXAMPLE 8.5 Comparison of Laminar or Turbulent Pressure Drop

GIVEN Air under standard conditions flows through a 4.0-mm-diameter drawn tubing with an average velocity of $V = 50$ m/s. For such conditions the flow would normally be turbulent. However, if precautions are taken to eliminate disturbances to the flow (the entrance to the tube is very smooth, the air is dust free, the tube does not vibrate, etc.), it may be possible to maintain laminar flow.

FIND **(a)** Determine the pressure drop in a 0.1-m section of the tube if the flow is laminar.

(b) Repeat the calculations if the flow is turbulent.

SOLUTION

Under standard temperature and pressure conditions the density and viscosity are $\rho = 1.23$ kg/m^3 and $\mu = 1.79 \times 10^{-5}$ N · s/m^2. Thus, the Reynolds number is

$$\text{Re} = \frac{\rho V D}{\mu} = \frac{(1.23 \text{ kg/m}^3)(50 \text{ m/s})(0.004 \text{ m})}{1.79 \times 10^{-5} \text{ N} \cdot \text{s/m}^2} = 13{,}700$$

which would normally indicate turbulent flow.

(a) If the flow were laminar, then $f = 64/\text{Re} = 64/13{,}700 = 0.00467$ and the pressure drop in a 0.1-m-long horizontal section of the pipe would be

$$\Delta p = f \frac{\ell}{D} \frac{1}{2} \rho V^2$$

$$= (0.00467) \frac{(0.1 \text{ m})}{(0.004 \text{ m})} \frac{1}{2} (1.23 \text{ kg/m}^3)(50 \text{ m/s})^2$$

or

$$\Delta p = 0.179 \text{ kPa} \qquad \textbf{(Ans)}$$

COMMENT Note that the same result is obtained from Eq. 8.8:

$$\Delta p = \frac{32 \mu \ell}{D^2} V$$

$$= \frac{32(1.79 \times 10^{-5} \text{ N} \cdot \text{s/m}^2)(0.1 \text{ m})(50 \text{ m/s})}{(0.004 \text{ m})^2}$$

$$= 179 \text{ N/m}^2$$

(b) If the flow were turbulent, then $f = \phi(\text{Re}, \varepsilon/D)$, where from Table 8.1, $\varepsilon = 0.0015$ mm so that $\varepsilon/D = 0.0015 \text{ mm}/4.0 \text{ mm} = 0.000375$. From the Moody chart with $\text{Re} = 1.37 \times 10^4$ and $\varepsilon/D = 0.000375$ we obtain $f = 0.028$. Thus, the pressure drop in this case would be approximately

$$\Delta p = f \frac{\ell}{D} \frac{1}{2} \rho V^2 = (0.028) \frac{(0.1 \text{ m})}{(0.004 \text{ m})} \frac{1}{2} (1.23 \text{ kg/m}^3)(50 \text{ m/s})^2$$

or

$$\Delta p = 1.076 \text{ kPa} \qquad \textbf{(Ans)}$$

An alternate method to determine the friction factor for the turbulent flow would be to use the Colebrook formula, Eq. 8.35*a*. Thus,

$$\frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right) = -2.0 \log\left(\frac{0.000375}{3.7} + \frac{2.51}{1.37 \times 10^4 \sqrt{f}}\right)$$

or

$$\frac{1}{\sqrt{f}} = -2.0 \log\left(1.01 \times 10^{-4} + \frac{1.83 \times 10^{-4}}{\sqrt{f}}\right) \qquad \textbf{(1)}$$

Eq. 8.35*b* provides an alternate form to the Colebrook formula that can be used to solve for the friction factor directly.

$$\frac{1}{\sqrt{f}} = -1.8 \log\left[\left(\frac{\varepsilon/D}{3.7}\right)^{1.11} + \frac{6.9}{\text{Re}}\right] = -1.8 \log\left[\left(\frac{0.000375}{3.7}\right)^{1.11} + \frac{6.9}{1.37 \times 10^4}\right]$$
$$= 0.0289$$

Blasius formula, for turbulent flow in smooth pipes ($\varepsilon/D = 0$) with $\text{Re} < 10^5$ is

$$f = \frac{0.316}{\text{Re}^{1/4}}$$

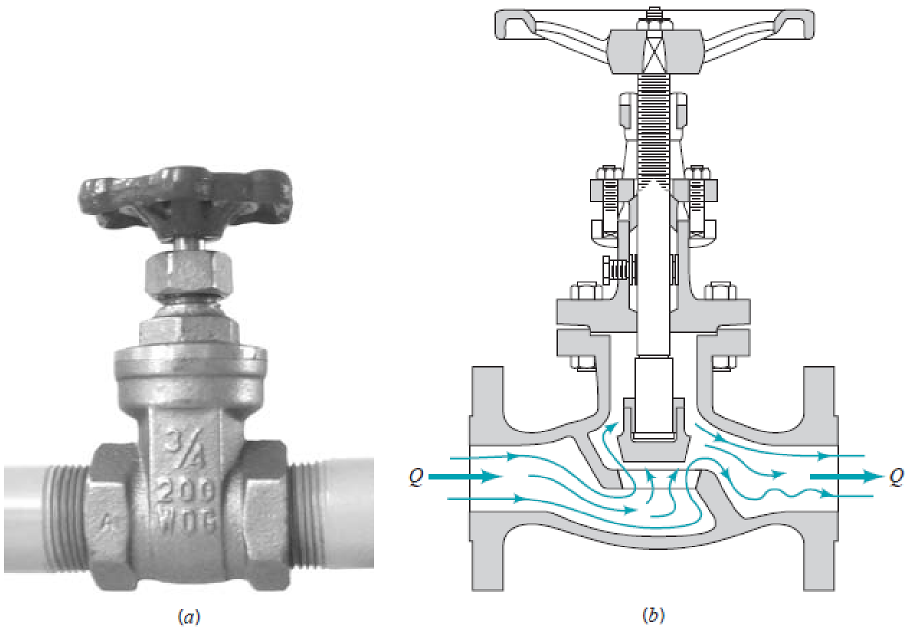
For our case this gives

$$f = 0.316(13{,}700)^{-0.25} = 0.0292$$

8.4.2 Minor Losses

As discussed in the previous section, the head loss in long, straight sections of pipe, the major losses, can be calculated by use of the friction factor obtained from either the Moody chart or the Colebrook equation.

$$K_L = \frac{h_{L\,\text{minor}}}{(V^2/2g)} = \frac{\Delta p}{\frac{1}{2}\rho V^2}$$



■ FIGURE 8.21 **Flow through a valve.**

so that

$$\Delta p = K_L \tfrac{1}{2}\rho V^2$$

or

$$h_{L\,\text{minor}} = K_L \frac{V^2}{2g} \qquad \textbf{(8.36)}$$

The actual value of K_L is strongly dependent on the geometry of the component considered. It may also be dependent on the fluid properties. That is,

$$K_L = \phi(\text{geometry}, \text{Re})$$

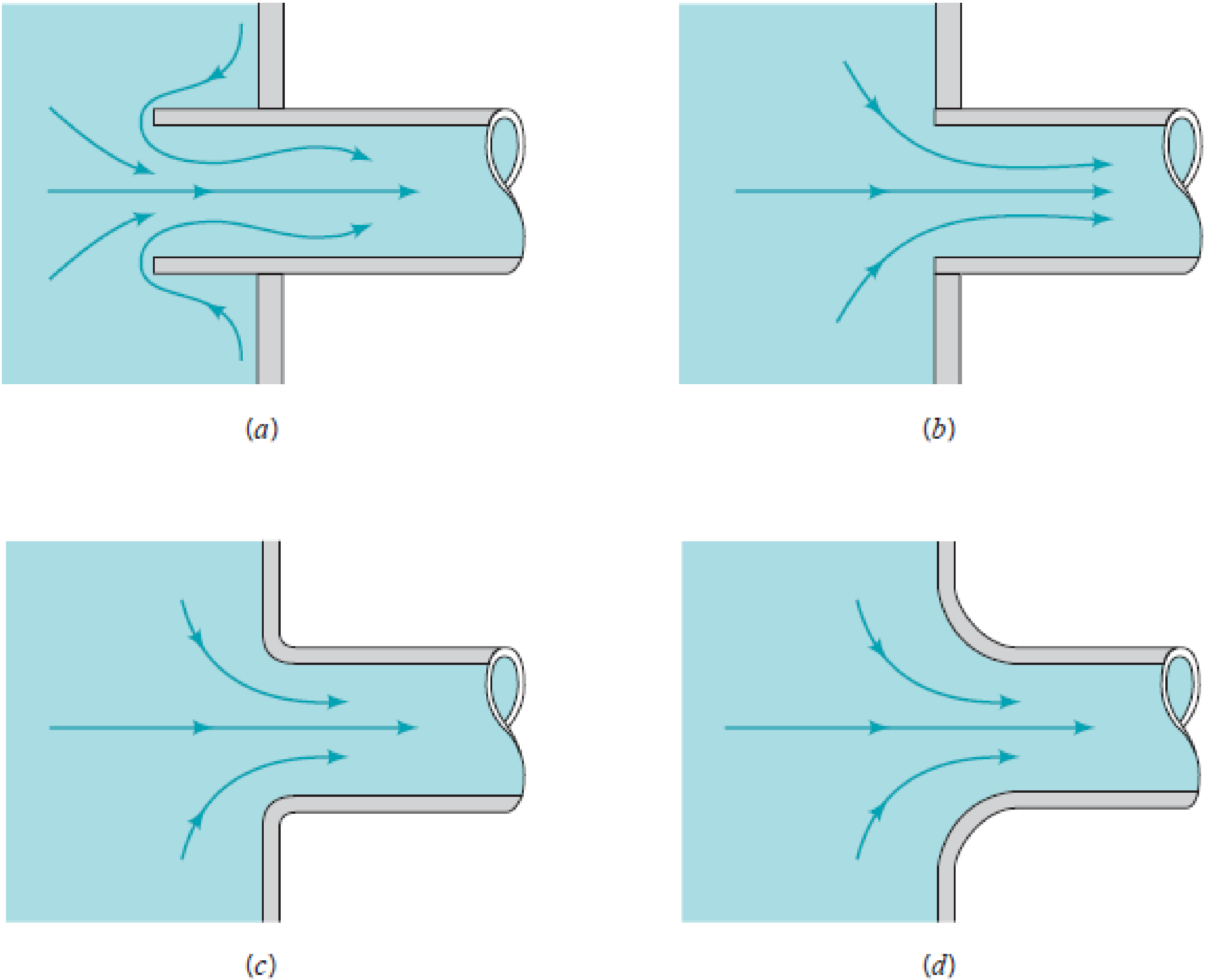
Minor losses are sometimes given in terms of an *equivalent length*, ℓ_{eq}. In this terminology, the head loss through a component is given in terms of the equivalent length of pipe that would produce the same head loss as the component. That is,

$$h_{L\,\text{minor}} = K_L \frac{V^2}{2g} = f \frac{\ell_{eq}}{D} \frac{V^2}{2g}$$

or

$$\ell_{eq} = \frac{K_L D}{f}$$

Minor head losses are often a result of the dissipation of kinetic energy.



■ FIGURE 8.22 Entrance flow conditions and loss coefficient (Refs. 28, 29). (*a*) Reentrant, $K_L = 0.8$, (*b*) sharp-edged, $K_L = 0.5$, (*c*) slightly rounded, $K_L = 0.2$ (see Fig. 8.24), (*d*) well-rounded, $K_L = 0.04$ (see Fig. 8.24).

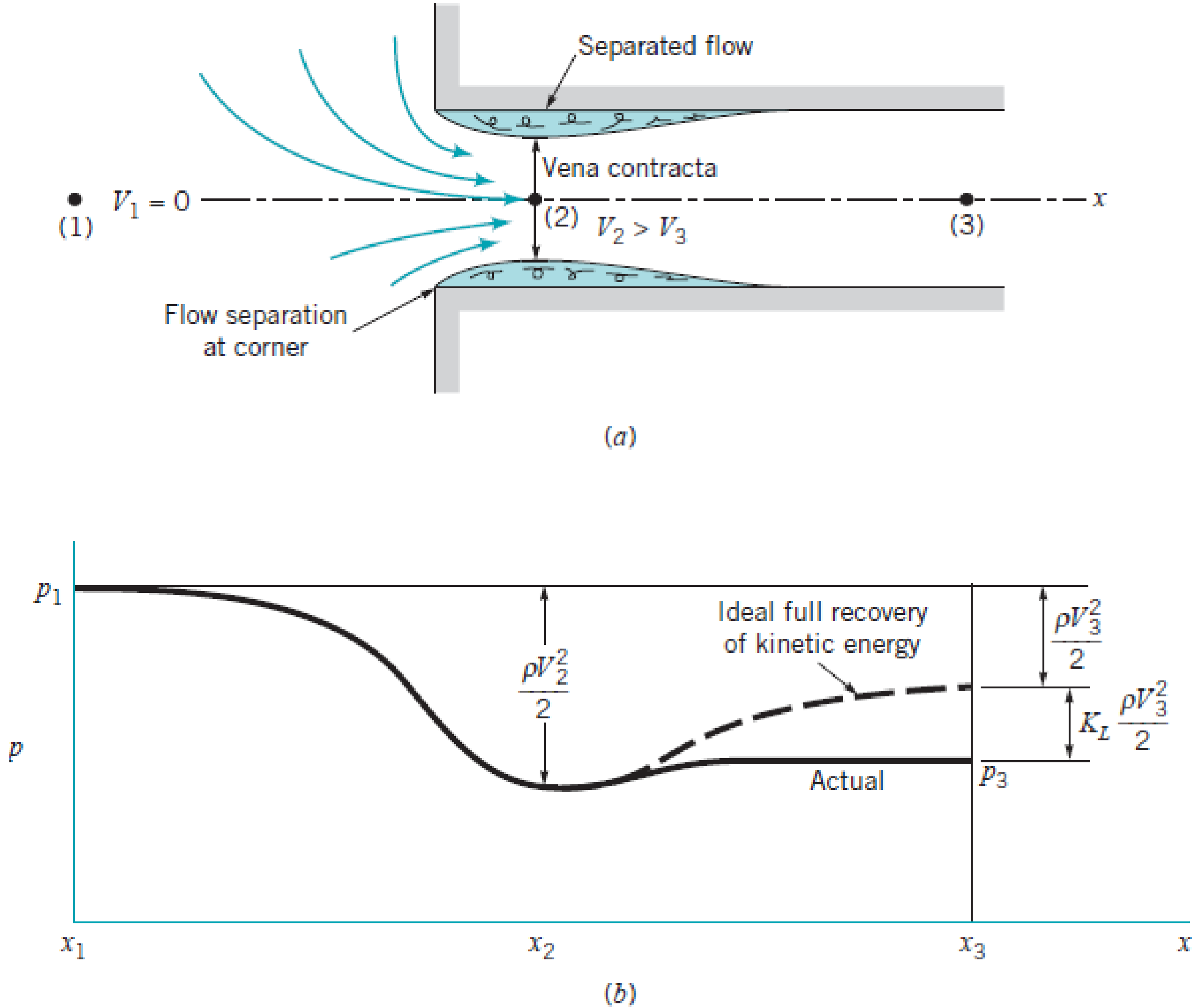


FIGURE 8.23 Flow pattern and pressure distribution for a sharp-edged entrance.

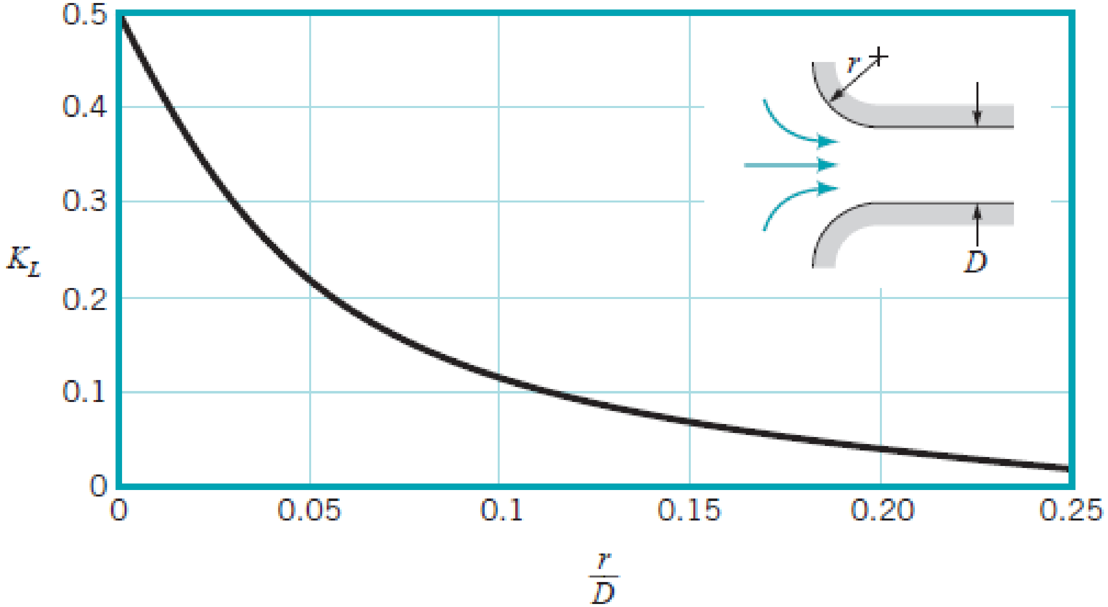


FIGURE 8.24 Entrance loss coefficient as a function of rounding of the inlet edge (Ref. 9).

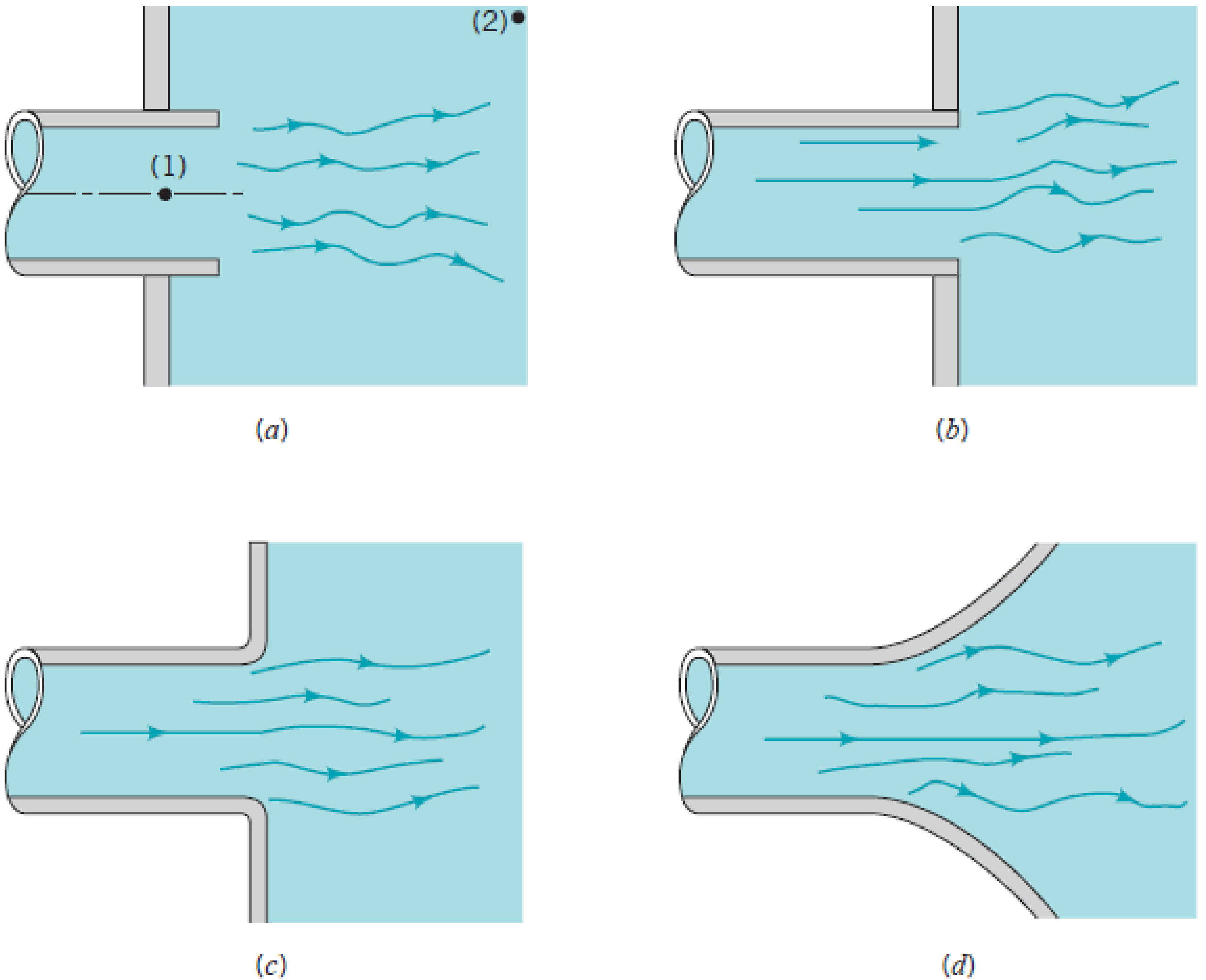
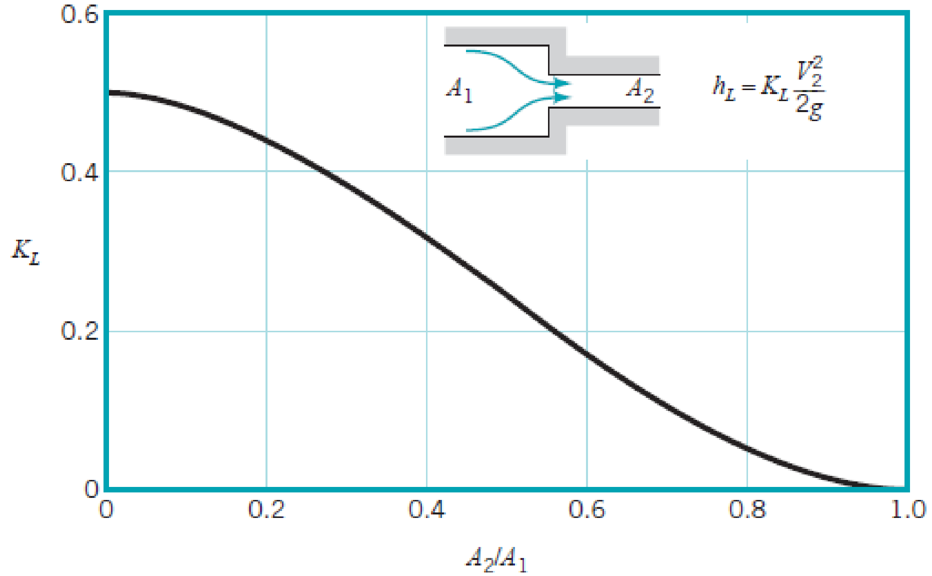
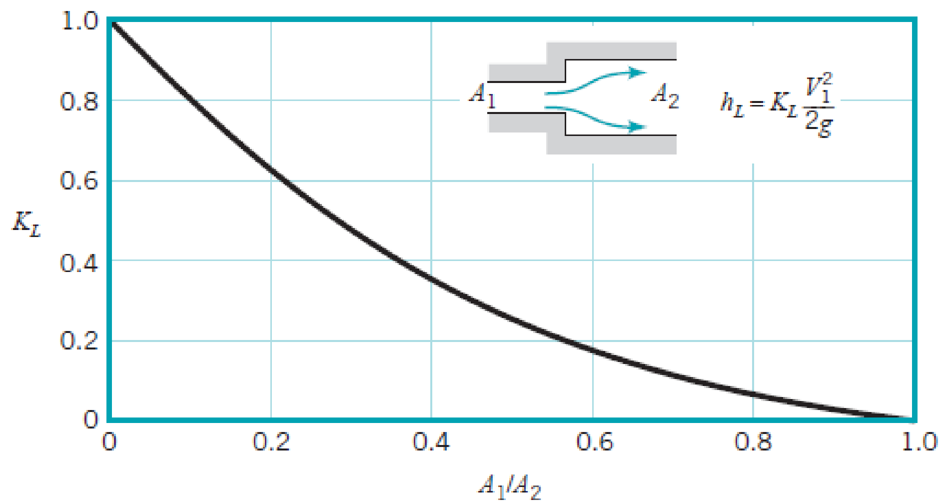


FIGURE 8.25 Exit flow conditions and loss coefficient. (*a*) Reentrant, $K_L = 1.0$, (*b*) sharp-edged, $K_L = 1.0$, (*c*) slightly rounded, $K_L = 1.0$, (*d*) well-rounded, $K_L = 1.0$.



■ FIGURE 8.26 Loss coefficient for a sudden contraction (Ref. 10).



■ FIGURE 8.27 Loss coefficient for a sudden expansion (Ref. 10).

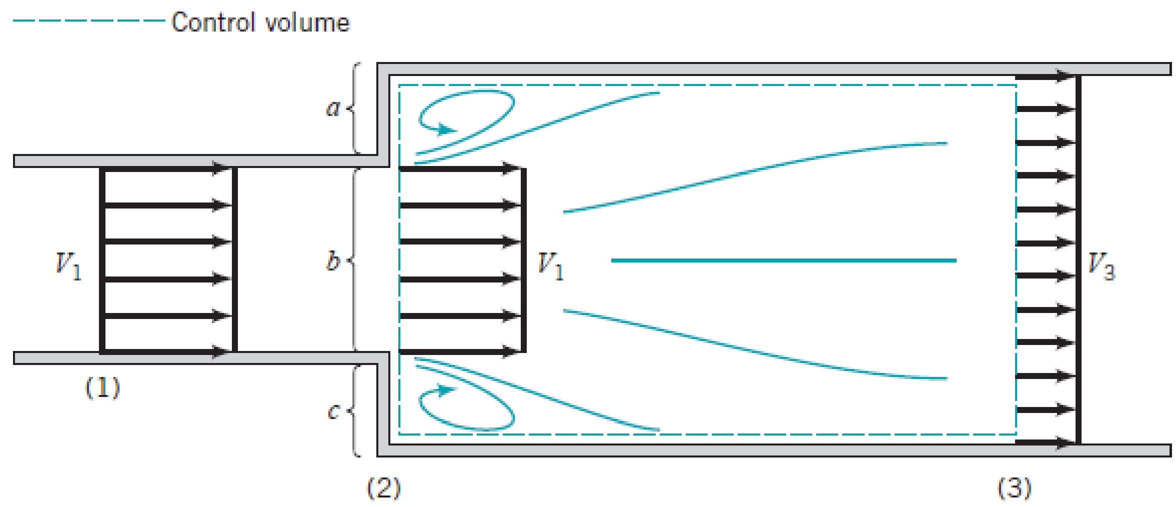


FIGURE 8.28 Control volume used to calculate the loss coefficient for a sudden expansion.

three governing equations (mass, momentum, and energy) are

$$A_1V_1 = A_3V_3$$

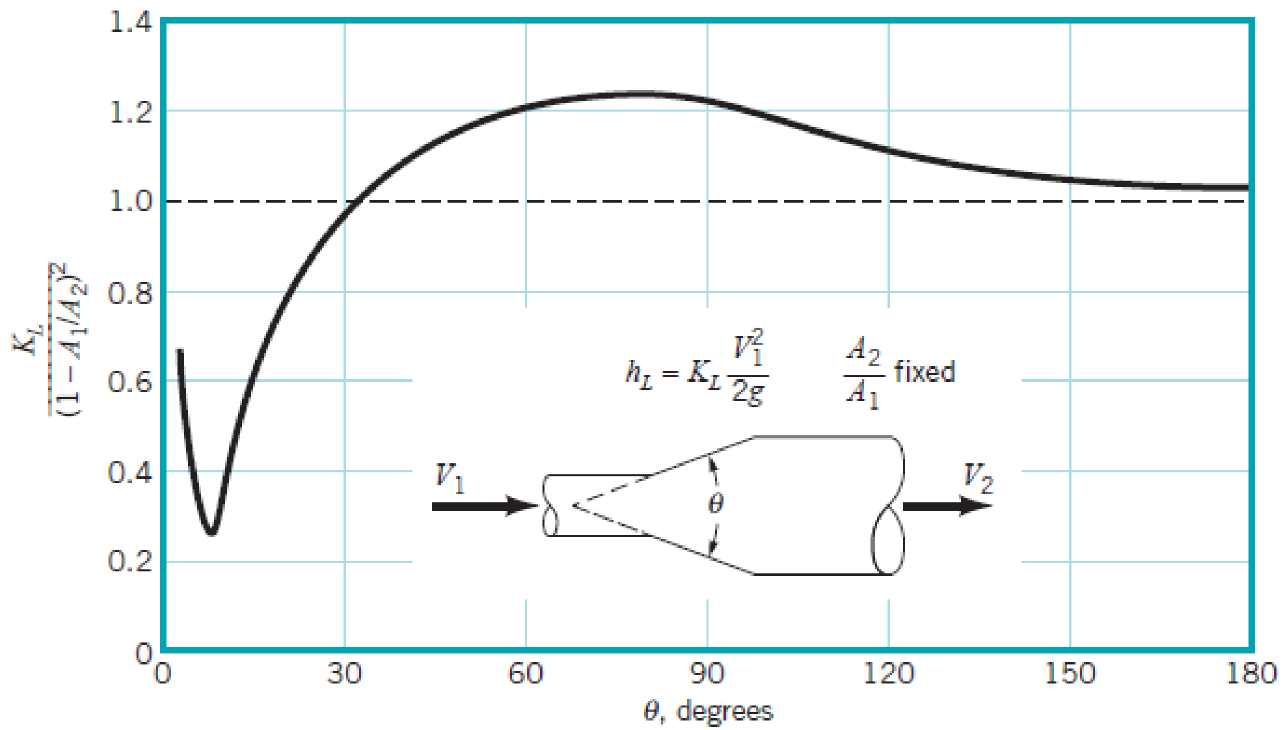
$$p_1A_3 - p_3A_3 = \rho A_3V_3(V_3 - V_1)$$

and

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} = \frac{p_3}{\gamma} + \frac{V_3^2}{2g} + h_L$$

These can be rearranged to give the loss coefficient, $K_L = h_L/(V_1^2/2g)$, as

$$K_L = \left(1 - \frac{A_1}{A_2}\right)^2$$



■ FIGURE 8.29 Loss coefficient for a typical conical diffuser (Ref. 5).

