**EASTERN MEDITERRANEAN UNIVERSITY**



**Department of Industrial Engineering**

**IENG514 Stochastic Processes and Applications**

**HOMEWORK 6 Fall 2019-20**

1. Assume that there are three production lines which produce memory chips as 2, 3 and 4 chips in each lot, respectively. Also suppose that the number of defective lots (more than half of its chips are defective) for each of production lines at each day distributed by Poisson distribution with parameter 2. The probability for producing the defective lot for each of the first production lines is tow time more than other production lines.
2. What is the probability that 3 defective chips produce in 5 hours of production?
3. What is the expected number of defective chips at each day?
4. Consider a two channel queuing system. Suppose that the service channels are numbered I and II and suppose that we are interested in whether particular channels are busy (B) or free (F). Let *(i,j)* denoted to state of the system. The demand for service arises in accordance with Poison process with parameter *a* also assume that when both the channels are free a demand may join either of the channels of service with equal probability. The service time in each channel is exponential with parameter *b* and we have loss system for servicing. Find the probability that both the channels be busy after a long time.
5. Suppose that the average time for calling Turkish Airline office distributed by exponential distribution with parameter 3 and also assume that the call duration after each connection behaved as exponential distribution with mean 0.2. Consider the evolution of the system as Markov process with two states.
6. Find transition density matrix.
7. Find transition probability matrix.
8. Now if we want to call this office for ticket reservation, with which probability the phone can be busy.
9. Consider a barber shop with two barbers and two waiting chairs. Customers arrive at a rate of five per hour. Each barber serves customers at a rate of two per hour. Customers arriving to a fully occupied shop leave without being served. When the shop opens at 8 a.m., there are already two waiting customers. We assume that arrivals are Poisson and service times are exponential and the arrival process is independent of service times. Hence the arrival rate is five per hour and the service rate is two per hour for each barber.
10. Model the problem as birth and rate process and classify it.
11. Show the rate matrix.
12. What will be the state of rate matrix if there are only one waiting chair, the rate of arrivals is two, the first barber completes serving a customer at a rate of 2 per hour and the second completes a serving a customer at a rate of 1 per hour.
13. Calculate the steady state probabilities.
14. Consider a scientist interesting to prevent some of organisms of Extinction. Assume the interested population of organisms has n size and depending on the scientist records it increased according to Poisson process with rate λ per a year for each individual of organisms and decreased according to the same type of increased process with rate μ per year for each individual.
15. Show if the process can be Markovian (birth and death) and classify it.
16. Show the rate matrix.
17. Assume n=10 and λ = 5 and μ = 3, what will be the population mean after 5 years.
18. Show that the probability of extinction is less than 1.
19. If μ = 5 and λ = 3, what is the probability of extinction.
20. Use Renewal Equation and find the expected number of renewals at [0,t] duration in terms of density function of time duration between two consecutive renewals.
21. In a renewal process let Xn have gamma distribution with parameters *a=2* and *k=2*. What is the probability that there is 5 renewals in first 10 minutes.
22. In a renewal process let Xn have gamma distribution with parameters *a=1* and *k=3*. Find the mean of renewals in first 5 minutes.
23. Assume that for complication of production of a batch each of them has to move from two sections. The service time duration at each of these sections has exponential distributions with mean 1/3 find the mean of completed batches after 6 unit of time. Fined the probability that 5 batches can complete in 4 unit of time. What is the probability that the waiting time for completing 6 batches be 8 units of time? (Hint: The service process has Erlang distribution with parameters λ =3 and *k=2*.)