**EASTERN MEDITERRANEAN UNIVERSITY**



**Department of Industrial Engineering**

**IENG514 Stochastic Processes and Applications**

**HOMEWORK 4 Fall 2017-18**

1. Consider a system with two components. We observe the state of the system every hour. A given component operating at time *n* has probability *p* of failing before the next observation at time *n + 1*. A component that was in a failed condition at time *n* has a probability *r* of being repaired by time *n + 1*, independent of how long the component has been in a failed state. The component failures and repairs are mutually independent events. Let *Xn* be the number of components in operation at time *n*. The process *{Xn; n = 0, 1, …}* is a discrete time homogeneous Markov chain with state space *I = 0, 1, 2*.
2. Determine its transition probability matrix, and draw the state diagram.
3. Obtain the steady state probability vector, if it exists.

1. A machine goes out of order whenever a component part fails. The failure of this part is in accordance with a Poisson process with mean rate of 2 per week.
2. What is the probability that three weeks pass after the last failure.
3. Suppose that there are 6 extra parts of the component in an inventory. What is the probability that the next supply for new component is not due in 12 weeks?
4. What is the expected time for running out those 6 extra parts?
5. Divided the interval *[0,1)* into a large number *n* of small intervals of length *h* and suppose that in each small interval, Bernoulli trails with probability of success are held this means that trial with only two outcomes, success with probability  and failure with probability *(1-**)*. Show that the number of successes in an interval of length *t* is a Poisson process with mean. State the assumptions you make.
6. The number of failures *N* (*t*), which occur in a computer network over the time interval [0*, t*), can be described by a homogeneous Poisson process. On an average, there is a failure after every 5 hours, i.e. the intensity of the process is equal = 0*.*2(*h*)-1.

(a) What is the probability of at most 1 failure in [0*;* 9), at least 2 failures in [9*,* 18), and at most 1 failure in [18,27) (time unit: hour)?

(b) What is the probability that the third failure occurs after 9 hours?

1. If *N1(t)*, *N2(t)* are two independent Poisson processes with parameters , respectively, then show that :



1. Suppose that *N1(t)*, *N2(t)* are two independent Poisson processes with parameters, respectively. Compute the expected value and variance of *N(t)=* *N1(t)*- *N2(t).*
2. Show that for large *t* the observation *N(t)/t* is a reasonable estimation for the mean of a Poisson process.
3. Assume that arrival of vehicles to a public transport station (buses and taxis) is according to a Poisson process with rate 20/hour. The station is starting to work from 6:00 AM.
4. Find the probability that there is no vehicle arriving in a 6 minute interval.
5. Given that a vehicle arrived at 10:00; find the probability that the next arrival happen before 10:07.
6. If the probability that a bus arrival be 0.2. What is the probability of seven buses arrived to the station during 2 hours given that 25 vehicles arrived in last hour.
7. Suppose that of each bus carry 16 passengers with variance 0.09 and each taxi carry 3 passengers with variance 0.05. How many passengers can carry from this public transportation station in 3 hours?
8. Given that the number of taxis which arrive until 12:00 is 180, what is the probability that 65 taxis arrive to the station until 9:30?
9. Show that an Erlang distribution with shape parameter *k* and rate parameter *kλ* has the same distribution as the random variable *X1+ X2+ …+Xk*, where each *Xi* is an exponential random variable with parameter *kλ*, and the *Xi* ‘s are independent random variables.
10. Suppose that defects in a sheet of material follow the Poisson model with an average of 3 defects per 3 square meters. Consider a 6 square meter sheet of material.
11. Find the probability that there will a. be at least 4 defects.
12. Find the mean and standard deviation of the number of defects.
13. Suppose that raisins in a cake follow the Poisson model with an average of 2 raisins per cubic inch. Consider a slab of cake that measures 3 by 4 by 1 inches.

a) Find the probability that there will be at no more than 20 raisins.

b) Find the mean and standard deviation of the number of raisins.