

# **MENG541**

# **Advanced**

# **Thermodynamics**

**CHAPTER 1 - FUNDAMENTALS**

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# Chapter 1

## The Fundamentals

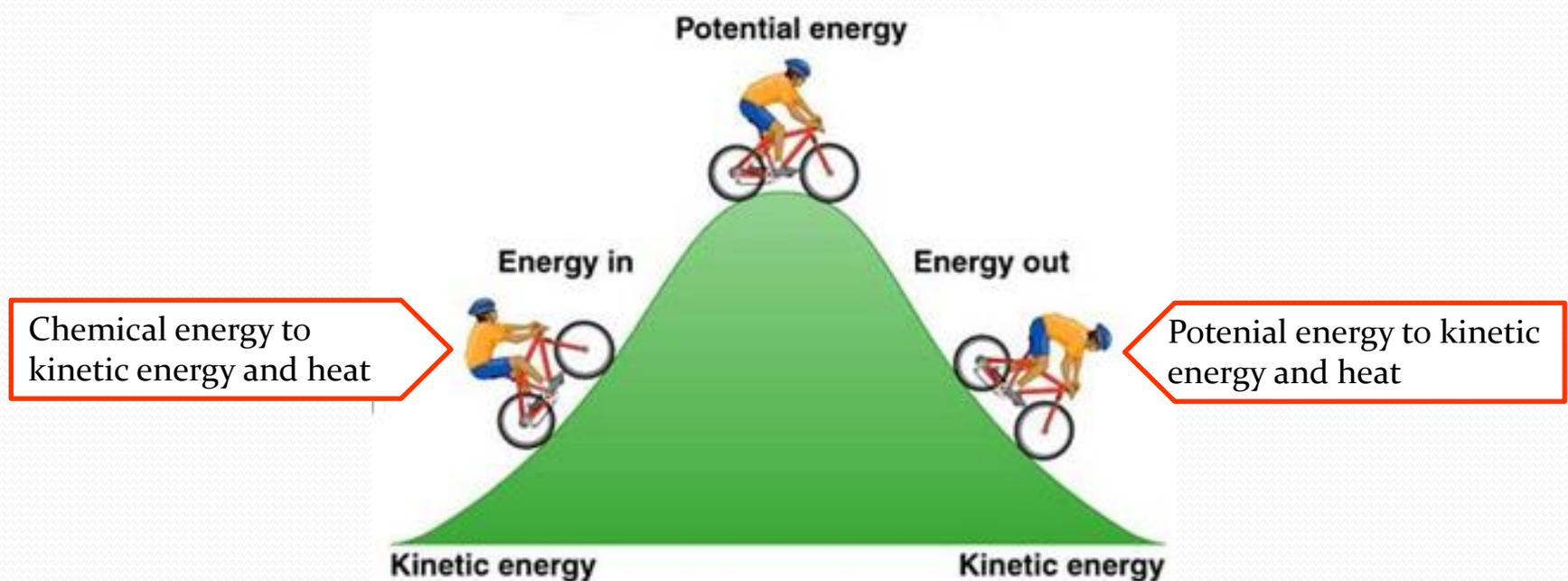
### Outline

- Fundamentals
- Closed Systems
- Work Transfer
- Heat Transfer
- Energy Equation
- Open Systems
- Steady Flow Energy Equation
- Second Law of Thermodynamics
- Cycles

# Conservation of Energy Principle (First Law of Thermodynamics)

## What is conservation of energy?

It simply states that during a process, energy can change from one state to another but that the total amount of energy remains constant. (Energy can not be created or destroyed)



# Basic Dimensions and their Units

Basic dimensions	Mass(m)	Length(L)	Time(t)
SI units	Kilogram (kg)	Meter (m)	Second (s)
English units	Pound-mass (lbm)	Foot (ft)	Second (s or sec)

$$1 \text{ lbm} = 0.45359 \text{ kg}$$

$$1 \text{ ft} = 0.3048 \text{ m}$$

# Units Derived from Basic Dimensions

Force = Mass  $\times$  Acceleration

$F = m(\text{kg}) \times a\left(\frac{\text{m}}{\text{s}^2}\right)$  In SI units the unit for force is Newtons (N)

In English units, Pound - force (lbf)

Work = Force  $\times$  Distance

$W = F(\text{N}) \times d(\text{m})$  In SI units the unit of work is Joules (J)

In English units, British Thermal Units (Btu)

Energy has the same units as Work.

1 Btu = 1.055 kJ

# Forms of Energy

- thermal, mechanical, kinetic, potential, electric, magnetic, chemical, nuclear
- Total energy in a process:

$E$  = the sum of all forms of energy

- Thermodynamics deals with the change of energy instead of its absolute value.
- It is appropriate to assign the total energy of a system a value of zero ( $E=0$ ) at some convenient reference point.

# Forms of Energy

## KINETIC ENERGY

$$KE = \frac{m \times v^2}{2} \quad (\text{kJ})$$

Rate of kinetic energy :

$$\dot{KE} = \frac{\dot{m} \times v^2}{2} \quad (\text{kJ/s} \equiv \text{kW})$$

## POTENTIAL ENERGY

$$PE = m \times g \times z \quad (\text{kJ})$$

Rate of potential energy :

$$\dot{PE} = \dot{m} \times g \times z \quad (\text{kJ/s} \equiv \text{kW})$$

## INTERNAL ENERGY

$U$  (kJ)

Is related to the degree of molecular activity in a system.

$$U = m \times C_v \times \Delta T$$

Nomenclature :

$KE$  : kinetic energy (kJ)

$PE$  : potential energy (kJ)

$U$  : internal energy (kJ)

$m$  : mass (kg)

$\dot{m}$  : mass flow rate (kg/s)

$v$  : velocity (m/s)

$g$  : gravitational acceleration ( $\text{m/s}^2$ )

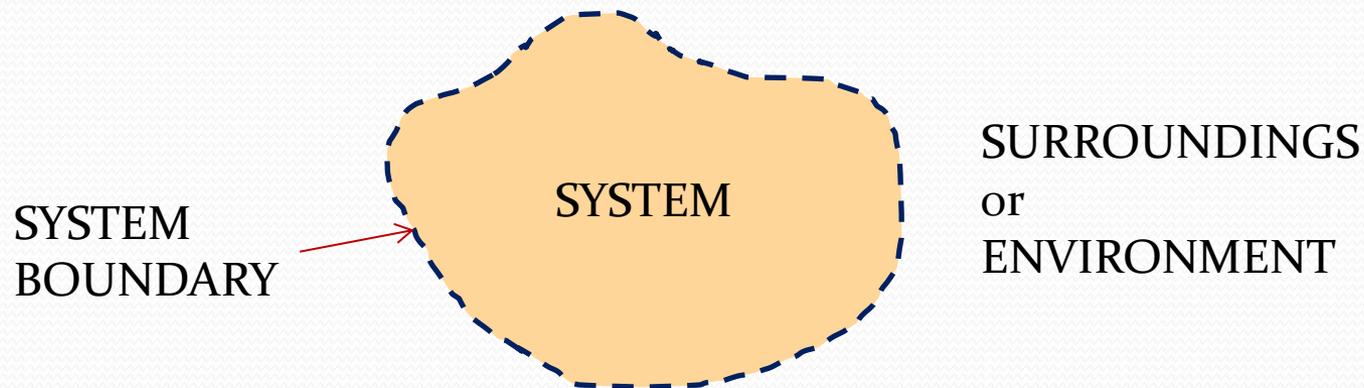
$z$  : elevation from a reference point (m)

$C_v$  : specific heat capacity at constant volume (kJ/kgK)

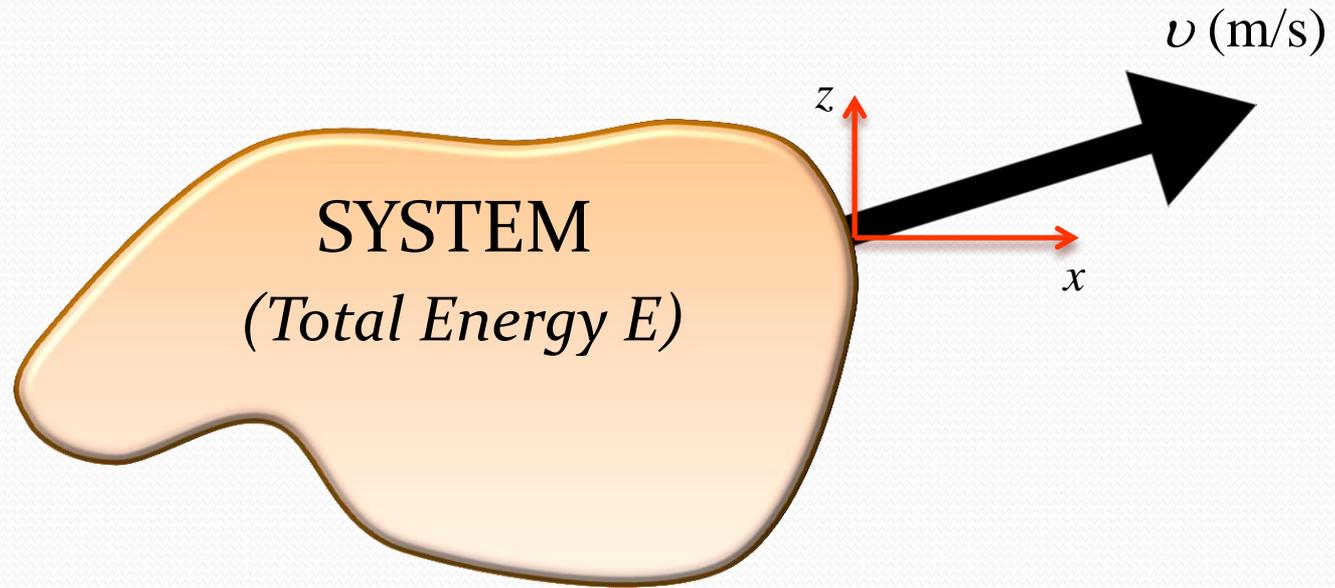
$T$  : Temperature

# Defining a System

- By defining a system we establish the *surroundings* of the system.
- The surface that separates the system from the surroundings is known as the *boundary*.



# Total Energy of the System



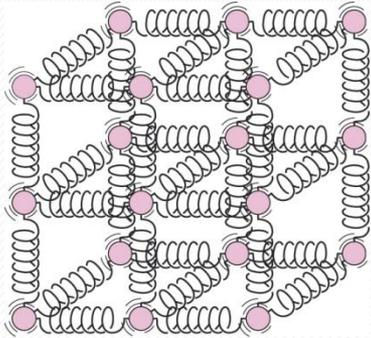
$E$  = the sum of all forms of energy

$$E = U + KE + PE = U + \frac{mv^2}{2} + mgz \quad (\text{kJ})$$

# Pure Substances

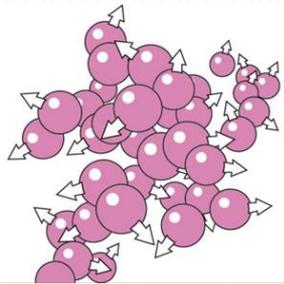
- A **Pure Substance** is a substance that is chemically homogenous and fixed in chemical composition.(e.g. water, nitrogen, air & etc.)
- mixture of oil and water is not a pure substance.
- a mixture of two or more phases of a pure substance is still a pure substance.
- The **phase** of a pure substance is the homogeneous, chemical, and physical of aggregation of its molecules.

# Three phases of a pure substance

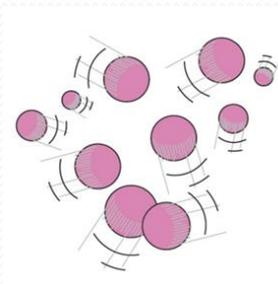


The molecules in a **solid** are kept at their positions by the large spring like inter-molecular forces.

At high temperatures, molecules overcome the inter molecules forces and break away.

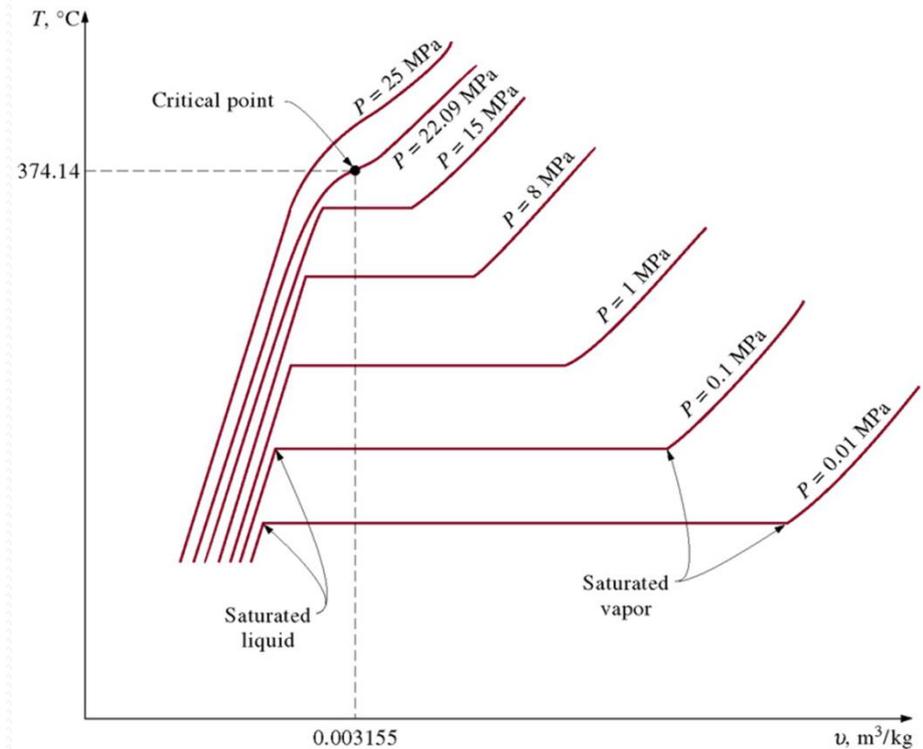
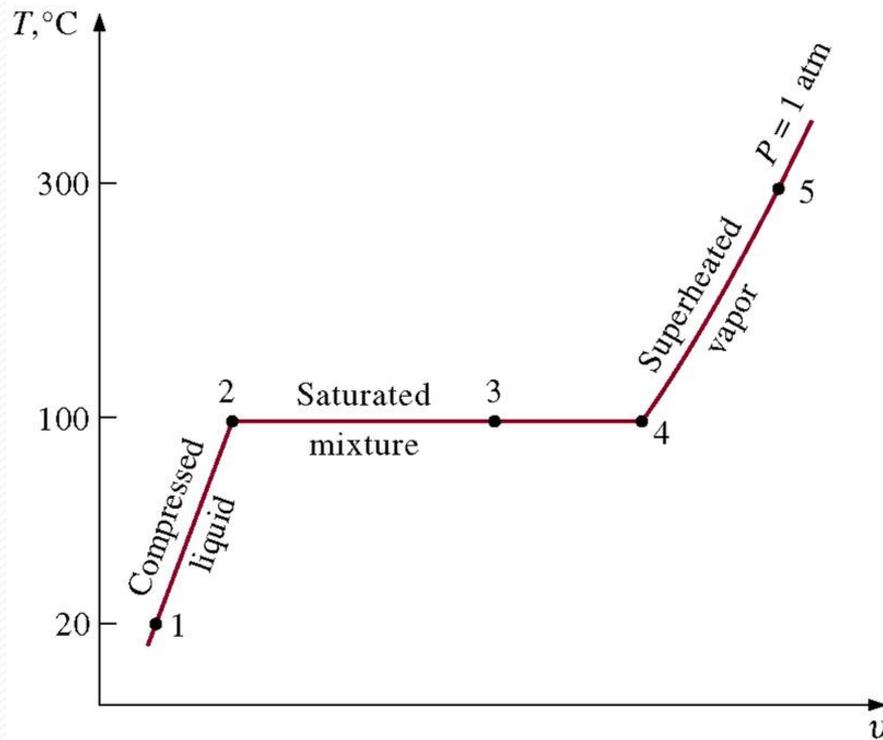


In the **liquid** phase the molecules are no longer at fixed positions, and chunks of the molecules float about each other.



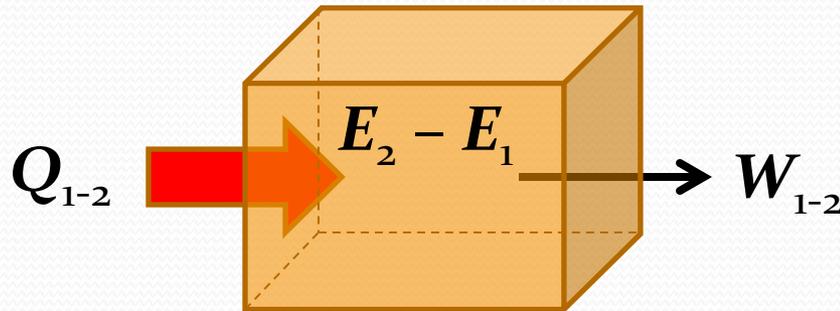
In the **gas** phase the molecules are far apart from each other, irregular and move about at random colliding with each other. Molecules are higher energy level than they are in liquid or solid phases.

# $T$ - $v$ diagram for the heating process of water at constant pressure



# First Law for Closed Systems

- Consider the following closed system:



$$Q_{1-2} - W_{1-2} = E_2 - E_1$$

Heat  
transfer

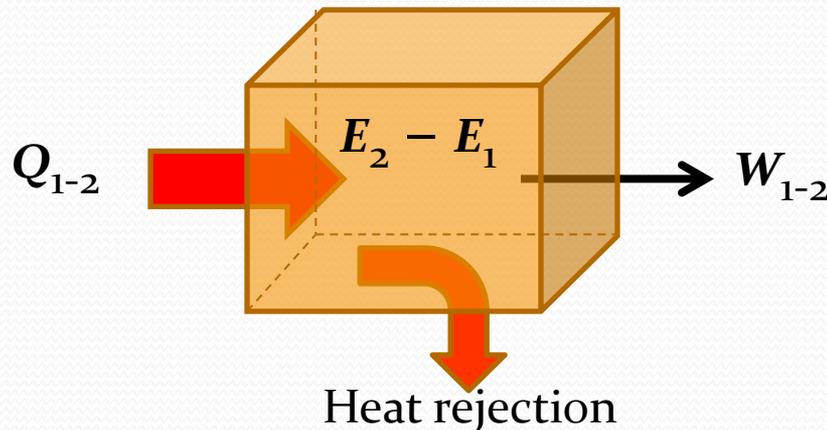
Work  
transfer

Energy change

Energy is a thermodynamic property

# Sign Convention

- Heat engine sign convention is used
- The purpose of a heat engine ( which is a closed system) is to deliver work to its environment while using up heat.
- Heat transfer into the system is (+) ve
- Work transfer out of the system is (+) ve