An alternative to using the Moody diagram that avoids any trial-and-error process is made possible by empirically derived formulas. Perhaps the best of such formulas were presented by Swamee and Jain (1976) for pipe flow; an explicit expression that provides an approximate value for the unknown in each category above is as follows:

$$h_L = 1.07\,\frac{Q^2 L}{g D^5}\left\{\ln\left[\frac{e}{3.7D} + 4.62\left(\frac{\nu D}{Q}\right)^{0.9}\right]\right\}^{-2} \qquad \begin{array}{l} 10^{-6} < e/D < 10^{-2} \\ 3000 < \mathrm{Re} < 3\times 10^{8} \end{array} \qquad (7.6.29)$$

$$Q = -0.965\left(\frac{g D^5 h_L}{L}\right)^{0.5}\ln\left[\frac{e}{3.7D} + \left(\frac{3.17\nu^2 L}{g D^3 h_L}\right)^{0.5}\right] \qquad \mathrm{Re} > 2000 \qquad (7.6.30)$$

$$D = 0.66\left[e^{1.25}\left(\frac{LQ^2}{g h_L}\right)^{4.75} + \nu Q^{9.4}\left(\frac{L}{g h_L}\right)^{5.2}\right]^{0.04} \qquad \begin{array}{l} 10^{-6} < e/D < 10^{-2} \\ 5000 < \mathrm{Re} < 3\times 10^{8} \end{array} \qquad (7.6.31)$$

Example

Water at 74°F is transported for 1500 ft in a $1\frac{1}{2}$-in.-diameter wrought iron horizontal pipe with a flow rate of 0.1 ft^3/sec. Calculate the pressure drop over the 1500-ft length of pipe, using (a) the Moody diagram and (b) the alternate method.

Solution

(a) The average velocity is

$$V = \frac{Q}{A} = \frac{0.1}{\pi \times 0.75^2/144} = 8.15 \text{ ft/sec}$$

The Reynolds number is

$$\text{Re} = \frac{VD}{\nu} = \frac{8.15 \times 1.5/12}{10^{-5}} = 1.02 \times 10^5$$

Obtaining e from Fig. 7.13, we have, using $D = 1.5/12$ ft,

$$\frac{e}{D} = \frac{0.00015}{0.125} = 0.0012$$

The friction factor is read from the Moody diagram to be

$$f = 0.023$$

The head loss is calculated as

$$h_L = f\frac{L}{D}\frac{V^2}{2g}$$

$$= \;= 0.023\,\frac{1500}{1.5/12}\,\frac{8.15^2\ \text{ft}^2/\text{sec}^2}{2\times 32.2\ \text{ft/sec}^2} = 280\ \text{ft}$$

This answer is given to two significant numbers since the friction factor is known to at most two significant numbers. The pressure drop is found by Eq. 7.6.22 to be

$$\Delta p = \gamma h_L$$

$$= 62.4\ \text{lb/ft}^3 \times 280\ \text{ft} = 17{,}500\ \text{psf} \quad \text{or} \quad 120\ \text{psi}$$

(b) The alternate method for this Category 1 problem uses Eq. 7.6.29, with $D = 1.5/12 = 0.125$ ft:

$$h_L = 1.07\,\frac{0.1^2\times 1500}{32.2\times 0.125^5}\left\{\ln\left[\frac{0.0012}{3.7} + 4.62\left(\frac{10^{-5}\times 0.125}{0.1}\right)^{0.9}\right]\right\}^{-2}$$

$$= 1.07\times 15{,}265\times 0.01734 = 280\ \text{ft}$$

Drawn tubing of what diameter should be selected to transport 0.002 m^3/s of 20°C water over a 400-m length so that the head loss does not exceed 30 m? (a) Use the Moody diagram and (b) the alternative method.

Solution

(a) In this problem we do not know D. Thus, a trial-and-error solution is anticipated. The average velocity is related to D by

$$V = \frac{Q}{A} = \frac{0.002}{\pi D^2/4} = \frac{0.00255}{D^2}$$

The friction factor and D are related as follows:

$$h_L = f\frac{L}{D}\frac{V^2}{2g}$$

$$30 = f\frac{400}{D}\frac{(0.00255/D^2)^2}{2 \times 9.8}$$

$$\therefore D^5 = 4.42 \times 10^{-6} f$$

The Reynolds number is

$$\text{Re} = \frac{VD}{\nu} = \frac{0.00255D}{D^2 \times 10^{-6}} = \frac{2550}{D}$$

Now, let us simply guess a value for f and check with the relations above and the Moody diagram. The first guess is $f = 0.03$, and the correction is listed in the following table. Note: the second guess is the value for f found from the calculations of the first guess.

f	D(m)	Re	e/D	f (Fig. 7.13)
0.03	0.0421	6.06×10^4	0.000036	0.02
0.02	0.0388	6.57×10^4	0.000039	0.02

The value of $f = 0.02$ is acceptable, yielding a diameter of 3.88 cm. Since this diameter would undoubtedly not be standard, a diameter of

$$D = 4 \text{ cm}$$

would be the tube size selected. This tube would have a head loss less than the limit of $h_L = 30$ m imposed in the problem statement. Any larger-diameter tube would also satisfy this criterion but would be more costly, so it should not be selected.

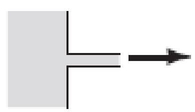
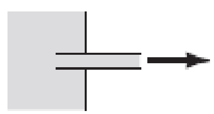
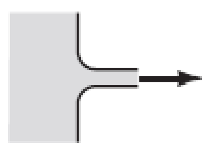
(b) The alternative method for this Category 3 problem uses the explicit relationship (7.6.31). We can directly calculate D to be

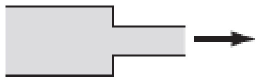
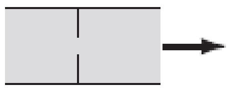
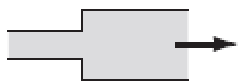
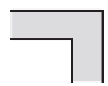
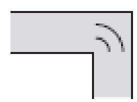
$$D = 0.66\left[(1.5 \times 10^{-6})^{1.25}\left(\frac{400 \times 0.002^2}{9.81 \times 30}\right)^{4.75} + 10^{-6} \times 0.002^{9.4}\left(\frac{400}{9.81 \times 30}\right)^{5.2}\right]^{0.04}$$

$$= 0.66[5.163 \times 10^{-33} + 2.102 \times 10^{-31}]^{0.04} = 0.039 \text{ m}$$

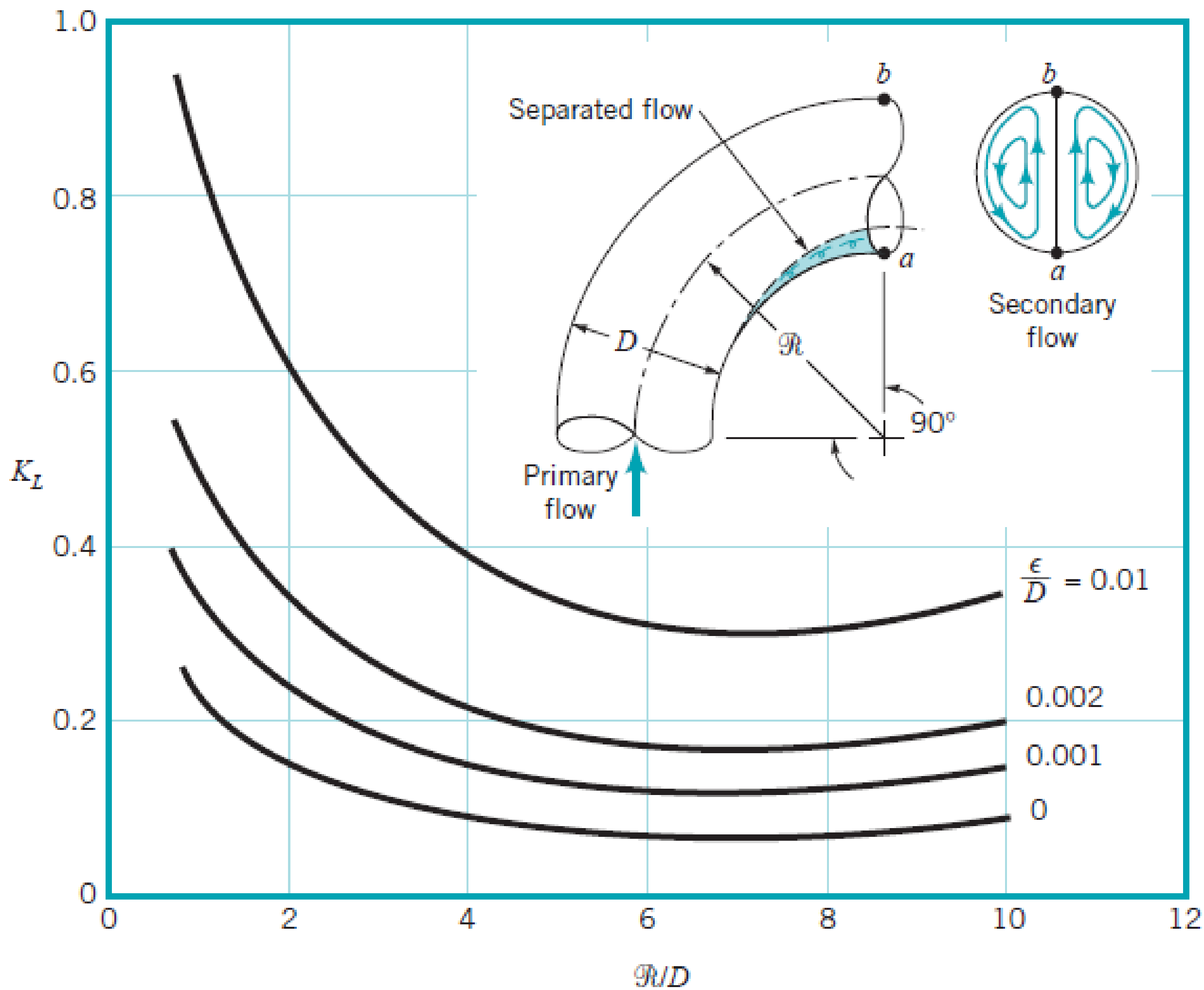
Hence $D = 4$ cm would be the tube size selected. This is the same tube size as that selected using the Moody diagram.

TABLE 7.2 Nominal Loss Coefficients K (Turbulent Flow)[a]

Type of fitting	*Screwed*			*Flanged*		
Diameter	2.5 cm	5 in.	10 cm	5 cm	10 cm	20 cm
Globe valve (fully open)	8.2	6.9	5.7	8.5	6.0	5.8
(half open)	20	17	14	21	15	14
(one-quarter open)	57	48	40	60	42	41
Angle valve (fully open)	4.7	2.0	1.0	2.4	2.0	2.0
Swing check valve (fully open)	2.9	2.1	2.0	2.0	2.0	2.0
Gate valve (fully open)	0.24	0.16	0.11	0.35	0.16	0.07
Return bend	1.5	0.95	0.64	0.35	0.30	0.25
Tee (branch)	1.8	1.4	1.1	0.80	0.64	0.58
Tee (line)	0.9	0.9	0.9	0.19	0.14	0.10
Standard elbow	1.5	0.95	0.64	0.39	0.30	0.26
Long sweep elbow	0.72	0.41	0.23	0.30	0.19	0.15
45° elbow	0.32	0.30	0.29			
Square-edged entrance 			0.5			
Reentrant entrance 			0.8			
Well-rounded entrance 			0.03			

Pipe exit		1.0
	Area ratio	
Sudden contraction[b]	2:1	0.25
	5:1	0.41
	10:1	0.46
	Area ratio A/A_0	
Orifice plate	1.5:1	0.85
	2:1	3.4
	4:1	29
	$\geq$ 6:1	$2.78\left(\frac{A}{A_0} - 0.6\right)^2$
Sudden enlargement[c]		$\left(1 - \frac{A_1}{A_2}\right)^2$
90° miter bend (without vanes)		1.1
(with vanes)		0.2
General contraction	(30° included angle)	0.02
	(70° included angle)	0.07

[a]Values for other geometries can be found in *Technical Paper 410,* The Crane Company, 1957.
[b]Based on exit velocity V_2.
[c]Based on entrance velocity V_1.

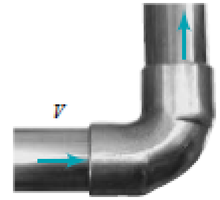


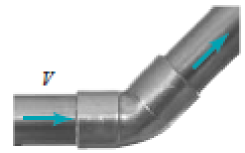
■ **FIGURE 8.30** Character of the flow in a 90° bend and the associated loss coefficient (Ref. 5).

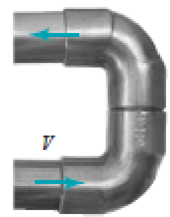
TABLE 8.2

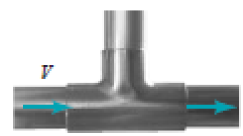
Loss Coefficients for Pipe Components $\left(h_L = K_L \frac{V^2}{2g}\right)$ **(Data from Refs. 5, 10, 27)**

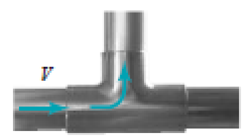
Component	K_L
a. Elbows	
Regular 90°, flanged	0.3
Regular 90°, threaded	1.5
Long radius 90°, flanged	0.2
Long radius 90°, threaded	0.7
Long radius 45°, flanged	0.2
Regular 45°, threaded	0.4
b. 180° return bends	
180° return bend, flanged	0.2
180° return bend, threaded	1.5
c. Tees	
Line flow, flanged	0.2
Line flow, threaded	0.9
Branch flow, flanged	1.0
Branch flow, threaded	2.0
d. Union, threaded	0.08
***e. Valves**	
Globe, fully open	10
Angle, fully open	2
Gate, fully open	0.15
Gate, $\frac{1}{4}$ closed	0.26
Gate, $\frac{1}{2}$ closed	2.1
Gate, $\frac{3}{4}$ closed	17
Swing check, forward flow	2
Swing check, backward flow	∞
Ball valve, fully open	0.05
Ball valve, $\frac{1}{3}$ closed	5.5
Ball valve, $\frac{2}{3}$ closed	210

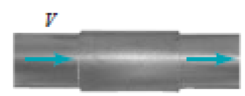












*See Fig. 8.32 for typical valve geometry.

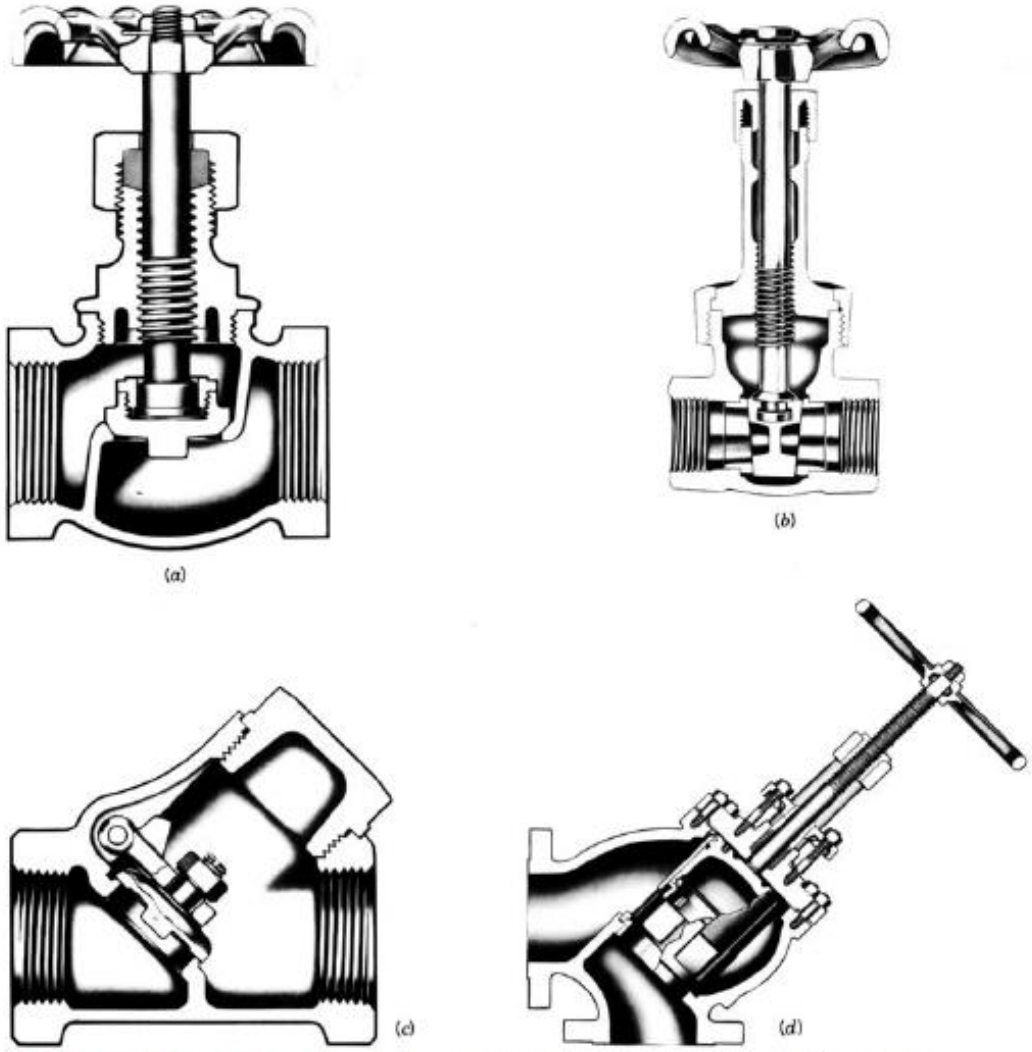


FIGURE 8.32 Internal structure of various valves: (*a*) globe valve, (*b*) gate valve, (*c*) swing check valve, (*d*) stop check valve. (Courtesy of Crane Co., Valve Division.)

8.4.3 Noncircular Conduits

Many of the conduits that are used for conveying fluids are not circular in cross section. Although the details of the flows in such conduits depend on the exact cross-sectional shape, many round pipe results can be carried over, with slight modification, to flow in conduits of other shapes.

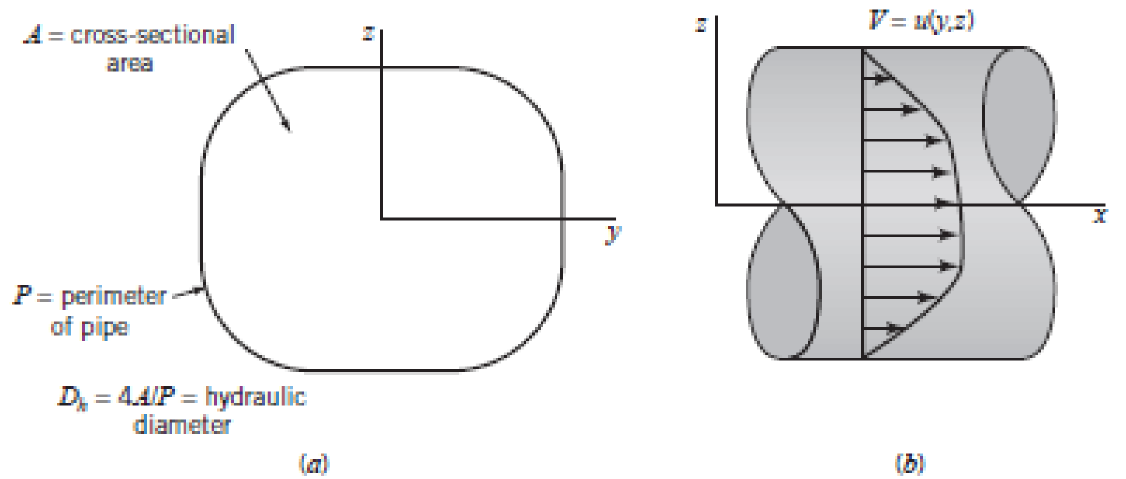


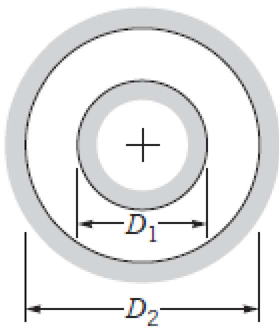
FIGURE 8.34 Noncircular duct.

hydraulic diameter defined as $D_h = 4A/P$

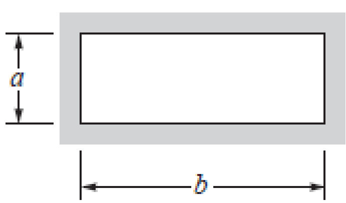
TABLE 8.3

Friction Factors for Laminar Flow in Noncircular Ducts (Data from Ref. 18)

Shape	Parameter	$C = f\,\text{Re}_h$
I. Concentric Annulus	D_1/D_2	
$D_h = D_2 - D_1$	0.0001	71.8
	0.01	80.1
	0.1	89.4
	0.6	95.6
	1.00	96.0



Shape	Parameter	$C = f\,\text{Re}_h$
II. Rectangle	a/b	
$D_h = \dfrac{2ab}{a+b}$	0	96.0
	0.05	89.9
	0.10	84.7
	0.25	72.9
	0.50	62.2
	0.75	57.9
	1.00	56.9



EXAMPLE 8.7 Noncircular Conduit

GIVEN Air at a temperature of 120 °F and standard pressure flows from a furnace through an 8-in.-diameter pipe with an average velocity of 10 ft/s. It then passes through a transition section similar to the one shown in Fig. E8.7 and into a square duct whose side is of length a. The pipe and duct surfaces are smooth ($\varepsilon = 0$). The head loss per foot is to be the same for the pipe and the duct.

FIND Determine the duct size, a.

■ FIGURE E8.7

SOLUTION

We first determine the head loss per foot for the pipe, $h_L/\ell = (f/D)\, V^2/2g$, and then size the square duct to give the same value. For the given pressure and temperature we obtain (from Table B.3) $\nu = 1.89 \times 10^{-4}\ \text{ft}^2/\text{s}$ so that

$$\text{Re} = \frac{VD}{\nu} = \frac{(10\ \text{ft/s})(\frac{8}{12}\ \text{ft})}{1.89 \times 10^{-4}\ \text{ft}^2/\text{s}} = 35{,}300$$

With this Reynolds number and with $\varepsilon/D = 0$ we obtain the friction factor from Fig. 8.20 as $f = 0.022$ so that

$$\frac{h_L}{\ell} = \frac{0.022}{(\frac{8}{12}\text{ ft})} \frac{(10\text{ ft/s})^2}{2(32.2\text{ ft/s}^2)} = 0.0512$$

Thus, for the square duct we must have

$$\frac{h_L}{\ell} = \frac{f}{D_h}\frac{V_s^2}{2g} = 0.0512 \qquad \textbf{(1)}$$

where

$$D_h = 4A/P = 4a^2/4a = a \quad \text{and}$$

$$V_s = \frac{Q}{A} = \frac{\frac{\pi}{4}\left(\frac{8}{12}\text{ ft}\right)^2(10\text{ ft/s})}{a^2} = \frac{3.49}{a^2} \qquad \textbf{(2)}$$

is the velocity in the duct.

By combining Eqs. 1 and 2 we obtain

$$0.0512 = \frac{f}{a}\frac{(3.49/a^2)^2}{2(32.2)}$$

or

$$a = 1.30 f^{1/5}$$

where a is in feet. Similarly, the Reynolds number based on the hydraulic diameter is

$$\text{Re}_h = \frac{V_s D_h}{\nu} = \frac{(3.49/a^2)a}{1.89 \times 10^{-4}} = \frac{1.85 \times 10^4}{a} \qquad \textbf{(4)}$$

We have three unknowns (a, f, and Re_h) and three equations—Eqs. 3, 4, and either in graphical form the Moody chart (Fig. 8.20) or the Colebrook equation (Eq. 8.35a).

If we use the Moody chart, we can use a trial and error solution as follows. As an initial attempt, assume the friction factor for the duct is the same as for the pipe. That is, assume $f = 0.022$. From Eq. 3 we obtain $a = 0.606$ ft, while from Eq. 4 we have $\text{Re}_h = 3.05 \times 10^4$. From Fig. 8.20, with this Reynolds number and the given smooth duct we obtain $f = 0.023$, which does not quite agree with the assumed value of f. Hence, we do not have the solution. We try again, using the latest calculated value of $f = 0.023$ as our guess. The calculations are repeated until the guessed value of f agrees with the value obtained from Fig. 8.20. The final result (after only two iterations) is $f = 0.023$, $\text{Re}_h = 3.03 \times 10^4$, and

$$a = 0.611\text{ ft} = 7.34\text{ in.} \qquad \textbf{(Ans)}$$

COMMENTS Alternatively, we can use the Colebrook equation (rather than the Moody chart) to obtain the solution as follows. For a smooth pipe ($\varepsilon/D_h = 0$) the Colebrook equation, Eq. 8.35a, becomes

$$\frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{\varepsilon/D_h}{3.7} + \frac{2.51}{\mathrm{Re}_h\sqrt{f}}\right)$$

$$= -2.0 \log\left(\frac{2.51}{\mathrm{Re}_h\sqrt{f}}\right) \qquad \textbf{(5)}$$

where from Eq. 3,

$$f = 0.269\, a^5 \qquad \textbf{(6)}$$

If we combine Eqs. 4, 5, and 6 and simplify, Eq. 7 is obtained for a.

$$1.928\, a^{-5/2} = -2 \log(2.62 \times 10^{-4}\, a^{-3/2}) \qquad \textbf{(7)}$$

By using a root-finding technique on a computer or calculator, the solution to Eq. 7 is determined to be $a = 0.614$ ft, in agreement (given the accuracy of reading the Moody chart) with that obtained by the trial and error method given above.

Note that the length of the side of the equivalent square duct is $a/D = 7.34/8 = 0.918$, or approximately 92% of the diameter of the equivalent duct. It can be shown that this value, 92%, is a very good approximation for any pipe flow—laminar or turbulent. The cross-sectional area of the duct ($A = a^2 = 53.9$ in.^{2}) is greater than that of the round pipe ($A = \pi D^2/4 = 50.3$ in.^{2}). Also, it takes less material to form the round pipe (perimeter $= \pi D = 25.1$ in.) than the square duct (perimeter $= 4a = 29.4$ in.). Circles are very efficient shapes.

TABLE 8.4
Pipe Flow Types

Variable	Type I	Type II	Type III
a. Fluid			
Density	Given	Given	Given
Viscosity	Given	Given	Given
b. Pipe			
Diameter	Given	Given	Determine
Length	Given	Given	Given
Roughness	Given	Given	Given
c. Flow			
Flowrate or Average Velocity	Given	Determine	Given
d. Pressure			
Pressure Drop or Head Loss	Determine	Given	Given

SEE EXAMPLES 8.9, 8.10, AND 8.11 FROM BOOK

EXAMPLE 8.8 FROM DIFFERENT BOOK

If the flow rate through a 10-cm-diameter wrought iron pipe (Fig. E7.15) is 0.04 m^3/s, find the difference in elevation H of the two reservoirs.

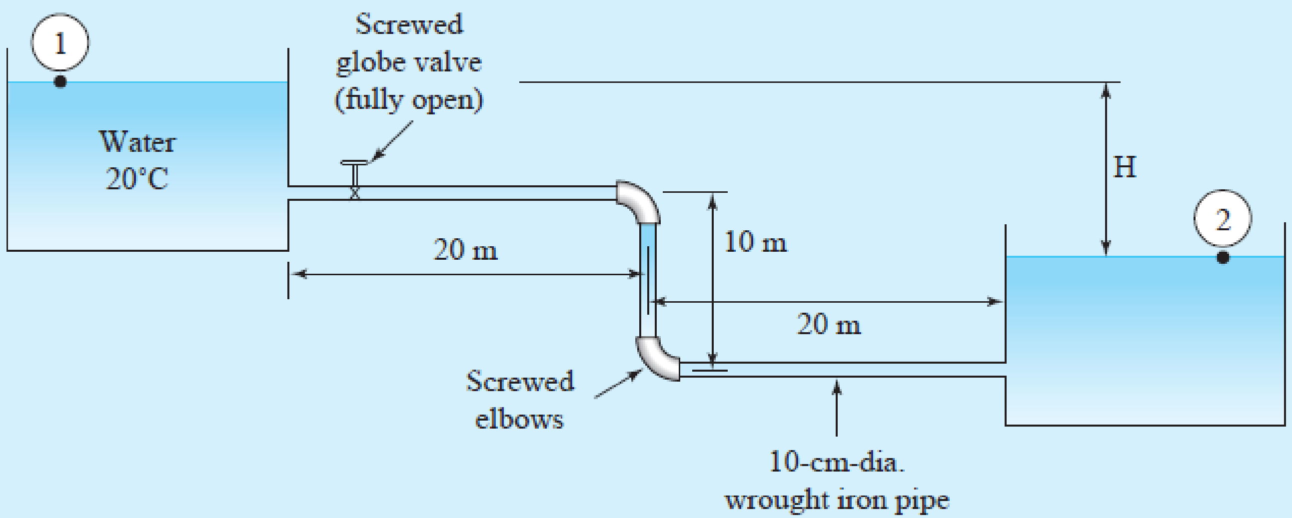


Fig. E7.15

Solution

The energy equation written for a control volume that contains the two reservoir surfaces (see Eq. 4.5.17), where $V_1 = V_2 = 0$ and $p_1 = p_2 = 0$, is

$$0 = z_2 - z_1 + h_L$$

Thus, letting $z_1 - z_2 = H$, we have

$$H = (K_{\text{entrance}} + K_{\text{valve}} + 2K_{\text{elbow}} + K_{\text{exit}})\frac{V^2}{2g} + f\frac{L}{D}\frac{V^2}{2g}$$

The average velocity, Reynolds number, and relative roughness are

$$V = \frac{Q}{A} = \frac{0.04}{\pi \times 0.05^2} = 5.09 \text{ m/s}$$

$$\text{Re} = \frac{VD}{\nu} = \frac{5.09 \times 0.1}{10^{-6}} = 5.09 \times 10^5$$

$$\frac{e}{D} = \frac{0.046}{100} = 0.00046$$

From the Moody diagram we find that

$$f = 0.0173$$

Using the loss coefficients from Table 7.2 for an entrance, a globe valve, screwed 10-cm-diameter standard elbows, and an exit there results

$$H = (0.5 + 5.7 + 2 \times 0.64 + 1.0)\frac{5.09^2}{2 \times 9.8} + 0.0173\frac{50}{0.1}\frac{5.09^2}{2 \times 9.8}$$

$$= 11.2 + 11.4 = 22.6 \text{ m}$$

Note: The minor losses are about equal to the frictional losses as expected, since there are five minor loss elements in 500 diameters of pipe length.

EXAMPLE 8.9 FROM DIFFERENT BOOK

Estimate the flow rate in the simple piping system of Fig. E7.18a if the pump characteristic curves are as shown in Fig. E7.18b. Also, find the pump power requirement.

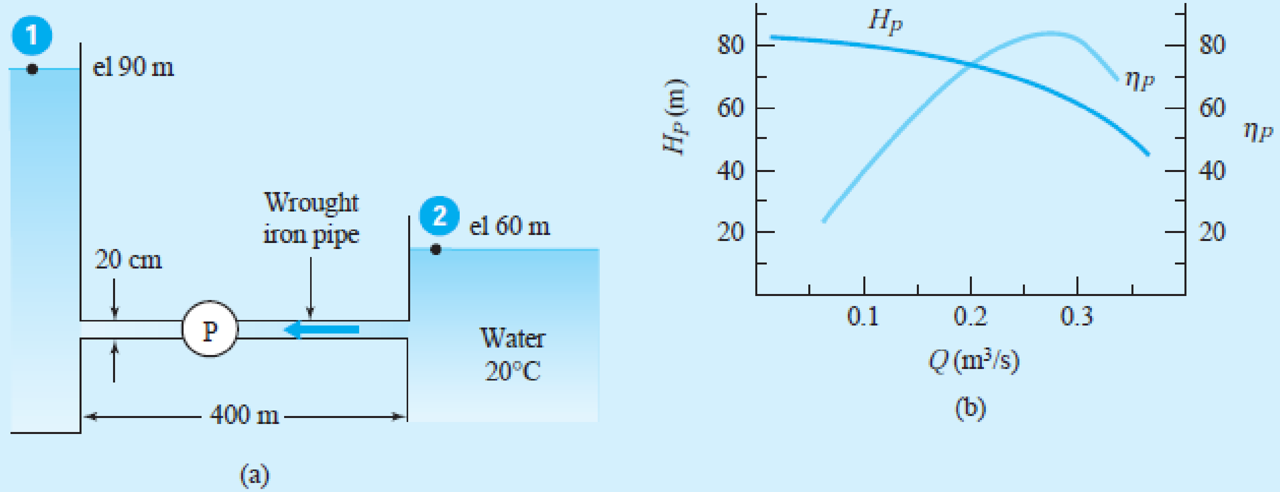


Fig. E7.18

Solution

We will assume that the Reynolds number is sufficiently large that the flow is completely turbulent. So, using $e/D = 0.046/200 = 0.00023$, the friction factor from the Moody diagram is

$$f = 0.014$$

The energy equation (see Eq. 7.6.40), with $H_P = -\dot{W}_S/\dot{m}g$, applied between the two surfaces, yields

$$H_P = \cancelto{0}{\frac{V_2^2 - V_1^2}{2g}} + z_2 - z_1 + \cancelto{0}{\frac{p_2 - p_1}{\gamma}} + h_L$$

or

$$H_P = 90 - 60 + \left(K_{\text{entrance}} + K_{\text{exit}} + f\frac{L}{D}\right)\frac{V^2}{2g}$$

$$= 30 + \left(0.5 + 1.0 + 0.014\,\frac{400}{0.2}\right)\frac{Q^2}{2 \times 9.8 \times [\pi \times 0.1^2]^2}$$

$$= 30 + 1520Q^2$$

This equation, the system demand curve, and the characteristic curve $H_P(Q)$ of the pump are now solved simultaneously by trial and error. Actually, the curve could be plotted on the same graph as the characteristic curve, and the point of intersection, the operating point, would provide Q. Try $Q = 0.2\ \text{m}^3/\text{s}$: $(H_P)_{\text{energy}} = 91$ m, $(H_P)_{\text{char}} \cong 75$ m. Try $Q = 0.15\ \text{m}^3/\text{s}$: $(H_P)_{\text{energy}} = 64$ m, $(H_P)_{\text{char}} \cong 75$ m. Try $Q = 0.17\ \text{m}^3/\text{s}$: $(H_P)_{\text{energy}} = 74$ m, $(H_P)_{\text{char}} \cong 76$ m. This is our solution. We have

$$Q = 0.17\ \text{m}^3/\text{s}$$

Check the Reynolds number: $\text{Re} = DQ/A\nu = 0.2 \times 0.17/(\pi \times 0.1^2 \times 10^{-6}) = 1.08 \times 10^6$. This is sufficiently large, but marginally so.

The power requirement of the pump is given by Eq. 4.5.26:

$$\dot{W}_P = \frac{Q\gamma H_P}{\eta_P}$$

$$= \frac{0.17 \times 9800 \times 75}{0.65} = 198\,000\ \text{W} \quad \text{or} \quad 198\ \text{kW}$$

where the efficiency $\eta_P = 0.63$ is found from the characteristic curve at $Q = 0.17\ \text{m}^3/\text{s}$.

Note: Since $L/D > 1000$, minor losses due to the entrance and exit could have been neglected.

8.5.2 Multiple Pipe Systems

In many pipe systems there is more than one pipe involved. The complex system of tubes in our lungs (beginning as shown by the figure in the margin, with the relatively large-diameter trachea and ending in tens of thousands of minute bronchioles after numerous branchings) and the maze of pipes in a city's water distribution system are typical of such systems. The governing mechanisms for the flow in *multiple pipe systems* are the same as for the single pipe systems discussed in this chapter. However, because of the numerous unknowns involved, additional complexities may arise in solving for the flow in multiple pipe systems. Some of these complexities are discussed in this section.

