

CMPE/CMSE-471 Assignment # 1

Last Due Time: 10/11/2020 – 16:30

Q.1) [20 pts] Given the set $A = \{1, 2, 3, 4\}$ and a relation R on A such that:
 $R = \{(1, 1), (1, 2), (3, 3), (2, 1)\}$, find

- a) The reflexive closure of R ,
 - b) The symmetric closure of R
 - c) The transitive closure of R .
 - d) The equivalence relation of R .
-

Q.2) [20 pts]

Let $A = \{1, 2, 3, 4, 5, 6\}$ and consider the following functions from $A \rightarrow A$:

$f(n) = n$; $g(n) = 6 - n$; $h(n) = \max\{3, n\}$; and $p(n) = \max\{1, n - 1\}$. Fill in the following table with **Yes/No** to indicate whether each of the above functions is *Total*, *onto*, *one-to-one*, and *bijection* or not.

Function	Total?	Onto?	One-to-one?	Bijection?
$f(n) = n$				
$g(n) = 6 - n$				
$h(n) = \max\{3, n\}$				
$p(n) = \max\{1, n - 1\}$				

Q.3)[20 pts] Prove by induction that

$$\sum_{i=1}^n i^3 = \frac{n^2(n+1)^2}{4} \quad (n = 1, 2, 3, \dots)$$

Q.4)[20 pts]

Consider the CFG grammar $G = (\{S, A\}, \{a, b, c, d\}, P, S)$ where P consists of the following productions:

$$\begin{aligned} S &\rightarrow aSd \mid A \\ A &\rightarrow bAc \mid \varepsilon \end{aligned}$$

Using set notation, describe the language generated by the above grammar.

Q.5)[20 pts]

Consider the CFL language $L = \{a^n b^m a^n \mid n, m \geq 1\}$.
Give a CFG grammar that generates L .