Hydraulic Jump Theory
Change of flow regime from sub-critical to super critical or vice versa occurs frequently in open channels. Such change is manifested by a corresponding change in depth of flow from a high stage to a low stage or vice versa.

**Hydraulic drop** is a change in the depth of flow from a high stage (high depth - low velocity) due to mild slope ($Fr < 1.00$) to a low stage (low depth - high velocity) lower than critical flow depth due to steep slope ($Fr > 1.00$).
Hydraulic jump is more important phenomenon from practical viewpoint. If a geomorphologic (cross-section, longitudinal channel bed slope, obstacles within the cross-section, etc.) changes are forcing the flow to change its regime from super critical \((Fr > 1.00\) where the depth of flow is at high velocity - low depth) to sub-critical \((Fr < 1.00\) where the depth of flow is at low velocity - high depth); this is only possible with an abrupt (rapid) jump, and it is known as Hydraulic Jump (HJ). ‘Sometimes it is known as surface roller’.
Hydraulic Jump at St. Anthony Falls on the Mississippi River
For the determination the hydraulic jump beginning and ending leg water depth values, the Momentum concept along the control volume is used. The SIMPLIFIED form for nearly horizontal slopes, rectangular channel cross-sections is:

\[ y_{\text{unknown}} = \frac{y_{\text{known}}}{2} \left( \sqrt{1+8Fr_{\text{known}}^2} - 1 \right) \]

where \( y_{\text{known}} \) and \( y_{\text{unknown}} \) are called the conjugate depths.

Note that, this equation is a symmetric equation and by applying one of the depths data to the equation, the other depth value is determined.

In literature this relationship is known as: “the equation of Bélanger”.
\[\frac{v_A^2}{2g}\]

\[\frac{v_B^2}{2g}\]

\[\Delta E = E_A - E_B\]

Beginning leg

Ending leg

\[v_A\]

\[v_B\]

\[\gamma_{cr}\]

\[E_A\]

\[E_B\]

\[L_{\text{jump}}\]
Due to energy loss always upstream energy $'E_A'$ > downstream energy $'E_B'$.
HYDRAULIC JUMP OCCURRENCE IN NATURE

Hydraulic Jumps may be classified due to their formations as:

1- **FREE JUMPS (PERFECT JUMPS)** (the jumps that occur fully)
   - WITH NO G.V.F. *(NO tail)* portion (very very rare)
   - WITH A G.V.F. portion *(a tail)* at one side (most common)
     i. Pulled Jump that occurs at the Upstream (HJ and S1 over Steep slope) i.e tail is S1
     ii. Repelled Jump that occurs at the Downstream (M3 and HJ over Mild slope) i.e tail is M3

   One of them is always occurring when the channel bed slope changes from STEEP to MILD.

   *(Jumps having GVF on both sides never occurs!)*
   *(NO TAILS ON BOTH SIDES NEVER OCCURS)*

2- **SUBMERGED (DROWNED) JUMPS**

   The jump is not fully established (formed) due to the upstream and/or the downstream constraints. **DOES NOT CONTAIN ANY GVF PART (NO TAIL).** Mainly occurring due to CHOKE phenomenon.
HYDRAULIC JUMP OCCURRENCE IN NATURE

Depending on upstream and downstream channels parameters, the HYDRAULIC JUMP location OCCURRENCE in NATURE varies.

This location of the jump depends on both the upstream and downstream conditions and the channels hydraulic parameters.

a) if the slope of the channel changes from STEEP to MILD DEFINITELY a HYDRAULIC JUMP occurs.

b) If CHOKE occurs, DEFINITELY a HYDRAULIC JUMP occurs.
a) If the longitudinal slope changes from STEEP to MILD definitely Hydraulic Jump (HJ) occur.

In this case, the formed HJ is always FREE (Not drown) type.

But
- either HJ with S1 curve
- or M3 curve with HJ occurs.

b) If CHOKE occurs, DEFINITELY a HYDRAULIC JUMP (HJ) occurs.

In this case, the formed HJ is may be FREE or DROWNED type.
HYDRAULIC JUMP OCCURRENCE IN NATURE

A FREE hydraulic jump may occur either on upstream (pulled) or on downstream (repelled) of the slope breaking point.

i. Pulled Jump that occurs at the Upstream (HJ and S1)

ii. Repelled Jump that occurs at the Downstream (M3 and HJ)
HYDRAULIC JUMP OCCURRENCE IN NATURE

- AFTER determining the normal depths of the upstream ‘\( y_1 \)’ and the downstream ‘\( y_2 \)’ and

- being sure about the upper slope is ‘steep’ and the lower slope is ‘mild’ then

- there exists two alternatives for the locational occurrence of the hydraulic jump.
HYDRAULIC JUMP OCCURRENCE IN NATURE

Alternative #1:
- Considering the possibility of the jump occurring on the upstream of the breaking point (forming pulled type HJ), calculate the conjugate depth \( y_{2\text{conj}} \) corresponding to the normal depth of the upstream slope \( y_1 \).
- If this conjugate depth \( y_{2\text{conj}} \) < the normal depth \( y_2 \) on the downstream after the breaking point,

Then

the jump will form on the upstream slope (forming pulled type HJ) and followed by an S1 curve (a tail) leading to the normal depth of the downstream ‘\( y_2 \)’.

Super critical (\( Fr > 1 \))

Sub-critical (\( Fr < 1 \))
(Pulled HJ.)

Length of Hydraulic Jump

S1 curve

\( y_1 \text{ upstream} \)

\( y_2 \text{ conjugate} \)

\( y_2 \text{ downstream} \)

STEEP SLOPE

MILD SLOPE
But if $y_{2\text{conj}} \nless y_2$; hydraulic jump cannot occur so the assumption is **WRONG**. Since the jump does not occurred on the previous assumption hence the remaining alternative should be checked.

**Alternative # 2:**
- using the the normal depth $y_2$ of the downstream, the corresponding conjugate depth $y_{1\text{conj}}$ is determined.

Hence;
if $y_{1\text{conj}} > y_1$ (upstream flow depth before the slopes breaking point), then definitely the hydraulic jump (**repelled**) occurs.
This HJ occurs on the downstream reach; but immediately before the jump, M3 curve (**a tail**) as well is expected to formed between the jump and just after the slope break where the upstream section ends.
(Repelled HJ.)

Length of Hydraulic Jump

M3 curve

$y_1$ upstream

$y_1$ conjugate

$y_2$ downstream

STEEP SLOPE

MILD SLOPE
2- Submerged (Drowned) Jump

The jump that is not fully established (formed) due to upstream and/or downstream constraints forms a submerged jump. A submerged hydraulic jump, or shortly called *submerged jump*, is defined as the jump where the **toe is covered by water** and the atmosphere has no direct access to the body of the jump. As a result, a submerged jump entrains much less air than the non-submerged jump.
Length of hydraulic jump for rectangular channel cross-section ‘L_HJ’

1- Free (Perfect) Jump length can be determined by using:

a) the equation: defined based on upstream depth and upstream Froude number:

\[ L_{HJ} = 9.75 \, y_{\text{super cr.}} (Fr_{\text{super cr.}} - 1)^{1.01} \]

b) the below graph
Length of hydraulic jump for **rectangular channel cross-section** ‘$L_{HJ}$’

2- *Drown (Submerged) Jump* length can be determined by using the equation:

$$L_{HJ} = y_2 (6.1+4.9S) \text{ where } S = \frac{y_4 - y_2}{y_2} \text{ (Submergence ratio)}$$

$y_2$: the theoretical conjugate (sub-critical) depth of the hydraulic jump at the downstream (m)

$y_4$: existing water depth at the downstream (m)

**ENERGY LOSS EQUATIONS FOR HYDRAULIC JUMPS**

1- $\Delta E_{loss} = E_{upstream} - E_{downstream} \text{ [m]}$

2- Loss ratio = $(\Delta E_{loss} / E_{upstream})$

3- Power Loss = $\gamma Q \Delta E_{loss} \text{ [Watt]}$
Question 10.1:

A long rectangular channel of width is 2.5 m carries a discharge of $Q = 18 \text{ m}^3/\text{s}$. If it is observed that, a Hydraulic Jump occurs having the depth of water at one of its conjugate depth to be 3.0 m. Determine:

i- suggest the possible location for this given conjugate depth (leg),

ii- the other conjugate depth;

iii- the energy head loss and the loss ratio due to the jump,

iv- the length of the jump,

v- draw the specific energy curve and show these values,

vi - draw the longitudinal cross-section and show all the important points.

Answer:

Given depth (leg) is for downstream part since $Fr = 0.44 < 1.0$, so $h' = 3.0 \text{ m}$

$y_{\text{conj}} = 0.90 \text{ m} ; \Delta E=0.87 \text{ m}, \text{Loss ratio}= 0.209; L_{\text{HJ}} = 14.92 \text{ m}$
Hydraulic Jump

**Y (m)**

- \( Y_1 \)
- \( Y_{cr} \)
- \( Y_2 \)

**E (m)**

- \( E_{min} \)
- \( E_1 \)
- \( E_2 \)

- \( Q = 18 \text{ m}^3/\text{s} \)
$\frac{v_1^2}{2g} = 3.26$

$E_1 = 4.16\, \text{m}$

$E_2 = 3.29\, \text{m}$

$\Delta E = E_1 - E_2$

$\Delta E = 0.87\, \text{m}$

$\frac{v_2^2}{2g} = 0.29$

$L_{\text{HJ}} = 14.92\, \text{m}$
Question 10.2:

Two successive long reaches of common rectangular channel cross-section of width B=2.75 m carries a uniform discharge Q=12.130 m³/s. The equivalent Manning’s roughness coefficient is n=0.016 for both cross-sections. The first reach bottom slope is \( s_{b1} = 0.173 \) and the second reach bottom slope is \( s_{b2} = 0.25 \% \). Determine:

i- the uniform flow depths \( y_1 \) and \( y_2 \) of the reaches,

ii- determine the critical flow depth \( y_{cr} \),

iii- classify the reaches,

iv- check the occurrence possibility of the hydraulic jump,

v- establish the hydraulic jump ‘H.J.’ (if occurs),

vi- suggest the possible curve type cooperated with hydraulic jump (if exits),

vii- calculate the length of the H.J. (if occurs),

viii- determine the specific energies,

ix- obtain the minimum specific energy \( E_{min} \),

x- calculate the energy loss due H.J. (if exists),

xi- calculate the energy loss due non uniform flow portion (if exists),

xii- draw the specific energy curve of this slope break zone,

xiii- show the flow variations on the given reaches.
$s_b_1 = 0.173$

$s_b_2 = 0.25 \%$

$Q = 12.13 \, m^3/s$

$n = 0.016$

$B = 2.75 \, m$
\[ Q = \frac{A}{n} \left( \frac{R^{1/3}}{S} \right) \]

\[ 12.13 = \frac{2.75 \gamma_n}{0.016} \left( \frac{2.75 \gamma_n}{2.75 + 2\gamma_n} \right)^{2/3} \sqrt{0.173} \]

\[ y_n = 0.380 \text{m} \]

\[ y_{n_{\text{upstream}}} = 0.016 \left( \frac{2.75 \gamma_n}{2.75 + 2\gamma_n} \right)^{2/3} \sqrt{0.0025} \]

\[ y_n = 1.696 \text{m} \]

\[ y_{n_{\text{downstream}}} \]
ii - \[ \frac{q^2T}{gA^3} = 1 \]
\[ y_{cr} = \sqrt[3]{\frac{12.13^2}{9.81 \times 2.75^2}} \]
\[ y_{cr} = 1.256 \text{ m} \]

Note that \( y_{cr} \) is independent on \( h \) or \( S_b \).

iii - Since \( y_1 < y_{cr} \), \( 0.380 < 1.256 \)

slope 1 is steep.

and \( y_2 > y_{cr} \), \( 1.696 > 1.256 \)

slope 2 is mild.

Note that \( S_{cr} = 0.0058 \) also indicating that slope 1: STEEP and slope 2: MILD

iv - Since the first reach (slope 1) is steep and the second reach (slope 2) is mild

definitely a hydraulic jump occurs.
Since the cross-section is rectangular
the H.J. conjugate equation can be used.

\[ y_2 = \frac{y_1}{2} \left( \sqrt{1 + 8 \pi r_1^2} - 1 \right) \]

In fact the location of the H.J. is not
known. Need trial.
Assume that H.J. occurs at the upstream (steep) slope.

\[ y_2 = 1.696 \text{ m} \]

\[ y_1 = 0.380 \text{ m} \]

\[ F_{r1} = \frac{12.13 \times (2.75)}{9.81 \times (0.38 \times 2.75)^3} = 36.144 \]

\[ y_2 = \frac{0.380}{2} \left( \sqrt{1 + (8 \times 36.144)} - 1 \right) = 3.046 \text{ m} \]

Since \( 3.046 \geq y_{n2} = 1.696 \text{ m} \), this jump not occur.

