CMPE471 – Tutorial 1

Q1. Using the principle of mathematical induction, prove that:

a: $\sum_{i=1}^{n} i^3 = (\sum_{i=1}^{n} i)^2$

b: $1 + 2^n < 3^n, for\ all\ n \geq 2, n \in N$

Q2. Prove that, if $(1 + x) > 0$ then, $(1 + x)^n \geq 1 + nx, for\ all\ n \in N$

Q3. Let $\Sigma = \{a,b\}$, in each of the following three cases, give an example of languages L_1 and L_2 that satisfy the stated conditions. (Note that $L_1 \subseteq \Sigma^*$, and $L_2 \subseteq \Sigma^*$).

I. $L_1L_2 = L_2L_1$, such that $L_1 \subseteq L_2$, $L_2 \not\subset L_1$, and $L_1 \neq \{\varepsilon\}$
II. $(L_1 \cup L_2)^* \neq L_1^* \cup L_2^*$
III. $(L_1 \cup L_2)^* = L_1^* \cup L_2^*$, such that $L_1 \not\subset L_2$, and $L_2 \not\subset L_1$

Q4. If r is a real number not equal to 1, prove the following formula:

$$\sum_{i=0}^{n} r^i = (1 - r^{n+1}) / (1 - r) \qquad \text{for } n \geq 0$$

Q5. In each case give an example of languages L_1 and L_2 that satisfy the stated conditions.

i) $L_1L_2 = L_2L_1$, $L_1 \subseteq L_2$, $L_2 \not\subset L_1$, $L_1 \neq \{\lambda\}$.
ii) $L_1L_2 = L_2L_1$, $L_1 \not\subset L_2$, $L_2 \not\subset L_1$, $L_1 \neq \{\lambda\}$, $L_2 \neq \{\lambda\}$.
iii) $L_1 \subseteq \{a,b\}^*$, $L_2 \subseteq \{a,b\}^*$, $(L_1 \cup L_2)^* \neq L_1^* \cup L_2^*$.
iv) $L_1 \subseteq \{a,b\}^*$, $L_2 \subseteq \{a,b\}^*$, $(L_1 \cup L_2)^* = L_1^* \cup L_2^*$, $L_1 \not\subset L_2$, $L_2 \not\subset L_1$.