CMPE471 Tutorial Booklet

Theory of Automata

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CMPE471 – Tutorial 1

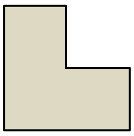
Q1. Using the principle of mathematical induction, prove that:

a: $\sum_{i=1}^{n} i^3 = (\sum_{i=1}^{n} i)^2$

b: $1 + 2^n < 3^n, for\ all\ n \geq 2, n \in N$

Q2. Prove that, if $(1 + x) > 0$ then, $(1 + x)^n \geq 1 + nx, for\ all\ n \in N$

Q3. A triomino is an L-shaped figure formed by juxtaposition of three unit squares:



An arrangement if triominoes is a tiling of a shape if it covers the shape exactly without overlaps. In tiling step, all 90 degrees rotations are possible. Prove that any $2^n \times 2^n$ grid that is missing one square can be tiled with triominoes, regardless of where the missing square is.

Q4. Let Σ be an alphabet. Prove that the relation:

R = { (x,y) | x is a prefix of y } is a partial ordering relation of Σ^*

Q5. Let $\Sigma = \{a,b\}$, in each of the following three cases, give an example of languages L_1 and L_2 that satisfy the stated conditions. (Note that $L_1 \subseteq \Sigma^*$, and $L_2 \subseteq \Sigma^*$).

I. $L_1L_2 = L_2L_1$, such that $L_1 \subseteq L_2$, $L_2 \not\subset L_1$, and $L_1 \neq \{\varepsilon\}$
II. $(L_1 \cup L_2)^* \neq L_1^* \cup L_2^*$
III. $(L_1 \cup L_2)^* = L_1^* \cup L_2^*$, such that $L_1 \not\subset L_2$, and $L_2 \not\subset L_1$

Q6. If r is a real number not equal to 1, prove the following formula:

$$\sum_{i=0}^{n} r^i = (1 - r^{n+1}) / (1 - r) \qquad \text{for } n \geq 0$$

Q7. In each case give an example of languages L_1 and L_2 that satisfy the stated conditions.

i) $L_1L_2 = L_2L_1$, $L_1 \subseteq L_2$, $L_2 \not\subset L_1$, $L_1 \neq \{\lambda\}$.
ii) $L_1L_2 = L_2L_1$, $L_1 \not\subset L_2$, $L_2 \not\subset L_1$, $L_1 \neq \{\lambda\}$, $L_2 \neq \{\lambda\}$.
iii) $L_1 \subseteq \{a,b\}^*$, $L_2 \subseteq \{a,b\}^*$, $(L_1 \cup L_2)^* \neq L_1^* \cup L_2^*$.
iv) $L_1 \subseteq \{a,b\}^*$, $L_2 \subseteq \{a,b\}^*$, $(L_1 \cup L_2)^* = L_1^* \cup L_2^*$, $L_1 \not\subset L_2$, $L_2 \not\subset L_1$.

CMPE471 – Tutorial 2

Q1. Let $L_1 = \{w \in \{a,b\}^* \mid n_a(w) > n_b(w)\}$ and $L_2 = \{w \in \{a,b\}^* \mid n_a(w) < n_b(w)\}$. Let $L = \{a,b\}^* - (L_1 \cup L_2)^*$. Describe L and justify your answer.

Q2. Construct CFGs for the following languages:

i) $L = \{a^j b^k a^n \mid k = j + n\}$

ii) $L = \{(a^n b^{n+2})^{3i} \mid n \geq 1, i \geq 0\}$
Assume that each repeatition of paranthesis is independent from others.

iii) $L = \{a^k b^{2k} c^n \mid k, n > 0\}$

iv) $L = \{a^j b^k c^n \mid 0 \leq j + k \leq n\}$

v) $L = \{a,b\}^* - \{a^n b^n \mid n \geq 0\}$

Q3. Find the languages generated by the following CFGs:

i) $S \rightarrow aSbb \mid aSb \mid aS \mid \varepsilon$

ii) $S \rightarrow aScc \mid aAcc$
$A \rightarrow bAc \mid bc$

iii) $S \rightarrow aSb \mid aSbb \mid aSbbb \mid \varepsilon$

iv) $S \rightarrow aSbS \mid bSaS \mid \varepsilon$

v) $S \rightarrow aS \mid cS \mid bA \mid \varepsilon$
$A \rightarrow aS \mid cS \mid \varepsilon$

vi) $S \rightarrow aSbb \mid A$
$A \rightarrow cA \mid c$

Q4. Show that the grammar S → aSb | bSa | SS | ε is ambiguous.

Q5. Consider the CFG with the following products. Find the derivation tree of *aababbbbb*.

$S \rightarrow AB \mid \varepsilon$

$A \to aB$
$B \to Sb$

CMPE471 – Tutorial 3

Q1. Consider the regular grammar G = ({S,A}, {a,b}, P, S) where P consists of the following productions:

$$G: S \rightarrow bS \mid aA \mid \varepsilon$$
$$A \rightarrow bA \mid aS$$

a) Describe the language generated by this grammar.

b) Give a minimal regular expression for L(G).

c) Find an equivalent grammar to G.

Q2. Give minimal regular expressions for the following languages:

a) The set of all strings over {a, b, c} that starts and end with the same symbol.

b) The set of all strings over {a, b} in which every pair of adjacent a's appears before any pair of adjacent b's.

c) The set of all strings over {0, 1} except for the two strings 11 and 111.

d) The set of all strings over {0, 1} that have an even number of 0's or exactly three 1's.

e) The set of all strings over {a, b, c} such that every a is followed by at least two c's.

Q3. Determine whether each of the following statements is true (T) or false (F). In case of being false write a short comment, or give a counter example.

	Statement	T/F	Comment / Counter Example
1)	a*(ba*)* = (a + b)*		
2)	L[a*b*] ∩ L[c*d*] = { }		
3)	If L_1 and L_2 are not regular then $L_1 \cup L_2$ is also not regular.		
4)	If L_1 is regular and $L_1 \cup L_2$ is also regular, then L_2 must be regular.		
5)	The set of even integers is closed under division.		

CMPE471 – Tutorial 4

Q1. Consider the set of strings over the alphabet {0,1} obeying the following conditions:

a) The number of 1's in a string is even and at least two.
b) There are no more than two 1's successively.
c) 01 is always followed by 1.
d) The strings always start with 0.

Find a regular expression that denotes the language described by the above set of strings.

Q2. Find a DFA that accepts the language denoted by 0* 1*.

Q3. Describe using set notation the language denoted by aa(a+bφ)*.

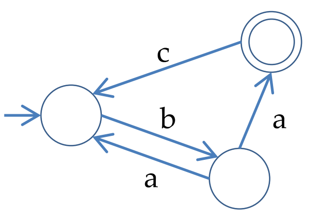
Q4. Find a Deterministic Finite Automaton that accepts the language generated by the following grammar.

S → 0S | 0A
A → 0B | 0C | 1S | 1A | 0
B → 0C
C → 0B | 1C | 0

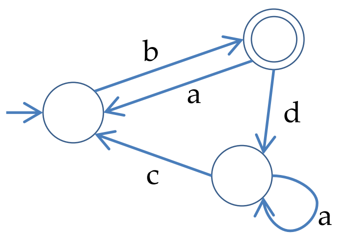
Q5. Give the language generated by the following grammar as a regular expression.

S → 0A | 1C | 0
A → 1B
B → 0A | 1B | 0
C → 0D | 1E | 1
D → 0E | 0
E → 1D

Q6. We are given the following non-deterministic finite automation M. Find a deterministic automation D that accepts the same language as M.



Q7. Find a regular expression that denotes the same languages accepted by the finite automaton below.

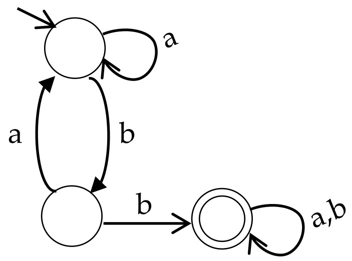

b
a
d
c
a

CMPE471 – Tutorial 5

Q1. For each of the following languages, construct a finite automaton that accepts the language:

a) L[$a^* (b^+a^+)^* b^*$]
b) L[$b (ab)^* bb$]
c) L = {w ∈ {0, 1}* | w contains odd number of 1's}
d) L[$(ba + bb)^* + (ab + aa)^*$]
e) L = {w ∈ {a, b }* | aa is a substring of w but aab is not}
f) L = {w ∈ {a, b, c}* | the number of a's in w is multiple of three}

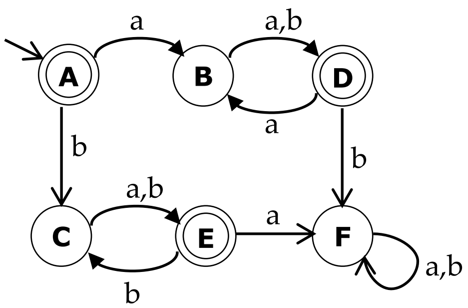
Q2. Consider the following DFA, M:



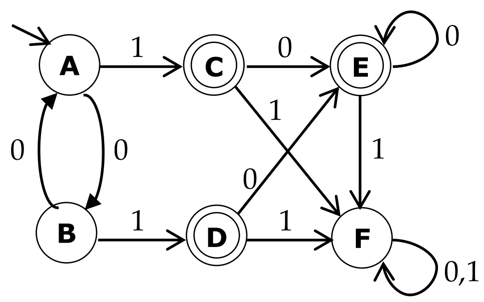
a) Give a minimum regular expression for this L(M).
b) Give a regular grammar for L(M).

CMPE471 – Tutorial 6

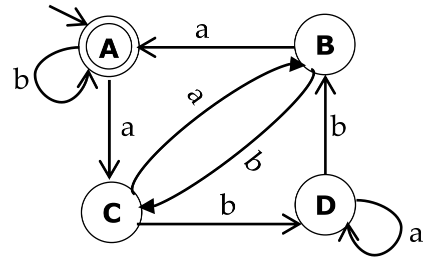
Q1. Minimize the following DFA:



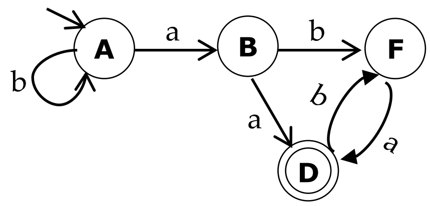
Q2. Minimize the following DFA.



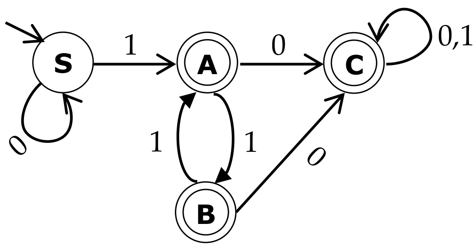
Q3. Write the equivalent Regular Expression for the following DFA. What would be the equivalent Regular Expression if state B was a final state.



Q4. Write the equivalent Regular Expression for the following DFA. What would be the equivalent Regular Expression if state B was a final state.



Q5 (Homework). Minimize the following DFA.



CMPE471 – Tutorial 7

Q1. Consider the following CLF over the alphabet {a,b}:

$L = \{ a^i b^j a^k \mid j = i + k; i > 0 \}$

a) Give a context-free grammar CFG that generates L.

b) Construct a PDA M for L.

c) Show that M accepts the string "aabbba" by starting with the configuration (s, aabbba, #) where "s" is the start state of M.

Q2. Convert the following CFG into a minimal Chomsky Normal Form (CNF).

$S \rightarrow aAbB \mid ABC \mid a$
$A \rightarrow aA \mid a$
$B \rightarrow bBcC \mid b$
$C \rightarrow abc$

Q3. For part (a), find the language that is accepted by the given PDA in the table below. Fort (b) and part (c), construct a **minimal** PDA for each of the given languages. (Use intuition. **Do not use** the theorem of CFG → PDA given in the class).

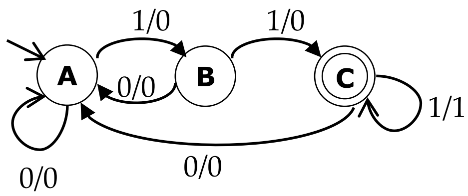
a) $L_1 = \{ ab^nac^{2n} \mid n > 0 \}$, over the alphabet $\sum = \{a, b, c\}$.
b) $L2 = \{ a^mb^nc^{3m+n} \mid m, n > 0 \}$, over the alphabet $\sum = \{a, b, c\}$.
c) $L3 = \{ w \mid w \in \sum^*, n_a(w) + n_b(w) = n_c(w) \}$, over the alphabet $\sum = \{a, b, c\}$.

Q4. Show that the language $L = \{ 0^m1^n0^{m+n} \mid m, n > 0 \}$ is not regular.

Q5. Let G be the CFG given by the below rules. Using intuition, construct an equivalent grammar in GNF (Greibach Normal Form).

$S \rightarrow SaA \mid A$
$A \rightarrow AbB \mid B$
$B \rightarrow cB \mid c$

Q6. Convert the following Mealy DFA to its Moore equivalent:



Q7. Convert the following Moore DFA to its Mealy equivalent: