T 7

Q.1) Consider the following CLF over alphabet $\{a, b\}$:

$$L = \{ a^i b^j a^k \mid j = i + k ; i > 0 \}$$

a) Give a context-free grammar CFG that generates L.

$S \rightarrow AB$

$A \rightarrow aAb \mid ab$

$B \rightarrow bBa \mid \varepsilon$

b) Construct a PDA M for L:

$\delta(s, a, \#) = (s, a\#)$

$\delta(s, a, a) = (s, aa)$

$\delta(s, b, a) = (q, \varepsilon)$

$\delta(q, b, a) = (q, \varepsilon)$

$\delta(q, \varepsilon, \#) = (f, \#)$

$\delta(q, b, \#) = (q, b\#)$

$\delta(q, b, b) = (q, bb)$

$\delta(q, a, b) = (f, \varepsilon)$

$\delta(f, a, b) = (f, \varepsilon)$

c) Show that M accepts the string "aabbba" by starting with the configuration (s, aabbba, #) where "s" is the start state of M.

$(s, aabbba, \#) \longrightarrow (s, abbba, a\#) \longrightarrow$

$(s, bbba, aa\#) \longrightarrow (q, bba, a\#) \longrightarrow$

$(q, ba, \#) \longrightarrow (q, a, b\#) \longrightarrow (f, \varepsilon, \#)$

Q2) Give the PDA for:

a) $L_1 = \{ab^n a c^{2n} \mid n > 0\}$, over the alphabet $\{a, b, c\}$.

$\delta(s, a, \#) = (s, k\#) \rightarrow$ push 1k

$\delta(s, b, k) = (q, kk) \rightarrow$ " "

$\delta(q, b, k) = (q, kkk) \rightarrow$ push 2k

$\delta(q, a, k) = (p, k) \rightarrow$ do nothing

$\delta(p, c, k) = (f, \varepsilon) \rightarrow$ pop 1k

$\delta(f, c, k) = (f, \varepsilon) \rightarrow$ " "

b) $L_2 = \{a^m b^n c^{3m+n} \mid m, n > 0\}$, over alphabet $\{a, b, c\}$.

$\delta(s, a, \#) = (s, kkk\#)$

$\delta(s, a, k) = (s, kkkk)$

$\delta(s, b, k) = (q, kk) \rightarrow \delta(q, b, k) = (q, kk)$

$\delta(q, c, k) = (f, \varepsilon)$

$\delta(f, c, k) = (f, \varepsilon)$

c) $L_3 = \{ w \mid w \in \Sigma^*, n_a(w) + n_b(w) = n_c(w) \}$, over alplabeta $\Sigma = \{a, b, c\}$

$\delta(s, a, \#) = (s, 1\#)$
$\delta(s, b, \#) = (s, 1\#)$
$\delta(s, c, \#) = (s, 0\#)$
$\delta(s, a, 0) = (s, \varepsilon)$
$\delta(s, b, 0) = (s, \varepsilon)$
$\delta(s, c, 0) = (s, 00)$
$\delta(s, a, 1) = (s, 11)$
$\delta(s, b, 1) = (s, 11)$
$\delta(s, c, 1) = (s, \varepsilon)$
$\delta(s, \varepsilon, \#) = (f, \#)$

Q3) Convert the following CFG into a minimal CNF.

$S \to aAbB \mid ABC \mid a$
$A \to aA \mid a$
$B \to bBcC \mid b$
$C \to abc$

$\Downarrow$

$S \to C_1 A C_2 B \mid ABC \mid a$
$A \to C_1 A \mid a$
$B \to C_2 B C_3 C \mid b$
$C \to C_1 C_2 C_3$

$C_1 \to a$
$C_2 \to b$
$C_3 \to c$

step 1: X
step 2:
$A \to \underbrace{a}_{C_1} A$
$C_1 \to a$
$S \to \underbrace{a}_{C_1} A \underbrace{b}_{C_2} B$
$C_2 \to b$
$C \to \underbrace{a}_{C_1} \underbrace{b}_{C_2} \underbrace{c}_{C_3}$
$C_3 \to c$

step 3:

$S \to \overbrace{C_1 A}^{D_1}\, \overbrace{C_2 B}^{D_2} \mid A\, \overbrace{BC}^{D_3}$

$D_1 \to C_1 A$

$D_2 \to C_2 B$

$D_3 \to BC$

$B \to \overbrace{C_2 B}^{D_2}\, \overbrace{C_3 C}^{D_4}$

$D_4 \to C_3 C$

$C \to \overbrace{C_1 C_2}^{D_5}\, C_3$

$D_5 \to C_1 C_2$

$\downarrow$

$S \to D_1 D_2 \mid A D_3$

$A \to C_1 A \mid a$

$B \to D_2 D_4 \mid b$

$C \to D_5 C_3$

$C_1 \to a$

$C_2 \to b$

$C_3 \to c$

$D_1 \to C_1 A$

$D_2 \to C_2 B$

$D_3 \to BC$

$D_4 \to C_3 C$

$D_5 \to C_1 C_2$

Q4) CFG $\to$ GNF:

$S \to SaA \mid A$

$A \to AbB \mid B$

$B \to cB \mid c$

by intution:

left recursive

$S \rightarrow SaA \mid A \iff S \rightarrow SaA \mid A$

$A \rightarrow AbB \mid B \qquad S \rightarrow A \mid AX$

$B \rightarrow cB \mid c \qquad X \rightarrow aA \mid aAX$

$A \rightarrow B \mid BY$

$Y \rightarrow bB \mid bBY$

$S \rightarrow A \mid AX \iff S \rightarrow cB \mid c \mid cBY \mid cY \mid cBX \mid cX \mid cBYX \mid cYX$

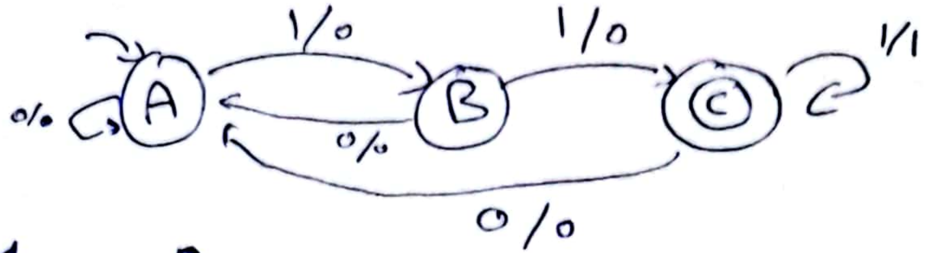
$X \rightarrow aA \mid aAX$

$A \rightarrow B \mid BY \longrightarrow A \rightarrow cB \mid c \mid cBY \mid cY$

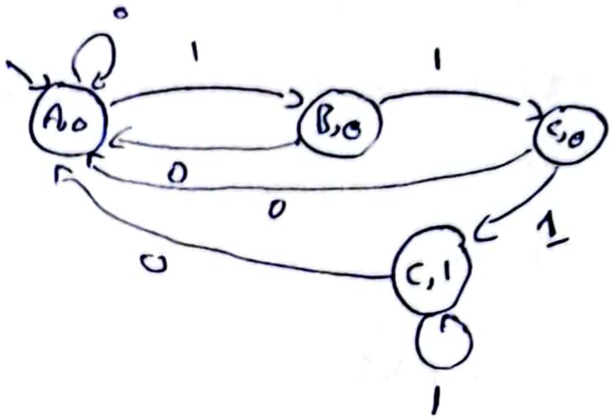
$Y \rightarrow bB \mid bBY$

$B \rightarrow cB \mid c$

Q5) Convert the following Mealy to its Moore equivalent



ps	1	0
A,0	B,0	A,0
B,0	C,0	A,0
C,0	C,1	A,0
C,1	C,1	A,0



Q6) Convert the following Moore to Mealy: