Entropy Balance

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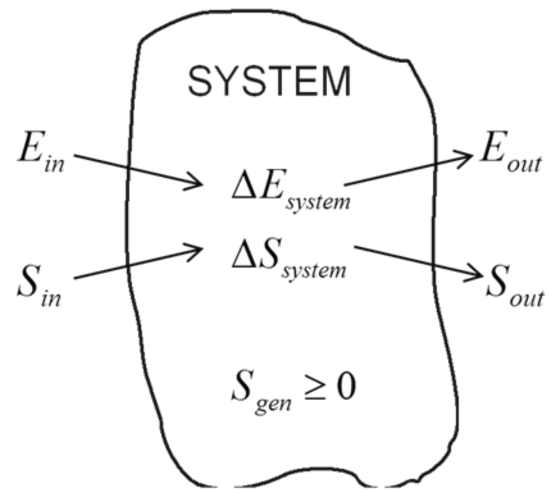
Entropy Balance

- The property *entropy* is a measure of molecular disorder or randomness of a system.
- Enropy can be created but it cannot be destroyed



$$\Delta E_{system} = E_{in} - E_{out}$$

$$\Delta S_{system} = S_{in} - S_{out} + S_{gen}$$

$$\begin{pmatrix} \text{Total} \\ \text{entropy} \\ \text{entering} \end{pmatrix} - \begin{pmatrix} \text{Total} \\ \text{entropy} \\ \text{leaving} \end{pmatrix} + \begin{pmatrix} \text{Total} \\ \text{entropy} \\ \text{generated} \end{pmatrix} = \begin{pmatrix} \text{Change in the} \\ \text{total entropy} \\ \text{of the system} \end{pmatrix}$$

or

$$S_{in} - S_{out} + S_{gen} = \Delta S_{system}$$

Entropy Change of a System ΔS_{sys}

Entropy change of a system = Entropy at final state – Entropy at initial state

$$\Delta S_{sys} = S_{final} - S_{initial}$$

Note: $\Delta S_{sys} = 0$ during steady state operation.

When the properties of the system are not uniform, the entropy of the system can be determined by:

$$S_{sys} = \int s\,\delta m = \int s\rho\,dV$$

volume

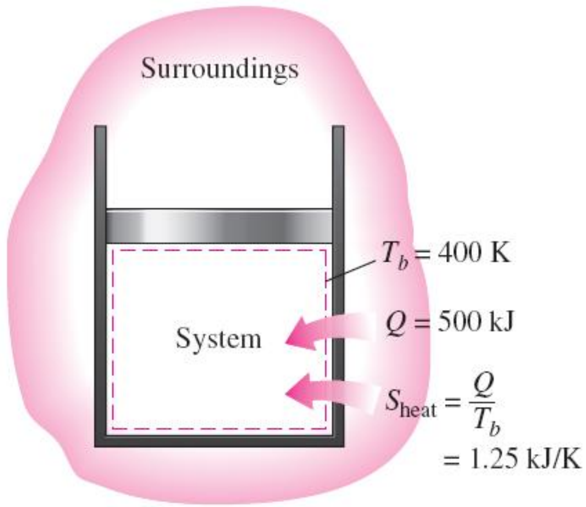
density

Mechanisms of entropy transfer, S_{in} and S_{out}

- Entropy can be transferred by the following two mechanisms:
  - **Heat transfer** ← Heat is a chaotic form of energy and some chaos (entropy) flows with heat
  - **Mass flow** ← Mass contains entropy and entropy is carried with it. Entropy increases with mass
- No entropy is transferred by work