It should be less than 1.696 m so that it will increase through \( S_1 \) curve to reach 1.696 m.
The remain possibility is:

H.J occurring on Mild slope (downstream)

i.e.

\[ y_2 = 1.696 \]

Using conjugate eqn

\[ y_{\text{conj}} = \frac{y_2^2}{2} \left( \sqrt{1 + 8F_{r_2}^2} - 1 \right) \]

\[ y_{\text{conj}} = \frac{1.696^2}{2} \left( \sqrt{1 + 8 \times 0.407^2} - 1 \right) \]
\[ y_2 = 1.696 \]
\[ F_{r2}^2 = \frac{12.13^2 \times 2.75}{9.81 (2.75 \times 1.696)} = 0.407 \]

\[ y_1 = \frac{1.696}{2} \left( \sqrt{1 + 8(0.407)} - 1 \right) = 0.900 \text{ m} \]

Since \( y_2 > y_1 \),

\[ 0.900 > 0.38 \]

possible

H. J occurs on M. Id slope flume and \( M_3 \) curve forms before 14.97
\[ F_1 = 0.38 + \frac{12.13^2}{19.62 \times (2.75 \times 0.38)^2} = 7.247 \text{ m} \]

\[ F_2 = 1.696 + \frac{12.13^2}{19.62 \times (2.75 \times 1.696)^2} = 2.040 \text{ m} \]

\[ F_{\text{conjugate}} = 0.9 + \frac{12.13^2}{19.62 \times (2.75 \times 0.9)^2} = 2.124 \text{ m} \]

\[ E_{\text{min}} = \frac{3}{2} \times y_{\text{cr}} \]

\[ E_{\text{min}} = 1.5 \times 1.256 = 1.884 \text{ m} \]
Energy loss in \( H_J \)

\[ E_{\text{con}} - E_2 = 2124 - 2.040 = 0.084 \text{ m} \]

Energy loss in \( M_3 \) curve

\[ E_1 - E_{\text{con}} = 7.247 - 2.124 = 5.123 \text{ m} \]

Length of \( H_J \)

\[ L_{H_J} = 9.75 \cdot y_i \left( \frac{F_{r_i}}{1} \right)^{1.01} \]

\[ y_i = y_{\text{con}} = 0.900 \text{ m} \]

\[ F_{r_{\text{con}}} = \sqrt{\frac{12.13 \cdot 2.25}{9.81 \cdot (2.25 \cdot 1.01)}} = 1.649 \]

\[ L_{H_J} = 9.75 \cdot 0.9 \left( 1.649 - 1 \right)^{1.01} = 5.67 \text{ m} \]
Question 10.3:
In a hydraulic laboratory, a rectangular channel cross-section of bottom width $B = 5.0 \, \text{m}$ that carries a discharge $Q=20 \, \text{m}^3/\text{s}$ is used. The flow passes under the sluice gate and just after it forms a uniform water depth of $y = 0.4 \, \text{m}$ where a hydraulic jump occurs afterwards. If at the downstream the flow depth is
i) $y=2.5 \, \text{m}$, ii) $y=3.0 \, \text{m}$

a) discuss the formation of HJ type and draw the longitudinal water surface profile for each case.
b) estimate the length of the jump occurring for case (i) and case (ii).
i) Taking $y_{2\text{conj}} = y_2 \text{ given} = 2.5 \text{ m}$ and applying the HJ equation for rectangular cross-sections $y_{1\text{conj}} = 0.442 \text{ m}$.

Hence since $y_{1\text{conj}} = 0.442 \text{ m} > y_1 = 0.40 \text{ m}$ (Free Hydraulic Jump with M3 curve occurs).

b) Free jump equation: $L_{HJ} = 9.75 \ y_1 (\text{Fr}_{1} - 1)^{1.01}$ so $L_{HJ} = 9.75 \times 0.442 (4.34 - 1.0)^{1.01} = 14.56 \text{ m}$.

A FREE HYDRAULIC JUMP OCCURS WITH M3 GVF!
ii) Taking \( y_{2\text{conj}} = y_2 \text{ given} = 3.0 \) m and applying the HJ equation for rectangular cross-sections \( y_{1\text{conj}} = 0.325 \) m.

But since \( y_{1\text{conj}} = 0.325 \) m \( < \) \( y_1 = 0.40 \) m (Drowned Hydraulic Jump occurs).

b) Submerged jump equation: \( L_{\text{HJ}} = y_2 (6.1 + 4.9S) \) where \( S = \frac{y_4 - y_2}{y_2} \)

\[
y_2 = \frac{0.40}{2} \left( \sqrt{1 + 8 \times (25.48)} - 1 \right) = 2.663 \text{ m}
\]

\[
L_{\text{HJ}} = 2.663 \times (6.1 + 4.9 \times \frac{3 - 2.663}{2.663}) = 17.90 \text{ m}
\]

SINCE DROWN JUMP OCCURS NO GVF FORM!
Two successive long reaches of rectangular channel cross-section of width $B=3.50$ m carries a uniform discharge $Q=1.85$ m$^3$/s. Manning’s roughness coefficients of the reaches are $n_1=0.023$ and $n_2=0.016$. The first reach bottom slope is $s_{b1}=0.02$ and the second reach bottom slope is $s_{b2}=0.0006$. Determine:

i- the uniform flow depths $y_1$ and $y_2$ of the reaches,
ii- determine the critical flow depth $y_{cr}$,
iii- classify the reaches,
iv- check the occurrence possibility of the hydraulic jump,
v- establish the hydraulic jump ‘H.J.’ (if occurs),
vi- suggest the possible curve type cooperated with hydraulic jump (if exits),
vii- calculate the length of the H.J. (if occurs),
viii- determine the specific energies,
ix- obtain the minimum specific energy $E_{min}$,
x- calculate the energy loss due H.J. (if exists),
xi- calculate the energy loss due non uniform flow portion (if exists),
xi- estimate the length of the occurring curve (is exists),

xi- draw the specific energy curve of this slope break zone,

xiv- show the flow variations on the given reaches.
\[ s_{b1} = 0.02 \]

\[ s_{b2} = 0.0006 \]

\[ Q = 1.85 \text{ m}^3/\text{s} \]

\[ B = 3.5 \text{ m} \]

\[ n = 0.023 \]

\[ n = 0.016 \]
Answers:
S1 with HJ.

$y_1 = 0.241$ m
$y_2 = 0.593$ m
$E_1 = 0.486$ m
$E_2 = 0.633$ m
$y_{cr} = 0.30$ m
$E_{\text{min}} = 0.45$ m
$L_{\text{HJ}} = 0.993$ m
$L_{S1 \text{ CURVE}} = 9.20$ m
Previous INTERM Exam

The given below is some part of a longitudinal section of a river channel with three reaches. This channel has a common rectangular cross-section of width \( B = 7.65 \) m and the other details are as shown over the relevant figure.

The flow depth at the first reach is \( y_1 = 1.22 \) m

i- determine the uniform discharge(s) ‘Q’, (3 p)

ii- find the flow depths \( y_2 \) and \( y_3 \) of the other reaches, (6 p)

iii- determine the critical flow depth(s) \( y_{cr} \) of these reaches, (6 p)

iv- classify (name) the reaches based on their slopes, (6 p)

v- check the occurrence possibility of the hydraulic jump(s) ‘HJ’ (5 p)

vi- establish the hydraulic jump ‘HJ’ wherever possible (if occurs), (20 p)

vii- suggest name(s) for the gradually varied flow (GVF) curve(s) (if exits), (9 p)

viii- calculate the length(s) of the hydraulic jump ‘HJ’ (if occurs), (at least one), (5 p)

ix- calculate the length(s) of the gradually varied flow GVF (if occur(s)) (at least one), (5 p)

x- draw the specific energy curve ‘E-y’ around each slope break zone separately and show all the relevant details, (20 p)

xi- show the flow variations on the given reaches with all the relevant details that you determined in the above sections. (15 p)
Depths in Open Channels

1- Normal depth ‘\( y_n \)’:
Obtained from Manning’s Equation;
\[
Q = \frac{A^{5/3}}{n P^{2/3}} \sqrt{S_0}
\]

2- Critical depth ‘\( y_{cr} \)’:
Obtained from Froude number;
\[
Fr^2 = \frac{Q^2 T}{g A^3}
\]

3- Alternate depth ‘\( y_{alt} \)’:
Obtained from Energy Equation;
\[
E = y_{alt} + \frac{Q^2}{2g A_{alt}^2}
\]

4- Conjugate depth ‘\( y_{conj} \)’:
Obtained from Hydraulic Jump Equation;(for rectangular case)
\[
y_{unknown} = \frac{y_{known}}{2} \left[ \left( \sqrt{1 + 8Fr_{known}^2} \right) - 1 \right]
\]