**EASTERN MEDITERRANEAN UNIVERSITY**



**Department of Industrial Engineering**

**IENG513 Probabilistic Models**

**HOMEWORK 3 Fall 2017-18**

1. Let *X1, X2, … , Xn* be independent random variables, each having a uniform distribution over (0,1). Let M= maximum(*X1, X2, … , Xn*). Show that the distribution function of M, *FM(.)*, is given by *FM(x)=xn,* for  *x* belongs to *[0,1]*
2. What is the probability density function of M?
3. Calculate *E[M]*.
4. An urn contains *n+m* balls, of which *n* are red and *m* are black. They are withdrawn from the urn, one at a time and without replacement. Let *X* is the number of red balls removed before the first black is chosen. We are interested in determining *E[X].* To obtain this quantity, number the red balls from 1 to *n.* Now define the random variables *Xi*, *i=1,...,n*, by



if red ball *i* is taken before any black ball is chosen

Otherwise

1. Express *X* in terms of *Xi*s. b) Find *E[X].*
2. An urn contains *2n* balls, of which *r* are red. The balls are randomly removed in *n* successive pairs. Let *X* denoted the number of pairs in which both balls are red.
3. Find *E[X].* b) Find *Var(X).*
4. Calculate the moment generating function of a geometric random variable.
5. Calculate the moment generating function of the uniform distribution on (2*,*5). Obtain *E*[*X*] and Var[*X*] by differentiating.
6. Assume that a product is manufactured by two machines. First, Machine 1 must complete its work and after that Machine 2 can start to work on the product. Suppose that *X* is the number of products manufactured by Machine 1 distributed exponentially with mean 2 and *Y* is the number of products manufactured by Machine 2 has gamma distribution with parameters (2,4). If the number of productions for the mentioned machines are independent. What is the probability that Machine 2 wait for coming products from Machine 1 in production duration.
7. Suppose that *X* takes on each of the values 1, 2, 3 with probability . What is the moment generating function? Derive *E*[*X*]*, E*[*X*2]*,* and *E*[*X*3] by diffeentiating the moment generating function and then compare the obtained result with a direct derivation of these moments.
8. Assume that *X* and *Y* are independent random variables both uniformly distributed on *(0.1)*, then calculate the probability density of *X+Y*. Now if *X* and *Y* satisfy in the above conditions and distributed on *(2,5)*, find *P(6<X+Y<8)*.
9. Assume that the following function is the jointly density function of the random variables *X* and *Y*. Find *c, fX(x), fY(y)* and *P{X+Y 2}.*



1. Let Z1 and Z2 are two independent standard normal random variables. Assume that *X1 = 2Z1-3Z2+5*and *X2 =-Z1+2Z2+3* are two multivariate normal random variables. Find the joint moment generating function of *X1* and *X2*.
2. Let *X* and *Y* be independent normal random variables, each having parameter *µ* and *ơ2*. Show that *X+Y* is independent of *X-Y*.