Entropy transfer by heat transfer

$$S_{\text{heat}} = \frac{Q}{T} \qquad (T = \text{constant})$$

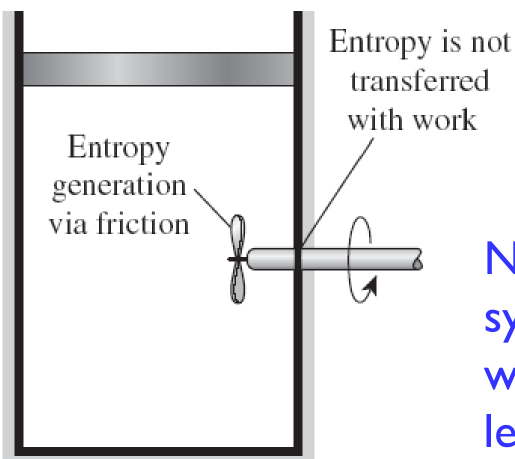


$$S_{\text{heat}} = \int_1^2 \frac{\delta Q}{T} \cong \sum \frac{Q_k}{T_k}$$

When temperature is not constant or different throughout the boundary

Entropy transfer by work:

$$S_{\text{work}} = 0$$



Heat transfer is always accompanied by entropy transfer in the amount of *Q*/*T*, where *T* is the boundary temperature.

No entropy accompanies work as it crosses the system boundary. But entropy may be generated within the system as work is dissipated into a less useful form of energy.

Entropy transfer by mass flow

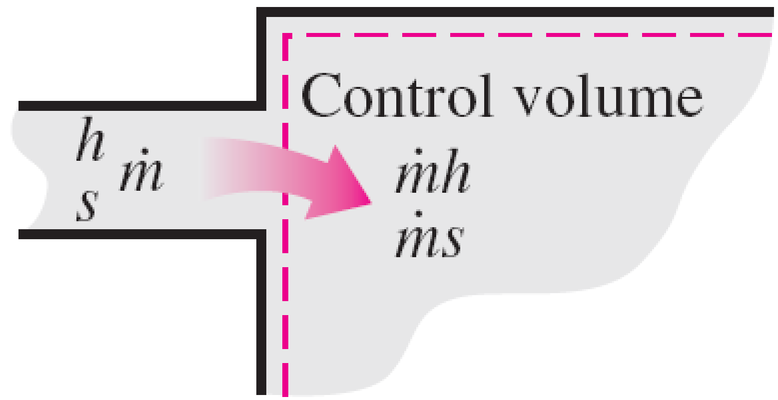
Entropy transfer by mass:

$$S_{\text{mass}} = ms$$

When the properties of the mass change during the process

$$\dot{S}_{\text{mass}} = \int_{A_c} s\rho V_n \, dA_c$$

$$S_{\text{mass}} = \int s\,\delta m = \int_{\Delta t} \dot{S}_{\text{mass}} \, dt$$



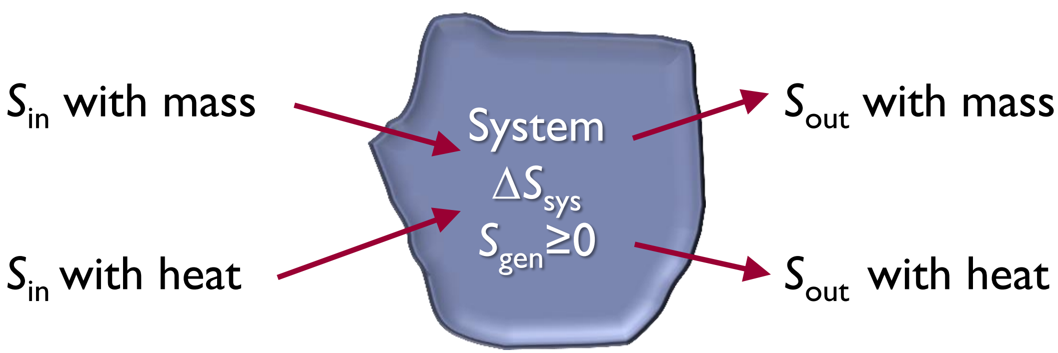
Mass contains entropy as well as energy, and thus mass flow into or out of system is always accompanied by energy and entropy transfer.

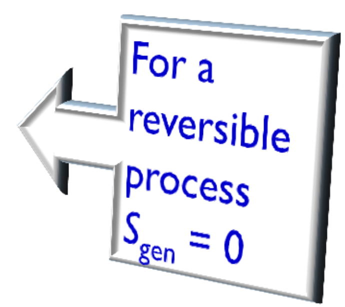
Entropy generation, S_{gen}

$$\underbrace{S_{in} - S_{out}}_{\substack{\text{Net entropy transfer}\\ \text{by heat and mass}}} + \underbrace{S_{gen}}_{\substack{\text{Entropy}\\ \text{generation}}} = \underbrace{\Delta S_{system}}_{\substack{\text{Change}\\ \text{in entropy}}} \qquad (\text{kJ/K})$$

$$\underbrace{\dot{S}_{in} - \dot{S}_{out}}_{\substack{\text{Rate of net entropy}\\ \text{transfer by heat}\\ \text{and mass}}} + \underbrace{\dot{S}_{gen}}_{\substack{\text{Rate of entropy}\\ \text{generation}}} = \underbrace{dS_{system}/dt}_{\substack{\text{Rate of change}\\ \text{in entropy}}} \qquad (\text{kW/K})$$

$$(s_{in} - s_{out}) + s_{gen} = \Delta s_{system} \qquad (\text{kJ/kg} \cdot \text{K})$$

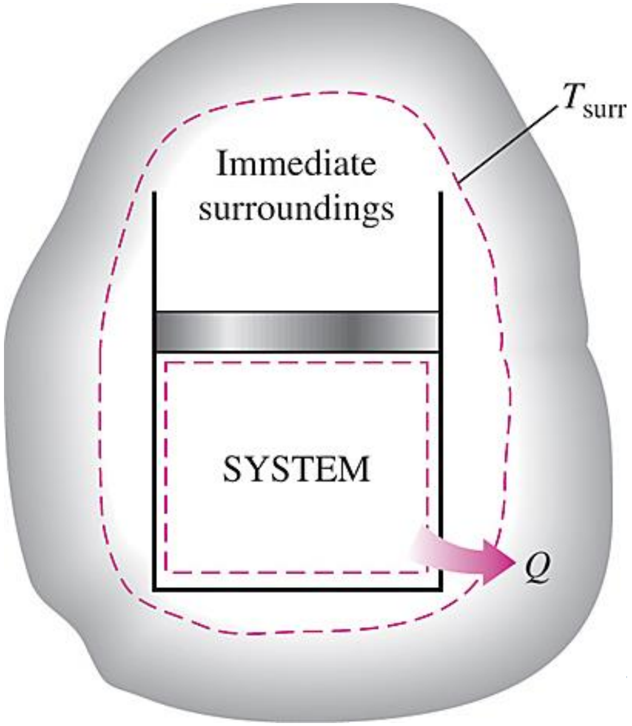




Entropy generation, S_{gen}

- The term S_{gen} represents the entropy within the system boundary only
- External irreversibilities are not accounted for in the term S_{gen}.

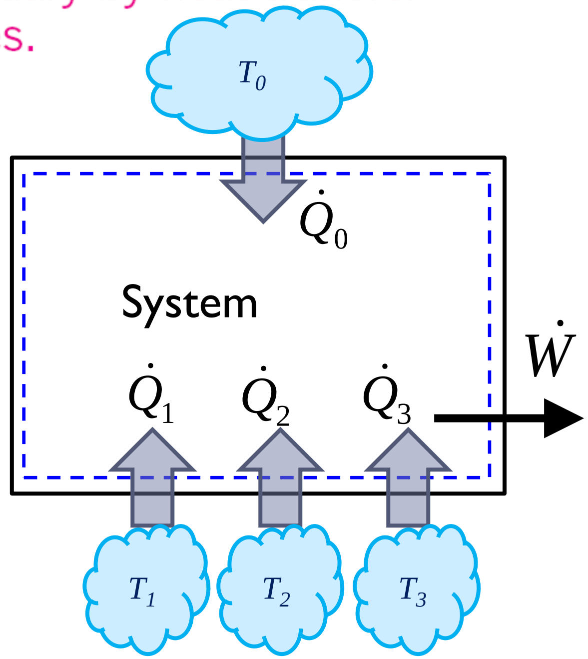
Entropy generation outside system boundaries can be accounted for by writing an entropy balance on an extended system that includes the system and its immediate surroundings.



Entropy balance of control masses (closed systems)

The entropy change of a closed system during a process is equal to the sum of the net entropy transferred through the system boundary by heat transfer and the entropy generated within the system boundaries.

$$\underbrace{\Delta S_{sys}}_{\text{Entropy change of a closed system}} = \underbrace{\sum \frac{Q_k}{T_k}}_{\text{Sum of net entropy transfer through the system boundary by heat transfer}} + \underbrace{S_{gen}}_{\text{Entropy generated}} \qquad \text{(kJ/K)}$$



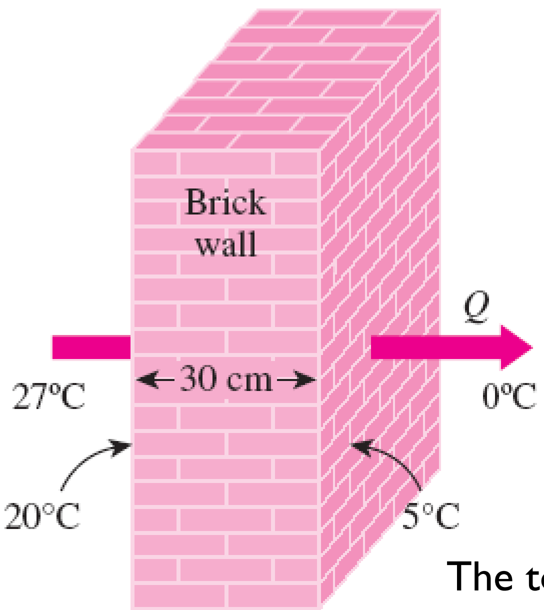
Adiabatic closed system: $S_{\text{gen}} = \Delta S_{\text{adiabatic system}}$

System + Surroundings: $S_{\text{gen}} = \sum \Delta S = \Delta S_{\text{system}} + \Delta S_{\text{surroundings}}$

$\Delta S_{\text{system}} = m(s_2 - s_1)$ $\qquad \Delta S_{\text{surr}} = Q_{\text{surr}}/T_{\text{surr}}$

Example:
Entropy generation in a wall

Determine the rate of entropy generation in a wall of 5-m x 7-m and thickness 30 cm. The rate of heat transfer through the wall is 1035 W.



$$\underbrace{\dot{S}_{in} - \dot{S}_{out}}_{\text{Rate of net entropy transfer by heat and mass}} + \underbrace{\dot{S}_{gen}}_{\text{Rate of entropy generation}} = \underbrace{\frac{dS_{sys}}{dt}}_{\text{Rate of change in entropy}}$$

0 (steady heat flow)

$$\left(\frac{\dot{Q}}{T}\right)_{in} - \left(\frac{\dot{Q}}{T}\right)_{out} + \dot{S}_{gen} = 0$$

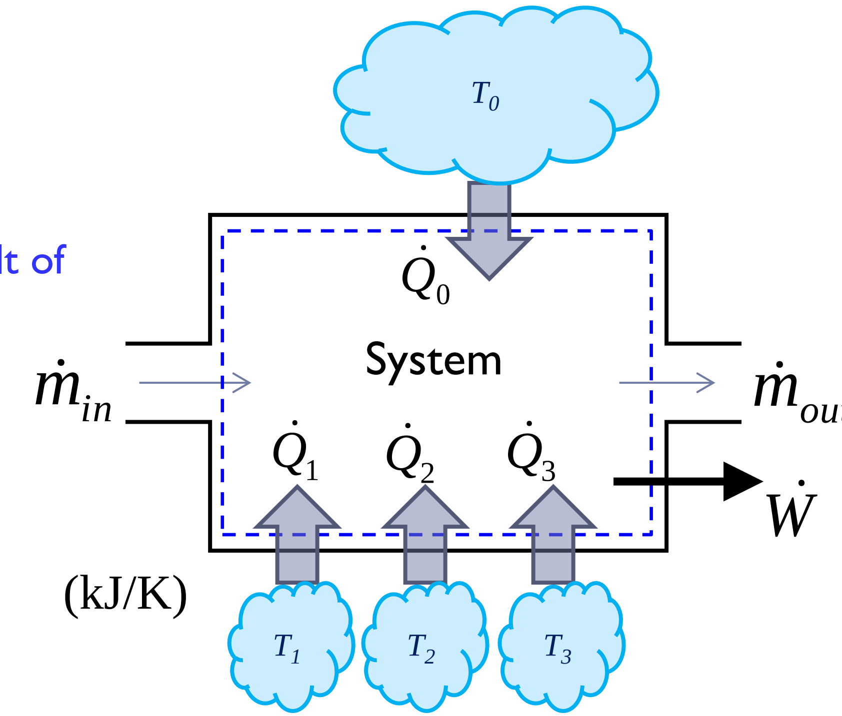
$$\frac{1035\ \text{W}}{293\ \text{K}} - \frac{1035\ \text{W}}{278\ \text{K}} + \dot{S}_{gen} = 0 \quad \Rightarrow \text{therefore } \dot{S}_{gen,\ wall} = 0.191\ \text{W/K}$$

The total rate of entropy generation (including the indoors and outdoors) can be found by taking into account the indoors and outdoors temperatures (extended system):

$$\frac{1035\ \text{W}}{300\ \text{K}} - \frac{1035\ \text{W}}{273\ \text{K}} + \dot{S}_{gen} = 0 \quad \Rightarrow \text{therefore } \dot{S}_{gen,\ total} = 0.341\ \text{W/K}$$

Entropy balance of control volumes (open systems)

The entropy of a control volume changes as a result of mass flow as well as heat transfer.



$$\sum \frac{Q_k}{T_k} + \sum m_i s_i - \sum m_e s_e + S_{gen} = \underbrace{(S_2 - S_1)_{\text{CV}}}_{\Delta S_{\text{CV}}} \qquad \text{(kJ/K)}$$

or in the rate form :

$$\underbrace{\sum \frac{\dot{Q}_k}{T_k}}_{\substack{\text{Entropy transfer}\\\text{rate by heat}\\\text{transfer}}} + \underbrace{\sum \dot{m}_i s_i - \sum \dot{m}_e s_e}_{\substack{\text{Net entropy flow rate}\\\text{out of the control volume}\\\text{via mass flow}}} + \underbrace{\dot{S}_{gen}}_{\substack{\text{Entropy}\\\text{generation}\\\text{rate}}} = \underbrace{\frac{dS_{\text{CV}}}{dt}}_{\substack{\text{Rate of entropy}\\\text{accumulation in}\\\text{the control volume}}} \qquad \text{(kW/K)}$$

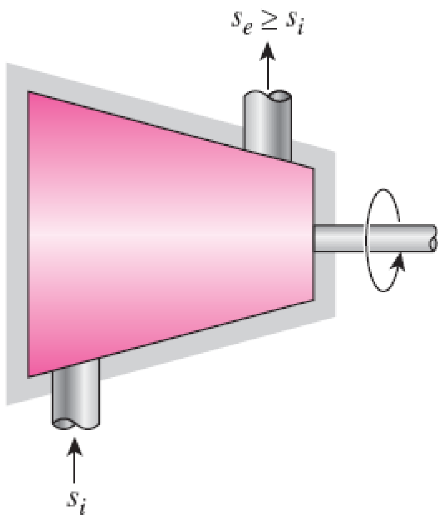
Entropy balance of control volumes (open systems)

$$\sum \frac{\dot{Q}_k}{T_k} + \sum \dot{m}_i s_i - \sum \dot{m}_e s_e + \dot{S}_{gen} = \frac{dS_{CV}}{dt}$$

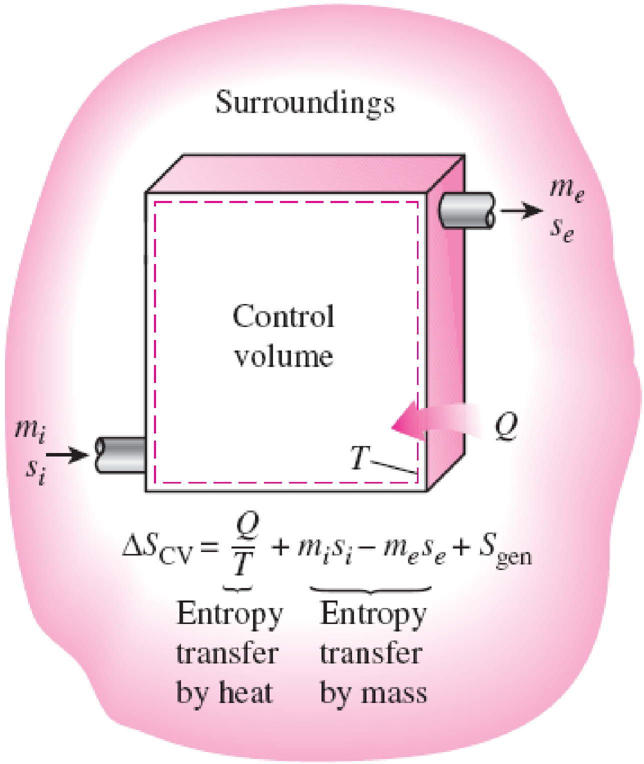
Steady-flow: $\dot{S}_{gen} = \sum \dot{m}_e s_e - \sum \dot{m}_i s_i - \sum \frac{\dot{Q}_k}{T_k}$

Steady-flow, single-stream: $\dot{S}_{gen} = \dot{m}(s_e - s_i) - \sum \frac{\dot{Q}_k}{T_k}$

Steady-flow, single-stream, adiabatic: $\dot{S}_{gen} = \dot{m}(s_e - s_i)$



The entropy of a substance always increases (or remains constant in the case of a reversible process) as it flows through a single-stream, adiabatic, steady-flow device.

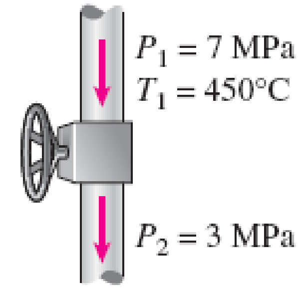


The entropy of a control volume changes as a result of mass flow as well as heat transfer.

Example: Entropy generation during a throttling process

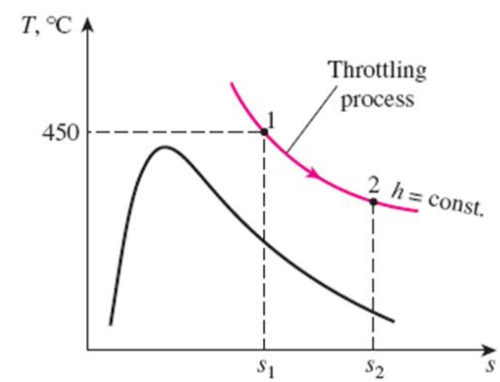
Determine the rate of entropy generation in a steady-state throttling process of steam shown in the diagram.

Use the tables to determine the entropy at the inlet and the exit states:



$$\text{State 1}: \left.\begin{array}{l} P_1 = 7\ \text{MPa} \\ T_1 = 450°\text{C} \end{array}\right\} h_1 = 3288.3\ \text{kJ/kg},\ s_1 = 6.6353\ \text{kJ/kg.K}$$

$$\text{State 2}: \left.\begin{array}{l} P_2 = 3\ \text{MPa} \\ h_2 = h_1 \end{array}\right\} s_2 = 7.0046\ \text{kJ/kg.K}$$



0 (negligible heat transfer) 0 (steady flow process)

$$\sum \frac{\dot{Q}_k}{T_k} + \sum \dot{m}_i s_i - \sum \dot{m}_e s_e + \dot{S}_{gen} = \frac{dS_{CV}}{dt}$$

$$\underbrace{\dot{S}_{in} - \dot{S}_{out}}_{\text{Rate of entropy transfer by mass flow}} + \underbrace{\dot{S}_{gen}}_{\text{Rate of entropy generation}} = \underbrace{\frac{dS_{sys}}{dt}}_{\text{Rate of change in entropy in the control volume}}$$

$$\dot{m}s_1 - \dot{m}s_2 + \dot{S}_{gen} = 0$$

$$\dot{S}_{gen} = \dot{m}(s_2 - s_1)$$

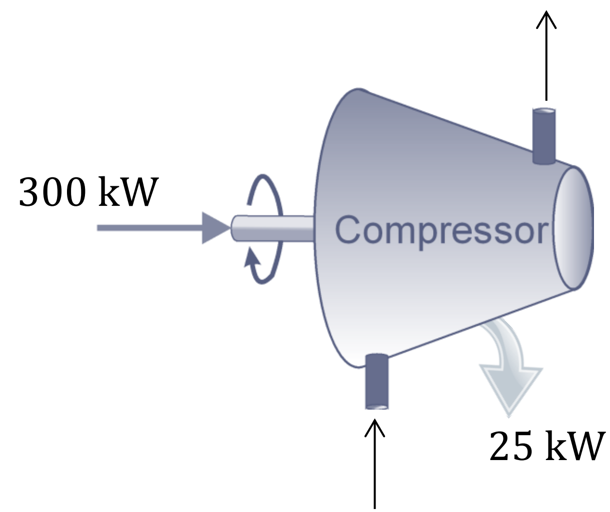
Dividing by mass flow rate:

$$s_{gen} = s_2 - s_1 = 7.0046 - 6.6353 = 0.3693\ \text{kJ/kg.K}$$

Example: Entropy generation in a compressor

$$\underbrace{\dot{S}_{in} - \dot{S}_{out}}_{\substack{\text{Rate of net entropy}\\ \text{transfer by heat}\\ \text{and mass}}} + \underbrace{\dot{S}_{gen}}_{\substack{\text{Rate of entropy}\\ \text{generation}}} = \underbrace{\frac{dS_{sys}}{dt}}_{\substack{\text{Rate of}\\ \text{change in}\\ \text{entropy}}} \to 0 \text{ (steady flow process)}$$

$P_1 = 1$ MPa
$T_1 = 327°\text{C}$
$s_2^o = 2.40902$ kJ/kg.K



$$\dot{m}s_1 - \dot{m}s_2 - \frac{\dot{Q}_{out}}{T_{b,surr}} + \dot{S}_{gen} = 0$$

$$\dot{S}_{gen} = \dot{m}(s_2 - s_1) + \frac{\dot{Q}_{out}}{T_{b,surr}}$$

$$\text{For ideal gases: } s_2 - s_1 = s_2^0 - s_1^0 - R\ln\frac{P_2}{P_1}$$

$$\dot{m}(s_2 - s_1)_{air} = 0.853 \text{ kg/s } (2.40902 - 1.66802)\frac{\text{kJ}}{\text{kg.K}} - 0.287 \ln\frac{1000 \text{ kPa}}{100 \text{ kPa}}$$

$$= 0.0684 \text{ kW/K}$$

$$\dot{S}_{gen} = 0.0684 \text{ kW/K} + \frac{25 \text{ kW}}{290 \text{ K}} = 0.155 \text{ kW/K}$$

Air
$\dot{m}_1 = 0.853$ kg/s
$P_1 = 100$ kPa
$T_1 = T_{amb} = 17°\text{C}$
$s_1^o = 1.66802$ kJ/kg.K