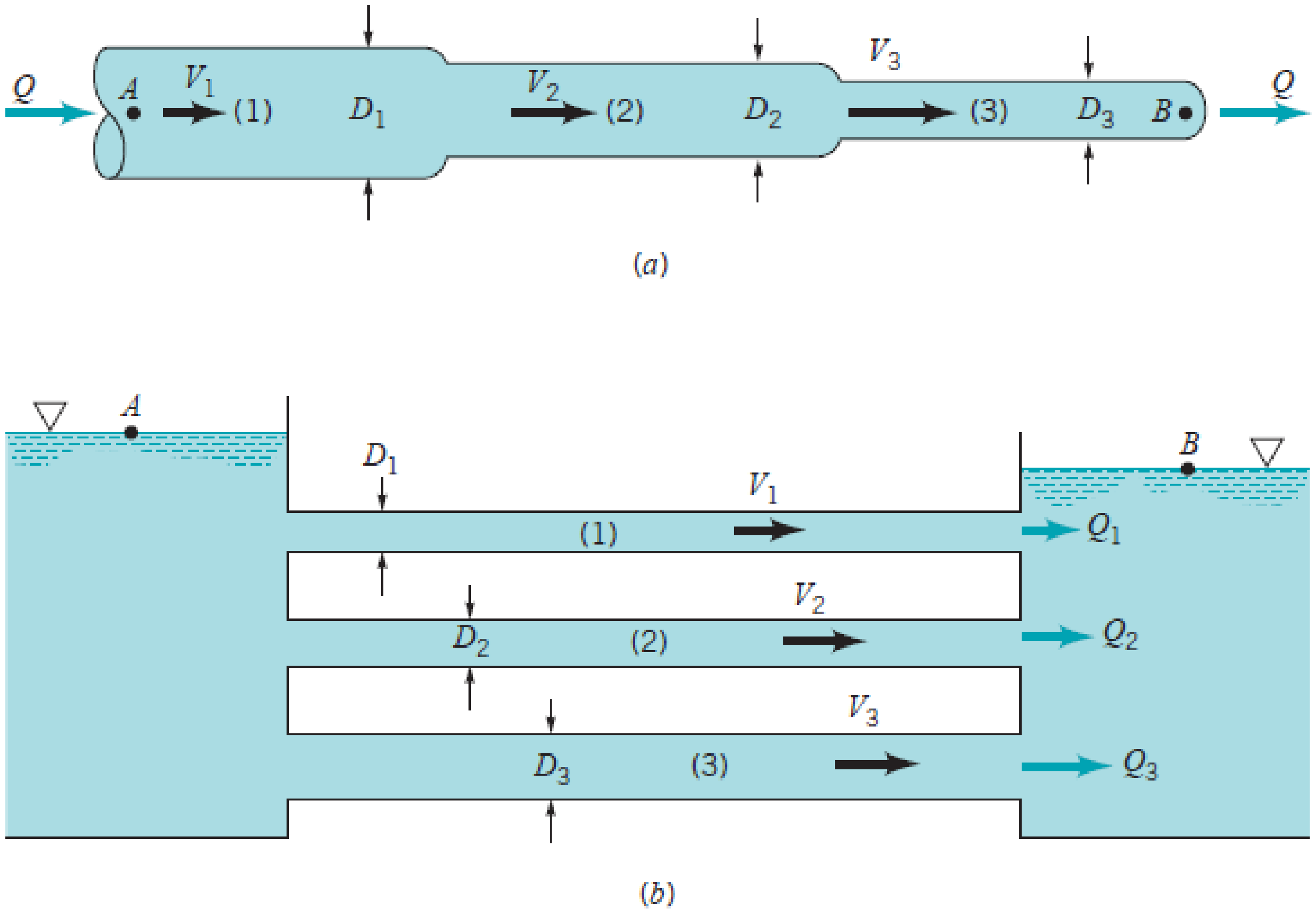


FIGURE 8.35 (*a*) Series and (*b*) parallel pipe systems.

The governing equations can be written as follows:

pipes in *series*,

$$Q_1 = Q_2 = Q_3$$

and

$$h_{L_{A-B}} = h_{L_1} + h_{L_2} + h_{L_3}$$

Thus, the governing equations for parallel pipes are

$$Q = Q_1 + Q_2 + Q_3$$

and

$$h_{L_1} = h_{L_2} = h_{L_3}$$

Another type of multiple pipe system called a *loop* is shown in Fig. 8.36. In this case the flowrate through pipe (1) equals the sum of the flowrates through pipes (2) and (3), or $Q_1 = Q_2 + Q_3$. As can be seen by writing the energy equation between the surfaces of each reservoir, the head loss for pipe (2) must equal that for pipe (3), even though the pipe sizes and flowrates may be different for each. That is,

$$\frac{p_A}{\gamma} + \frac{V_A^2}{2g} + z_A = \frac{p_B}{\gamma} + \frac{V_B^2}{2g} + z_B + h_{L_1} + h_{L_2}$$

for a fluid particle traveling through pipes (1) and (2), while

$$\frac{p_A}{\gamma} + \frac{V_A^2}{2g} + z_A = \frac{p_B}{\gamma} + \frac{V_B^2}{2g} + z_B + h_{L_1} + h_{L_3}$$

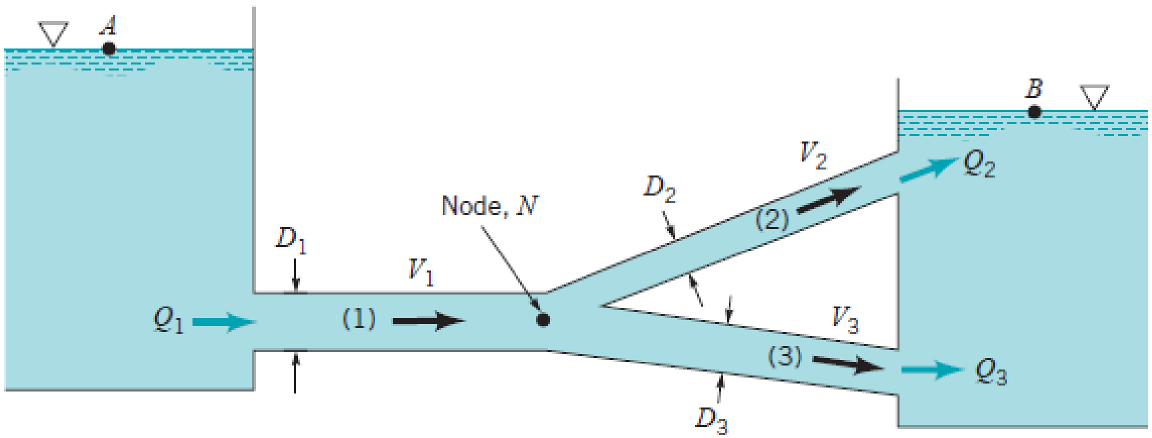


FIGURE 8.36 Multiple pipe loop system.

Example from William S. Jenna

Benzene flows through a 12-nominal, schedule 80 wrought iron pipe. The pressure drop measured at points 350 m apart is 34 kPa. Determine the flow rate through the pipe.

Solution

The method and calculations in the preceding example, where pressure drop is unknown, are quite straightforward. In this example, volume flow rate Q is the unknown; therefore, velocity V and friction factor f are also unknown. A trial-and-error solution method will be required to solve this problem, but the technique is simple. From property and data tables, we determine the following:

Benzene	$\rho = 0.876(1\,000)\ \text{kg/m}^3$ $\mu = 0.601 \times 10^{-3}\ \text{N} \cdot \text{s/m}^2$	[Table A.5]
12-nom, sch 80	$D = 28.89\ \text{cm}$ $A = 655.50\ \text{cm}^2$	[Table C.1]
Wrought iron	$\varepsilon = 0.004\ 6\ \text{cm}$	[Table 5.2]

The continuity equation is

$$Q = A_1V_1 = A_2V_2$$

Because diameter is constant, $A_1 = A_2$ and so $V_1 = V_2$. The Bernoulli equation with the friction factor term applies:

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + \sum \frac{fL}{D_h}\frac{V^2}{2g}$$

Evaluating properties,

$V_1 = V_2$ (from continuity)

$z_1 = z_2$ (for a horizontal pipe)

$L = 350\ \text{m}$ (given)

$p_1 - p_2 = 34\ \text{kPa}$ (given)

The Bernoulli equation becomes

$$p_1 - p_2 = \frac{fL}{D}\frac{\rho V^2}{2}$$

In problems of this type, where volume flow rate Q is the unknown, it is convenient to solve the Bernoulli equation for velocity:

$$V = \sqrt{\frac{2D(p_1 - p_2)}{\rho f L}}$$

Trial and error is necessary because velocity is unknown, but it is needed to calculate the Reynolds number, which in turn is needed to determine the friction factor. Substituting yields

$$V = \sqrt{\frac{2(0.288\ 9)(34\ 000)}{876 f(350)}}$$

or

$$V = \frac{0.253}{\sqrt{f}} \qquad \text{(i)}$$

The Reynolds number is

$$\mathrm{Re} = \frac{\rho V D}{\mu} = \frac{876V(0.288\ 9)}{0.601 \times 10^{-3}}$$

$$= 4.21 \times 10^{5} V \tag{ii}$$

In addition, we have

$$\frac{\varepsilon}{D} = \frac{0.004\ 6\ \text{cm}}{28.89\ \text{cm}} = 0.000\ 16$$

With reference to the Moody diagram, Figure 5.14, we know that our operating point is somewhere along the $\varepsilon/D = 0.000\ 16$ line. As our first estimate, we assume a value for the friction factor that corresponds to the fully turbulent value for this line. Thus we have the following first trial:

$f = 0.013$ (fully turbulent value for $\varepsilon/D = 0.000\ 16$)

Then

$$V = \frac{0.253}{\sqrt{f}} = \frac{0.253}{\sqrt{0.013}} = 2.22 \text{ m/s} \qquad \text{(from Equation i)}$$

$$\text{Re} = 4.21 \times 10^5 (2.22) \qquad \text{(from Equation ii)}$$

Thus

$$\left.\begin{aligned} \text{Re} &= 9.35 \times 10^5 \\ \frac{\varepsilon}{D} &= 0.000\ 16 \end{aligned}\right\} f = 0.014\ 5 \qquad \text{(from Figure 5.14)}$$

For the second trial,

$$f = 0.014\ 5 \quad V = \frac{0.253}{\sqrt{0.014\ 5}} = 2.10 \text{ m/s}$$

$$\left.\begin{aligned} Re &= 4.21 \times 10^5(2.10) = 8.85 \times 10^5 \\ \frac{\varepsilon}{D} &= 0.000\ 16 \end{aligned}\right\} \quad \begin{aligned} &f \approx 0.014\ 5 \\ &\text{(from Figure 5.14)} \end{aligned}$$

which agrees with our assumed value. So

$$V = 2.10 \text{ m/s}$$

$$Q = AV = 655.50 \times 10^{-4}(2.10)$$

or $Q = 0.138 \text{ m}^3/\text{s}$

The method converges very rapidly. Seldom are more than two trials necessary.