**EASTERN MEDITERRANEAN UNIVERSITY**

 **Department of Industrial Engineering**

**IENG314/ MANE314 Operations Research II**

**HOMEWORK 3 Spring 2021-22**

1. A housewife buys three kinds of cereals (beans): A, B and C. She never buys the same cereal in successive weeks. If she buys cereal A, then the next week she buys C. However, if she buys either B or C, then the next week she is four times as likely to buy A as the other brands. What is the probability that she buys cereal A and switch to cereal C after 2 weeks? In the long run, how often does she buy each of the three brands?
2. Three children 1,2,3 are arranged in a circle playing a game of throwing a ball to each another. At each stage the child having the ball is equally likely to throw it in to anyone of the other. What is the probability that first child takes the ball and don’t throw it to anyone.
3. A Toyota dealer consumes four kinds of engine oil A, B, C and D. This dealer buys its consumption each week. The dealer never buys the same brand in successive weeks, except brand D. If the dealer buys engine oil D then with same probability it can buy all kinds of engine oils next week. If the dealer perches brand C then next week it will buy D. However, if the dealer buys engine oil B then the next week it is four times as likely to perches A as the other brands and finally if the dealer buy engine oil A it will buy D or C with same probabilities. What is the probability that mentioned dealer buys oil C when we know that oil B was purchased three weeks ago?
4. Let *Xn* be a Markov chain with state space *{0,1,2}*, the initial probability vector and one step transition matrix 
5. Compute.
6. Compute.
7. Show that for any closed Markov chain , where *n* is the number of states. Is it true for n-transition probability matrix of this Markov chain.
8. Consider the following transition matrix:

|  |  |
| --- | --- |
|  | 1. Which states are transient?
2. Which states are recurrent?
3. Identify all closed sets of states?
4. Is this chain ergodic?
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1. Find communication classes for each of the following Markov chains. Which one is ergodic?

 

1. Find the steady state probabilities for the transition probability matrices in question 6.
2. Consider a Markov Chain which its transition probability matrix is P1 in question 7, show that after long time, the probability of leaving the second state is equal to the probability of entering to this state.
3. Draw the graph of the Markov chain associated with the following transition probability matrix. Classified the states. Is this Markov chain is ergodic? Why? Is it possible that,

i) ?

ii) ?