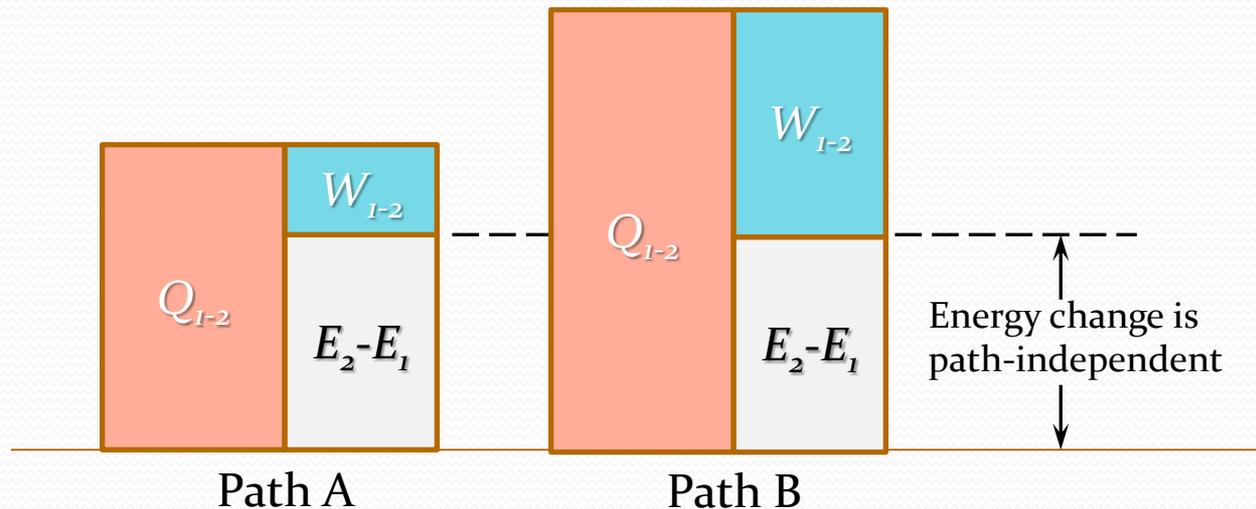
# Path dependence of $Q_{1-2}$ and $W_{1-2}$

- Energy interactions depend on the path followed but energy change is path independent

$$\dot{Q} - \dot{W} = \frac{dE}{dt}$$

- If the process in a closed system is a cycle then the energy change will reduce to zero such that:

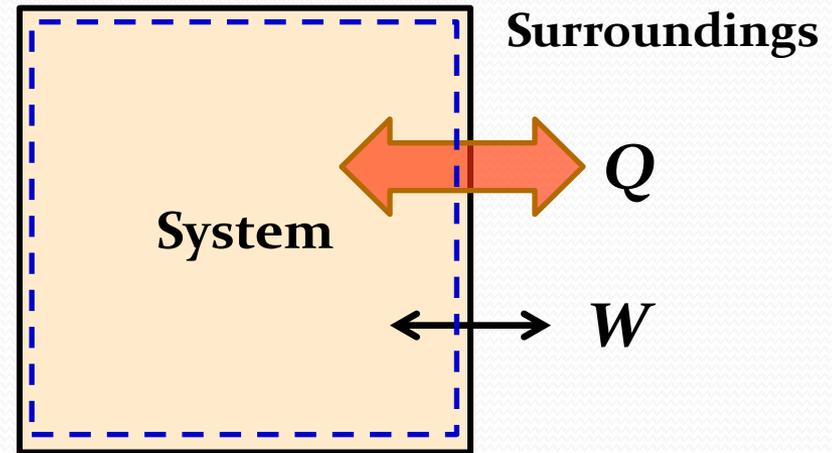
$$\oint \delta Q - \oint \delta W = 0$$



$$Q_{1-2} - W_{1-2} = E_2 - E_1$$

# Closed Systems

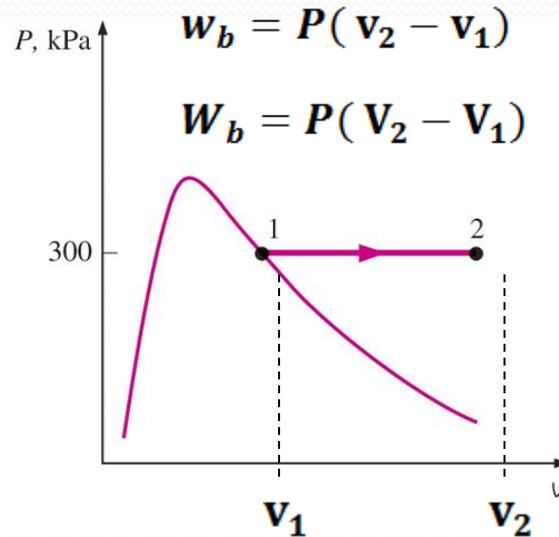
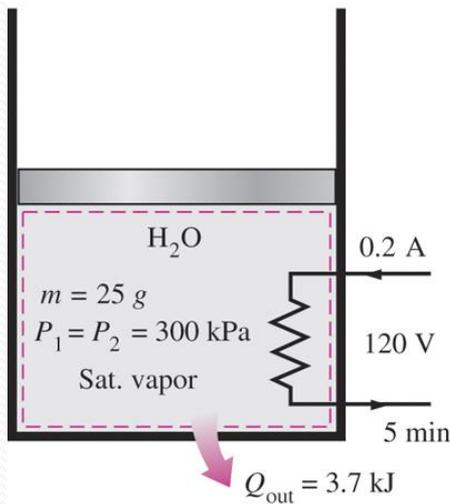
- Also known as *control mass*
- No mass can cross the boundary
- Energy (in the form of heat and work) can cross the boundary



$$\left\{ \begin{array}{l} \text{Net energy transfer} \\ \text{to (or from) the system} \\ \text{as heat and work} \end{array} \right\} = \left\{ \begin{array}{l} \text{Net increase (or decrease)} \\ \text{in the total energy} \\ \text{of the system} \end{array} \right\}$$

$$\begin{aligned} Q - W &= \Delta E = \Delta U + \Delta KE + \Delta PE \\ &= (U_2 - U_1) + \frac{m(v_2^2 - v_1^2)}{2} + mg(z_2 - z_1) \quad (\text{kJ}) \end{aligned}$$

**Example:** constant-pressure process, initially saturated water vapor. Evaluate  $T_2$



$$\underbrace{E_{\text{in}} - E_{\text{out}}}_{\text{Net energy transfer by heat, work, and mass}} = \underbrace{\Delta E_{\text{system}}}_{\text{Change in internal, kinetic, potential, etc., energies}}$$

$$Q - W = \Delta U + \cancel{\Delta KE}^0 + \cancel{\Delta PE}^0$$

$$Q - W_{\text{other}} - W_b = U_2 - U_1$$

$$Q - W_{\text{other}} - P_0(V_2 - V_1) = U_2 - U_1$$

$$Q - W_{\text{other}} = (U_2 + P_2V_2) - (U_1 + P_1V_1)$$

$$H = U + PV$$

$$Q - W_{\text{other}} = H_2 - H_1$$

$$Q - W_{\text{el}} = m(h_2 - h_1) =$$

$$-3.7 \text{ kJ} - (-7.2 \text{ kJ}) = 0.025 \text{ kg} (h_2 - 2725.3 \text{ kJ/kg})$$

$$h_2 = 2865.3 \text{ kJ/kg}$$

$$\left. \begin{array}{l} P_2 = 300 \text{ kPa} \\ h_2 = 2865.3 \end{array} \right\}$$

$$T_2 = 200^\circ\text{C} \quad \text{Table A-6}$$

NOTE:

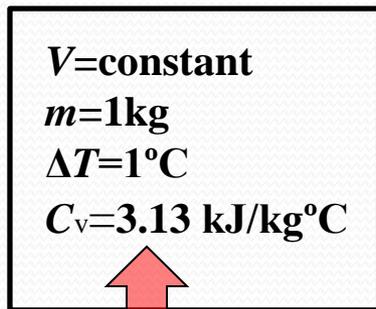
$W$  in energy equation means  $W_{\text{boundary}} + W_{\text{other}}$

For constant pressure case:  $Q - W_{\text{other}} = \Delta H$

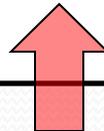
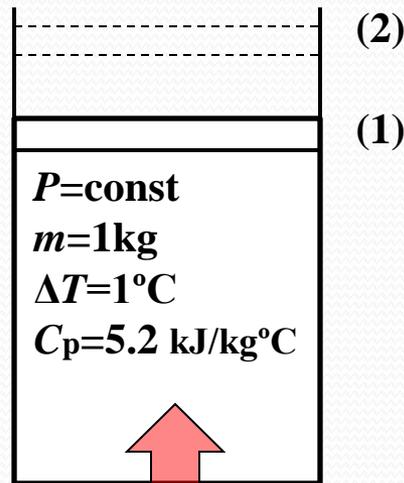
# Specific Heats

- The energy required to raise the temperature of a unit mass of a substance by one degree.
- $C_v$  : specific heat at constant volume
- $C_p$  : specific heat at constant pressure

**Helium gas:**



**3.13kJ**



**5.2kJ**

$$C_p > C_v$$

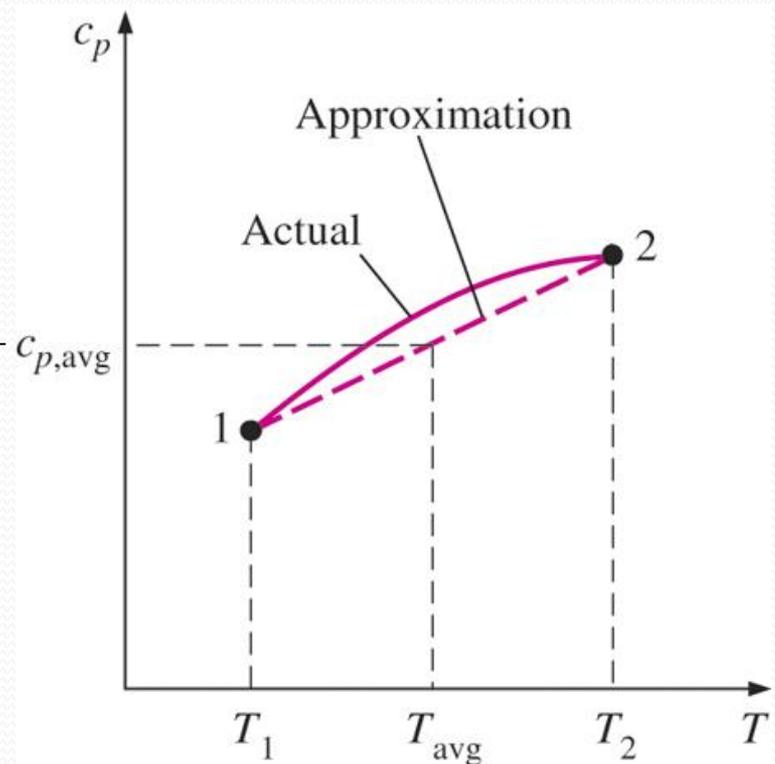
Because at constant pressure, the energy required for expansion work must also be supplied to system.

# Three ways of calculating $\Delta u$ and $\Delta h$

$$\Delta h = h_2 - h_1 \leftarrow \text{from tables}$$

$$\Delta h = \int_1^2 c_p(T) dT$$

$$\Delta h \cong c_{p,avg} \Delta T$$



Similarly:

$$\Delta u = u_2 - u_1 \text{ (table)}$$

$$\Delta u = \int_1^2 c_v(T) dT$$

$$\Delta u \cong c_{v,avg} \Delta T$$

# Work Transfer

Work interaction with the surroundings is experienced if

- A force is present on the boundary and
- Boundary must move

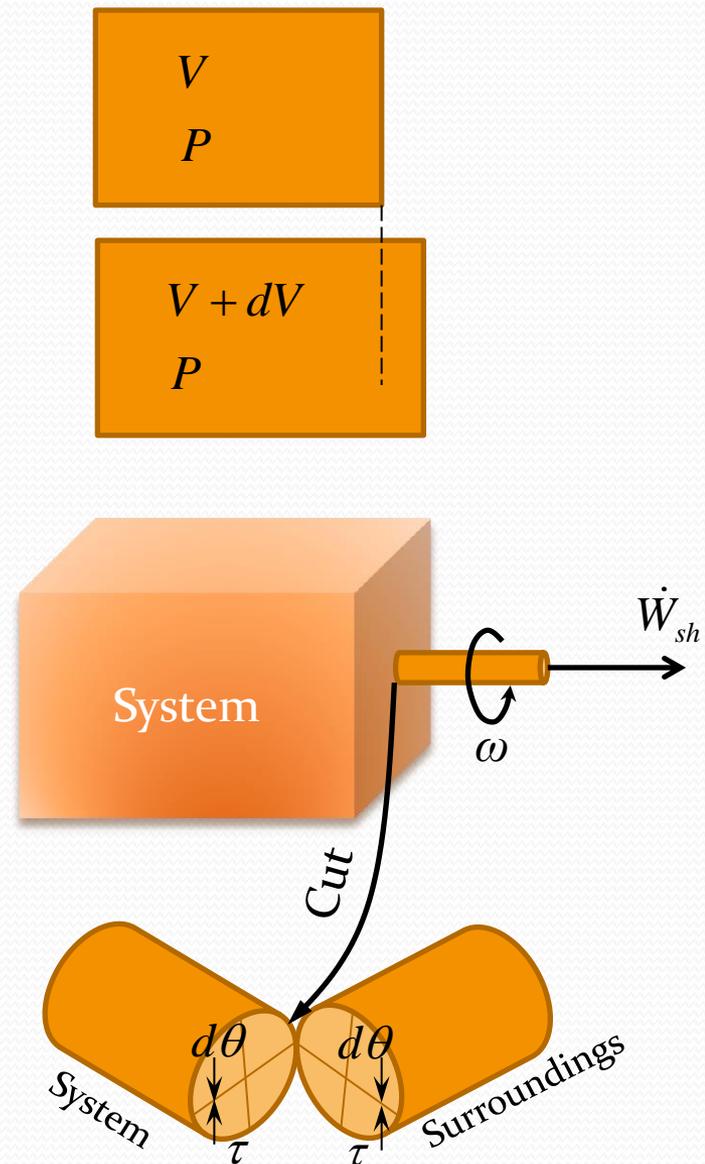
$$\delta W = -\mathbf{F} \cdot d\mathbf{r}$$

Work due to expansion of fluid

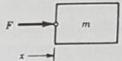
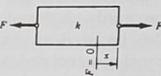
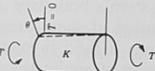
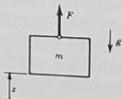
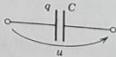
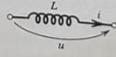
$$\delta W_{rev} = P \cdot dV$$

Shaft work

$$\delta W_{rev} = \tau \cdot d\theta$$



# Simple forms of energy storage and corresponding work interactions

Macroscopic Forms of Energy Storage, $(E_2 - E_1)$ , [Eq. (1.13)]	Relation Assumed in Writing Each Expression	Infinitesimal Work Transfer, $\delta W$ [Eq. (1.8)]	Notation
Kinetic, translational $\frac{1}{2} mV_2^2 - \frac{1}{2} mV_1^2$	$F = m \frac{dV}{dt}$	$-F dx$	
Kinetic, rotational $\frac{1}{2} J\omega_2^2 - \frac{1}{2} J\omega_1^2$	$T = J \frac{d\omega}{dt}$	$-T d\theta$	
Spring, translational $\frac{1}{2} kx_2^2 - \frac{1}{2} kx_1^2$	$F = kx$	$-F dx$	
Spring, rotational $\frac{1}{2} K\theta_2^2 - \frac{1}{2} K\theta_1^2$	$T = K\theta$	$-T d\theta$	
Gravitational spring (or constant-force translational spring) $mgz_2 - mgz_1$	$F = mg$	$-F dz$	
Electrical capacitance $\frac{1}{2} \frac{q^2}{C} = \frac{1}{2} Cu^2$	$u = \frac{q}{C}$	$-u dq$	
Electrical inductance $\frac{1}{2} Li^2 = \frac{1}{2} L\Phi^2$	$\Phi = Li$	$-i d\Phi$	

Systems can experience more than one form of work interaction. Therefore the previous equation ( $\delta W_{rev} = P dV$ ) can be replaced by:

$$\delta W_{rev} = -\sum_i Y_i dX_i$$

where

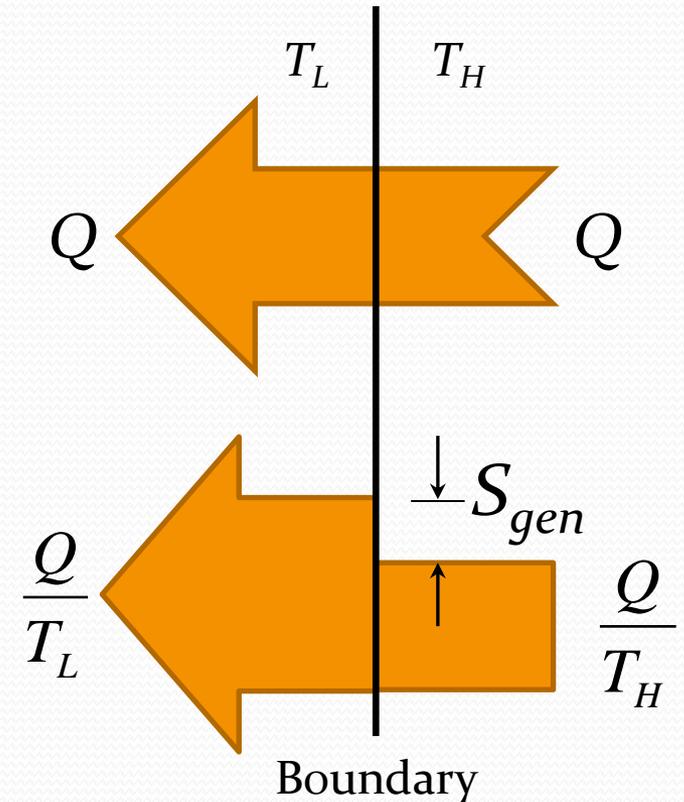
$Y_i$  are generalized forces and

$X_i$  are generalized displacements

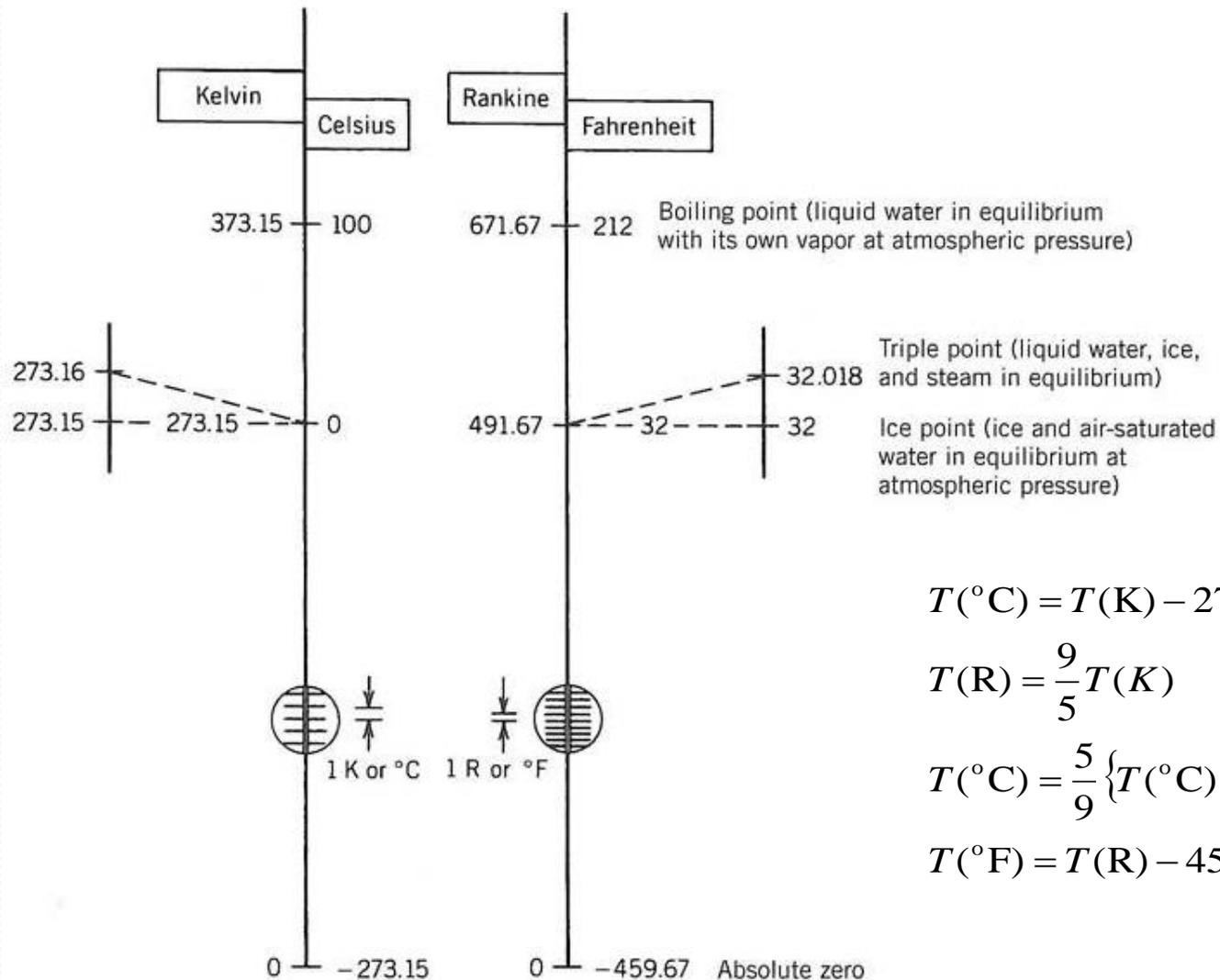
(Note: Negative of the pressure used in the previous equation due to work done on the system)

# Heat Transfer

- Heat transfer is the energy interaction determined by the temperature difference between the system and its surroundings.
- «Heat transfer is the energy interaction accompanied by entropy transfer, whereas work transfer is the energy interaction that takes place in the absence of entropy transfer.» *Adrian Bejan*



# Temperature scales



$$T(^{\circ}\text{C}) = T(\text{K}) - 273.15$$

$$T(\text{R}) = \frac{9}{5} T(\text{K})$$

$$T(^{\circ}\text{C}) = \frac{5}{9} \{T(^{\circ}\text{F}) - 32\}$$

$$T(^{\circ}\text{F}) = T(\text{R}) - 459.67$$

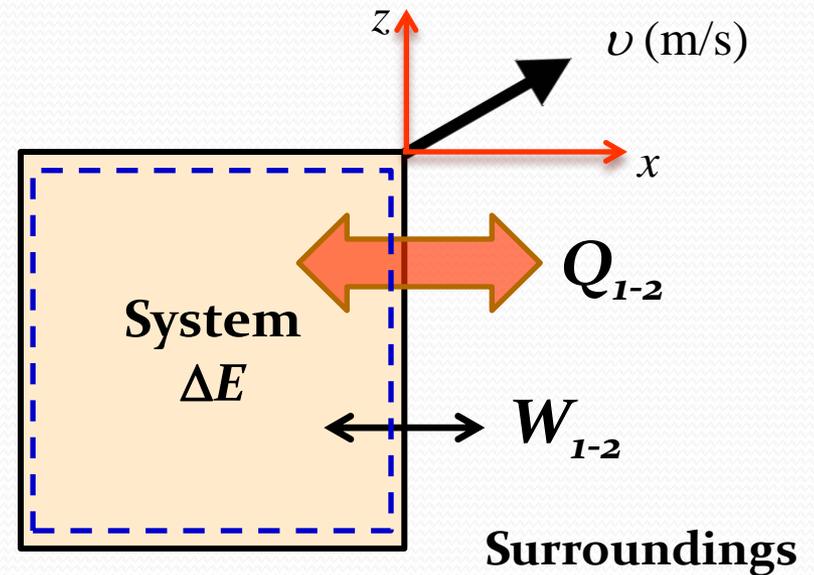
# Energy Change

First law for closed systems :

$$Q_{1-2} - W_{1-2} = \underbrace{E_2 - E_1}_{\text{Energy change}}$$

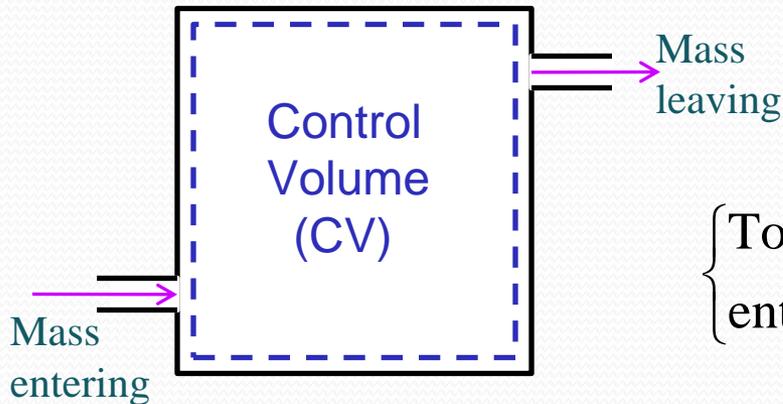
The general expression for energy change :

$$\underbrace{E_2 - E_1}_{\text{Energy change}} = \underbrace{U_2 - U_1}_{\text{Internal energy change}} + \underbrace{\frac{1}{2}m v_2^2 - \frac{1}{2}m v_1^2}_{\text{Kinetic energy change}} + \underbrace{mgz_2 - mgz_1}_{\text{Potential energy change}}$$



# Open Systems (or Control Volume)

**Control volume** can be thought of a region of space through which mass flows.



## Conservation of mass principle

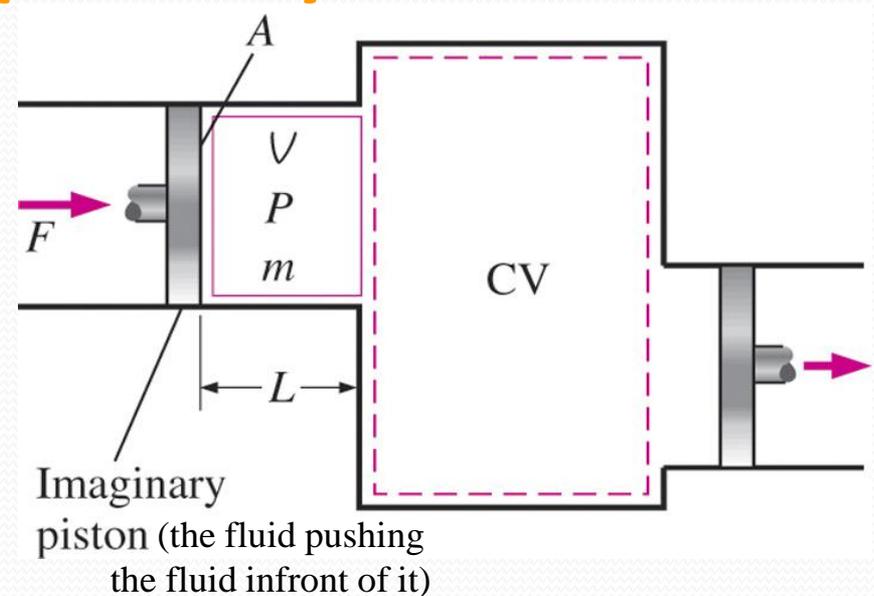
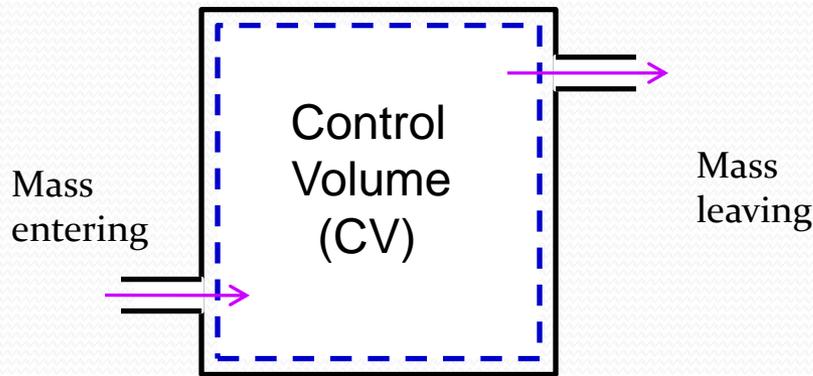
$$\left\{ \begin{array}{l} \text{Total mass} \\ \text{entering CV} \end{array} \right\} - \left\{ \begin{array}{l} \text{Total Mass} \\ \text{leaving CV} \end{array} \right\} = \left\{ \begin{array}{l} \text{Net change in} \\ \text{mass within CV} \end{array} \right\}$$

$$\sum m_i - \sum m_e = \Delta m_{CV}$$

**Note: *i*-inlet , *e*-exit , CV-control volume**

The rate form  $\rightarrow$  
$$\sum \dot{m}_i - \sum \dot{m}_e = \Delta \dot{m}_{CV}$$

# Flow Work in Open Systems



## Flow energy or flow work

The energy required to push fluid into or out of a control volume.

The force applied on the fluid element :  $F = P A$

The work done in pushing the fluid element into the control volume:

i.e. the flow work :  $W_{flow} = F L = P A L = P V$  (kJ)

On a unit mass basis  $w_{flow} = P v$  (kJ/kg)

# Energy Equation for Open Systems (also known as control volume)

$$\left\{ \begin{array}{l} \text{Total energy} \\ \text{crossing boundary} \\ \text{as heat and work} \end{array} \right\} + \left\{ \begin{array}{l} \text{Total energy of} \\ \text{mass entering the} \\ \text{control volume} \end{array} \right\} - \left\{ \begin{array}{l} \text{Total energy of} \\ \text{mass leaving the} \\ \text{control volume} \end{array} \right\} = \left\{ \begin{array}{l} \text{Net change} \\ \text{in Energy of} \\ \text{control volume} \end{array} \right\}$$



$$Q - W + \sum E_{in} - \sum E_{out} = \Delta E_{cv}$$

[remember  $h = u + Pv$  therefore total energy  $e = \underbrace{u + Pv}_h + \frac{1}{2}v^2 + gz$ ]

In the rate form the energy equation in full:

$$\frac{dE}{dt} = \dot{Q} - \dot{W} + \sum_{in} \dot{m} \left( h + \frac{1}{2}v^2 + gz \right) - \sum_{out} \dot{m} \left( h + \frac{1}{2}v^2 + gz \right)$$

# Energy Equation for Steady Flow Devices

## Steady flow energy equation (SFEE)

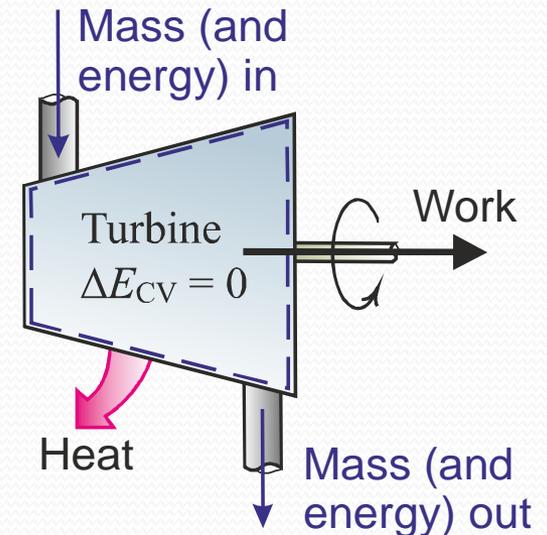
$$E_{CV} = \text{constant} \quad \text{or} \quad \Delta E_{CV} = 0$$

$$\left\{ \begin{array}{l} \text{Total energy} \\ \text{crossing boundary} \\ \text{as heat and work} \end{array} \right\} + \left\{ \begin{array}{l} \text{Total energy of} \\ \text{mass entering the} \\ \text{control volume} \end{array} \right\} - \left\{ \begin{array}{l} \text{Total energy of} \\ \text{mass leaving the} \\ \text{control volume} \end{array} \right\} = 0$$

$$Q - W + \sum E_{in} - \sum E_{out} = 0$$

In the rate form ( $i = \text{inlet}$ ,  $e = \text{exit}$ )

$$\Rightarrow \dot{Q} - \dot{W} = \sum \dot{m}_e \left( h_e + \frac{V_e^2}{2} + gz_e \right) - \sum \dot{m}_i \left( h_i + \frac{V_i^2}{2} + gz_i \right)$$



# Ex: Steam Turbine

For steady flow:

$$\dot{m}_1 = \dot{m}_2 = \dot{m}$$

$$\dot{Q} - \dot{W} = \dot{m} \left\{ h_2 - h_1 + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1) \right\}$$

$$\frac{\dot{Q}}{\dot{m}} - \frac{\dot{W}}{\dot{m}} + \left( h + \frac{V^2}{2} + gz \right)_1 - \left( h + \frac{V^2}{2} + gz \right)_2 = 0$$

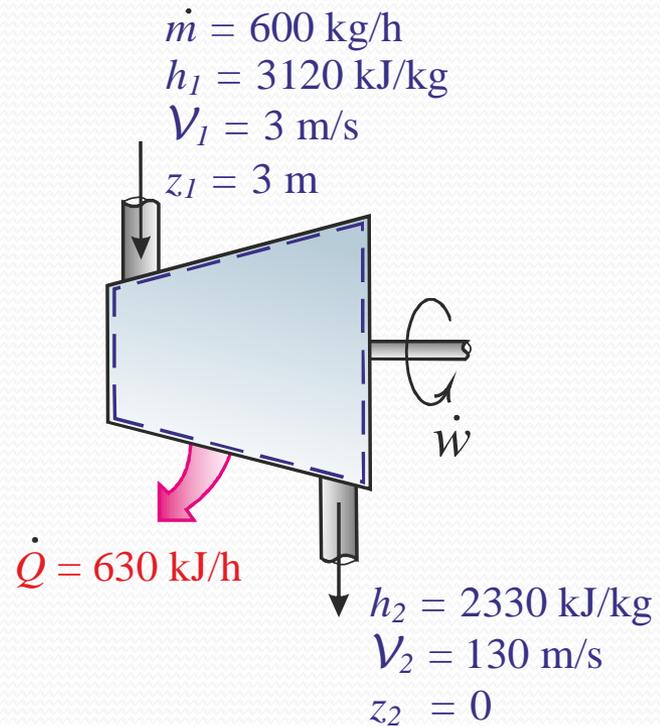
$$\dot{m} = \frac{600 \text{ kg/hr}}{3600 \text{ sec/hr}} = 0.167 \text{ kg/sec}$$

$$\dot{Q} = \frac{630 \text{ kJ/hr}}{3600 \text{ sec/hr}} = 0.175 \text{ kW}$$

$$-\frac{0.175 \text{ kJ/s}}{0.167 \text{ kg/s}} - \frac{\dot{W}}{0.167 \text{ kg/s}} + \left( 3120 \text{ kJ/kg} + \frac{(100 \text{ m/s})^2}{2(1000 \text{ J/kJ})} + \frac{9.81 \text{ m/s}^2}{1000 \text{ J/kJ}} \right) - \left( 2330 \text{ kJ/kg} + \frac{(130 \text{ m/s})^2}{2(1000 \text{ J/kJ})} + 0 \right) = 0$$

$$\Rightarrow -1.05 \text{ kJ/kg} - \frac{\dot{W}}{0.167 \text{ kg/s}} + \left( 3120 \text{ kJ/kg} + 5 \text{ kJ/kg} + 0.029 \text{ kJ/kg} \right) - (2330 + 8.45 + 0) = 0$$

$$\Rightarrow \dot{W} = +131.183 \text{ kW}$$



# Second Law of Thermodynamics: Is it required?

First Law says:

Potential energy lost = Kinetic energy gained

Can he fly backwards?

According to First Law:

Kinetic energy lost =  
Potential energy gained

First Law Satisfied!?!?



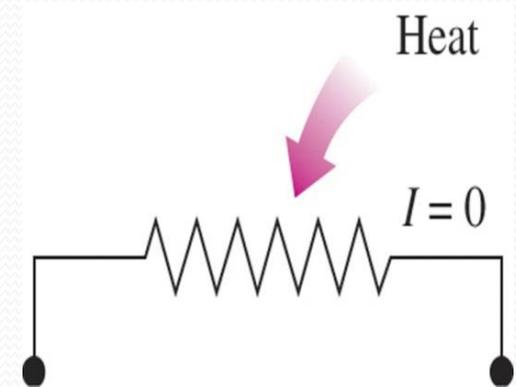
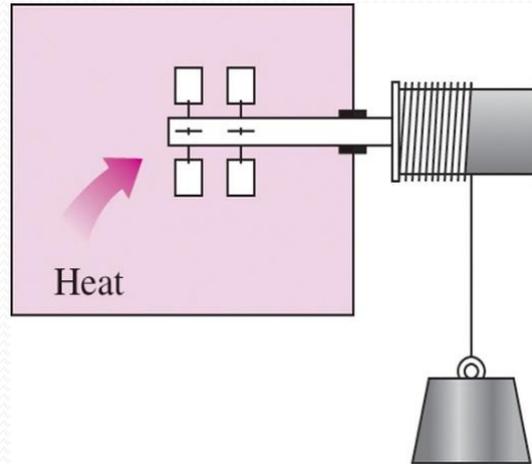
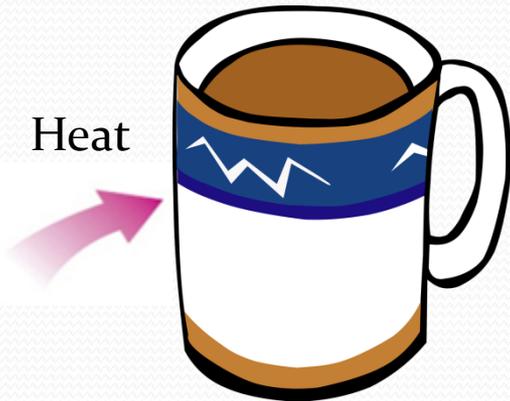
**We need a second law!  
Processes can only  
happen in one  
direction and not in  
the other.**

# Why 2nd Law?

A cup of hot coffee does not get hotter in a cooler room.

Transferring heat to a paddle wheel will not cause it to rotate.

Transferring heat to a wire will not generate electricity.



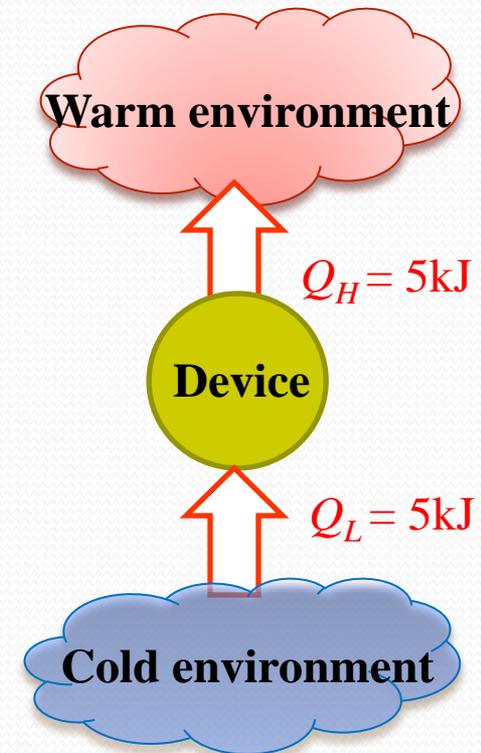
It is clear from the above examples that processes take place in a certain direction and not in the reverse direction. First law alone is not enough to determine if a process will actually occur.

Another principle is needed: Second law of Thermodynamics

# Clausius Statement of the 2nd Law

It is impossible for any device to operate in such a manner that it produces no effect other than the transfer of heat from one body to another body at a higher temperature.

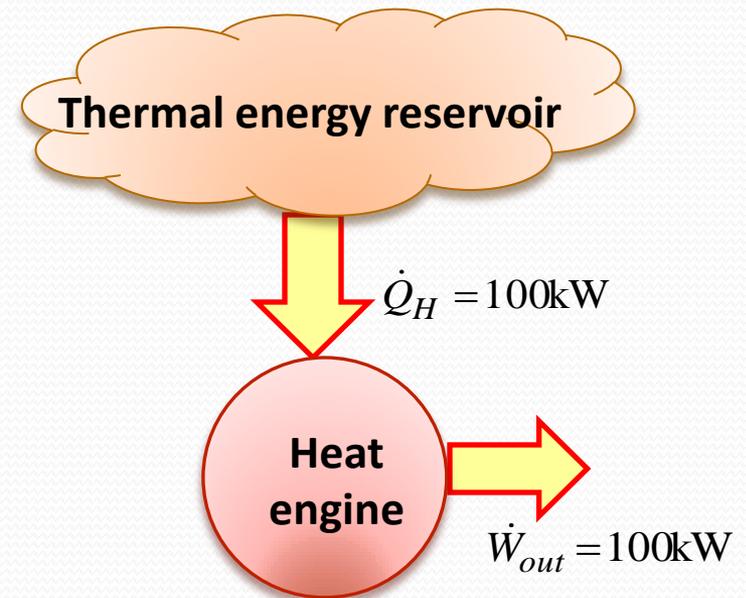
A refrigerator that violates the Clausius statement of the second law.



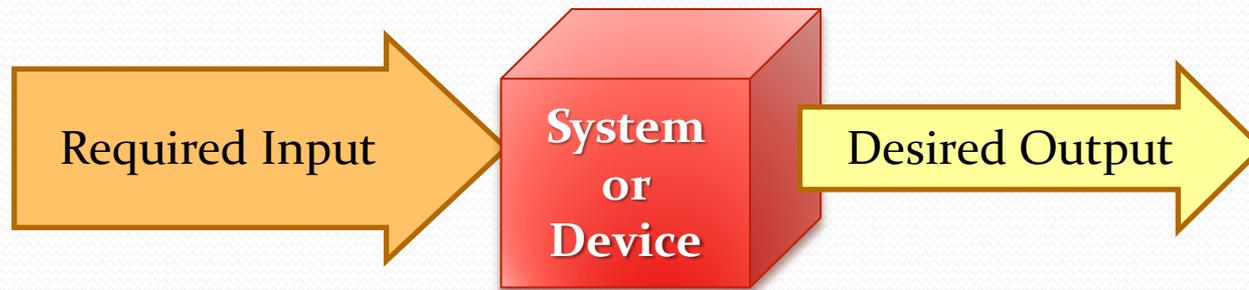
# Kelvin-Planck Statement of the 2nd Law

It is impossible for any device to operate in a cycle and produce work while exchanging heat only with a single reservoir (i.e. no engine can have 100% efficiency).

A heat engine that violates the Kelvin-Planck statement of the second law.



# Definition of Performance for Engineering Devices



$$\text{Performance} = \frac{\text{Desired Output}}{\text{Required Input}}$$

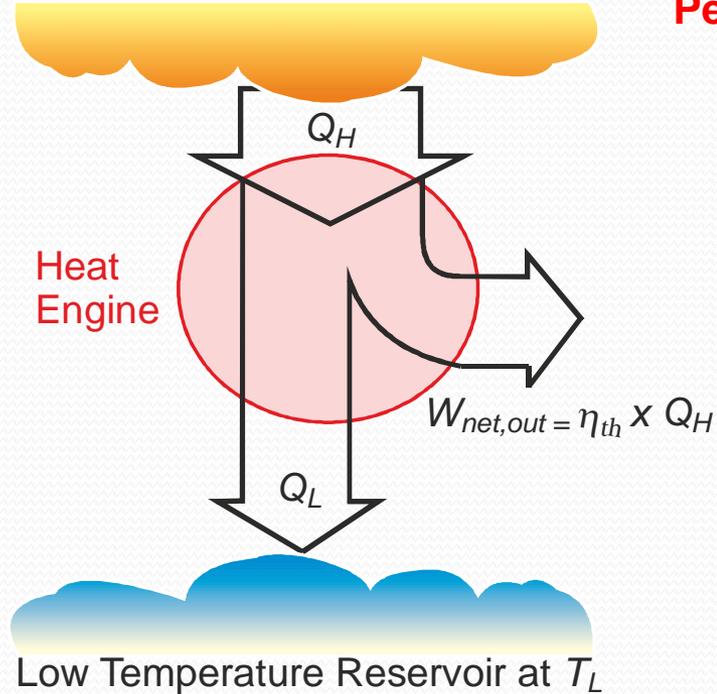
or easy to remember version :

(for devices consuming fuel or electricity)

$$\text{Performance} = \frac{\text{What I get}}{\text{What I pay for}}$$

# Performance of Heat Engines: Thermal Efficiency

High Temperature Reservoir at  $T_H$



$$\text{Performance} = \frac{\text{Desired output}}{\text{required input}} = \frac{\text{What I Get}}{\text{What I pay for}}$$

In heat engines:

the desired output = net work output =  $W_{net,out}$

the required input = heat supplied to system =  $Q_{in}$

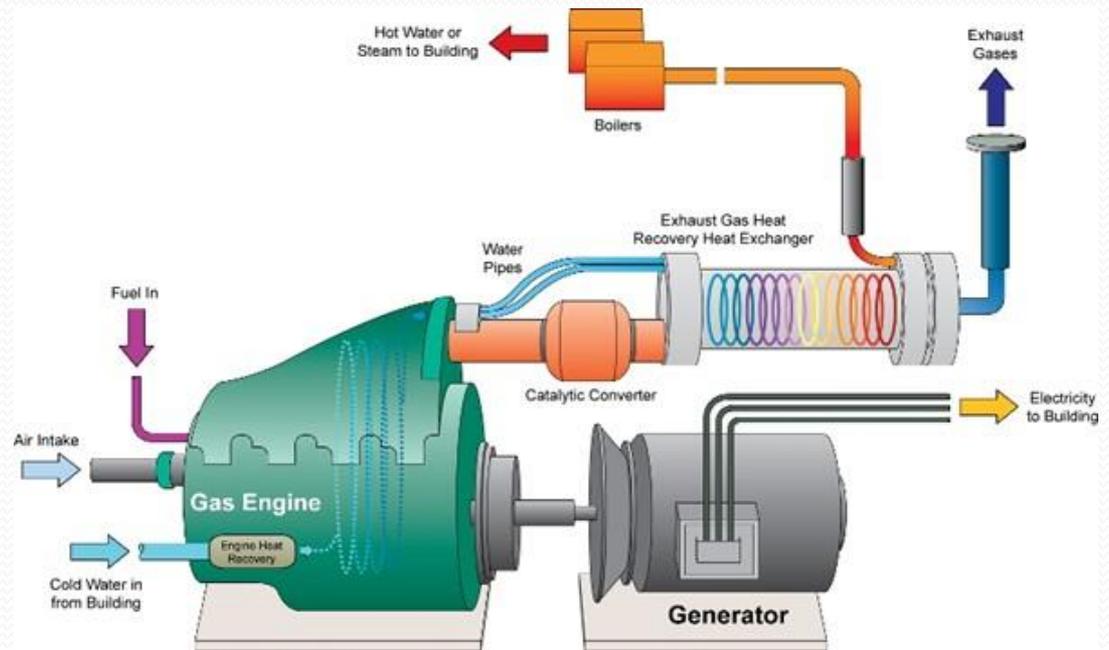
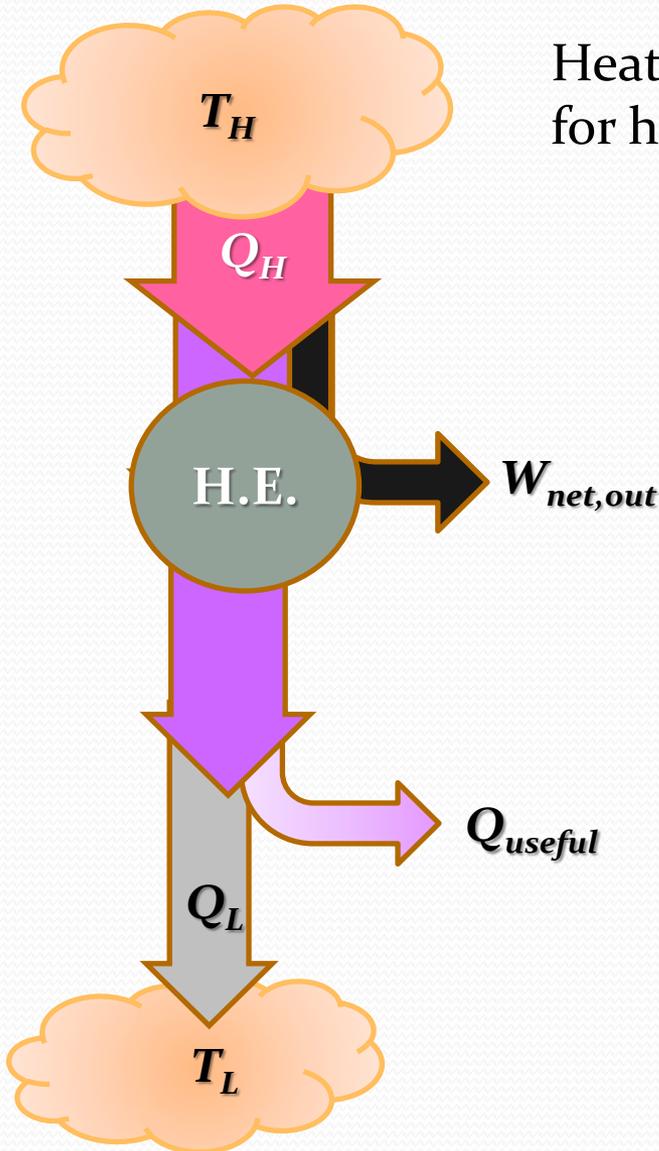
Thermal efficiency is denoted with  $\eta_{th}$

$$\eta_{th} = \frac{W_{net,out}}{Q_{in}} = \frac{Q_{in} - Q_{out}}{Q_{in}} = 1 - \frac{Q_{out}}{Q_{in}} \quad \text{or}$$

$$\eta_{th} = \frac{W_{net,out}}{Q_H} = 1 - \frac{Q_L}{Q_H}$$

# Performance of Cogeneration Units

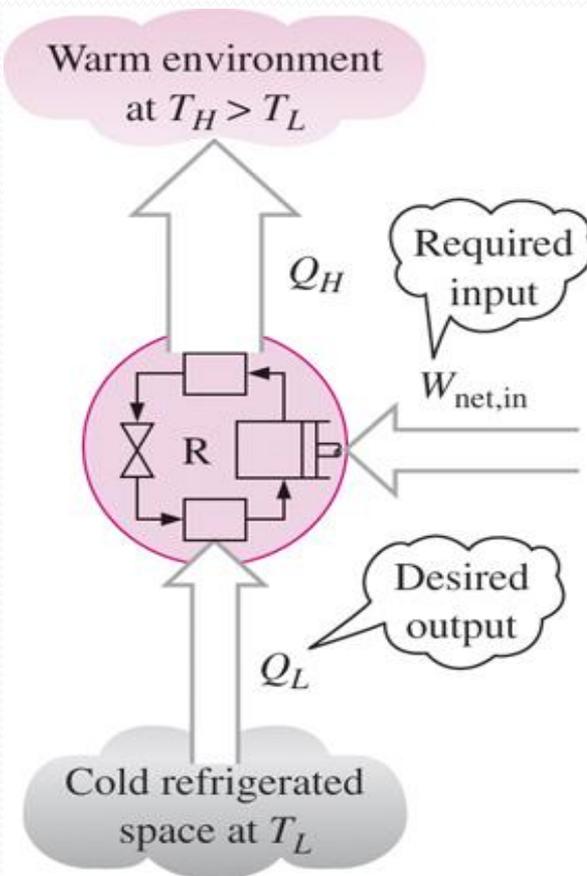
Heat rejection from a heat engine can be used for heating processes as well



Energy Utilization Factor:

$$\varepsilon = \frac{W_{net,out} + Q_{useful}}{Q_H}$$

# Performance of Refrigerators: Coefficient of Performance ( $COP$ )



Notice that  $COP_R$  can be greater than unity.

- The *efficiency* of a refrigerator is expressed in terms of the coefficient of performance (COP).
- The objective of a refrigerator is to remove heat ( $Q_L$ ) from the refrigerated space.

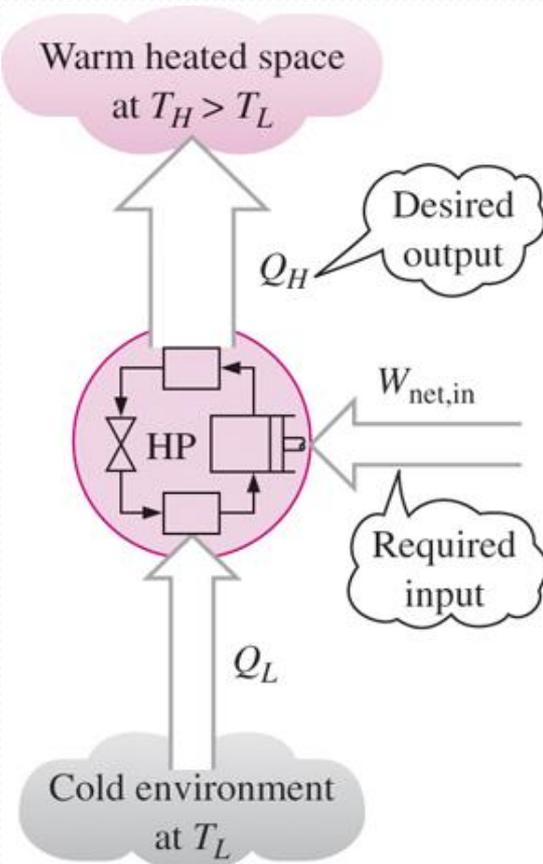
**For a refrigerator  $COP_R = \frac{\text{Desired output}}{\text{Required input}}$**

$$COP_R = \frac{Q_L}{W_{net,in}} \quad (\text{or} \quad \frac{\dot{Q}_L}{\dot{W}_{net,in}})$$

$$W_{net,in} = Q_H - Q_L \quad (\text{kJ})$$

$$COP_R = \frac{Q_L}{Q_H - Q_L} = \frac{1}{\frac{Q_H}{Q_L} - 1}$$

# Coefficient of Performance (*COP*) of Heat Pumps



The objective of a heat pump is to supply heat  $Q_H$  into the warmer space.

- another device that transfer heat from  $T_L$  to  $T_H$ .
- objective is different : maintain a heated space at high temperature.

$$COP_{HP} = \frac{\text{Desired output}}{\text{Required input}} = \frac{Q_H}{W_{net,in}}$$

$$COP_{HP} = \frac{Q_H}{Q_H - Q_L} = \frac{1}{1 - \frac{Q_L}{Q_H}}$$

# Heat Pump Example

$$COP_{HP} = \frac{\dot{Q}_H}{\dot{W}_{net,in}} \rightarrow \dot{W}_{net,in} = \frac{\dot{Q}_H}{COP_{HP}} = \frac{75,000}{2.5}$$

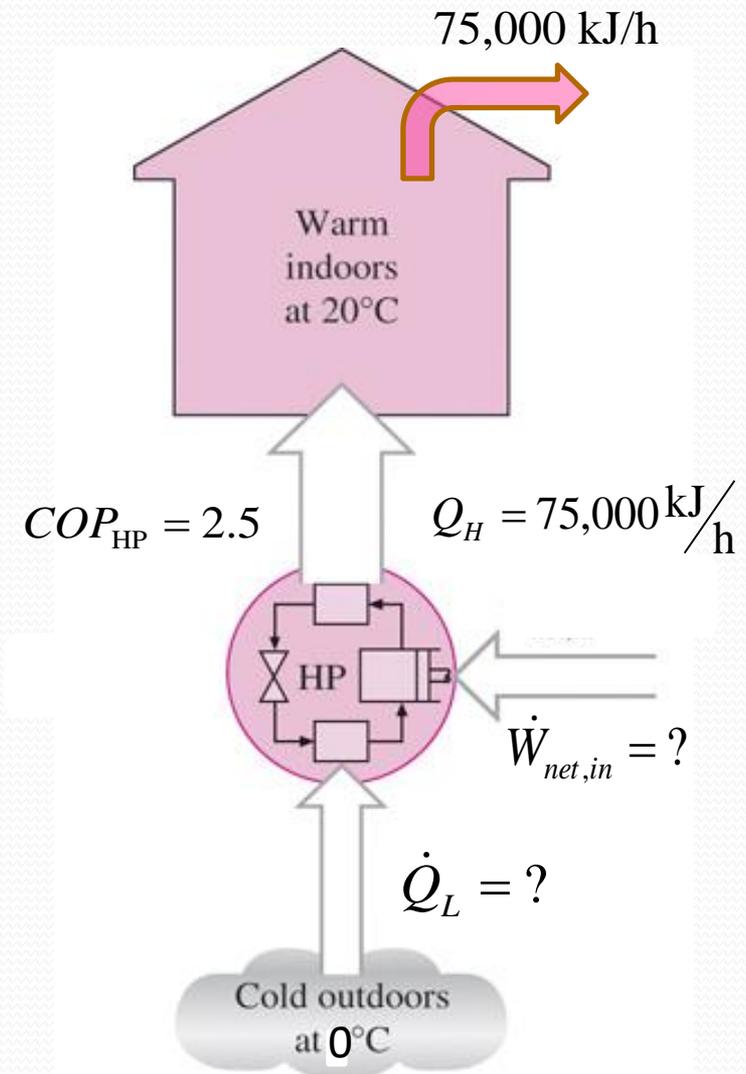
Net power consumed by the heat pump :

$$\dot{W}_{net,in} = 30,000 \text{ kJ/h (or 8.33 kW)}$$

How much heat extracted from outdoors?

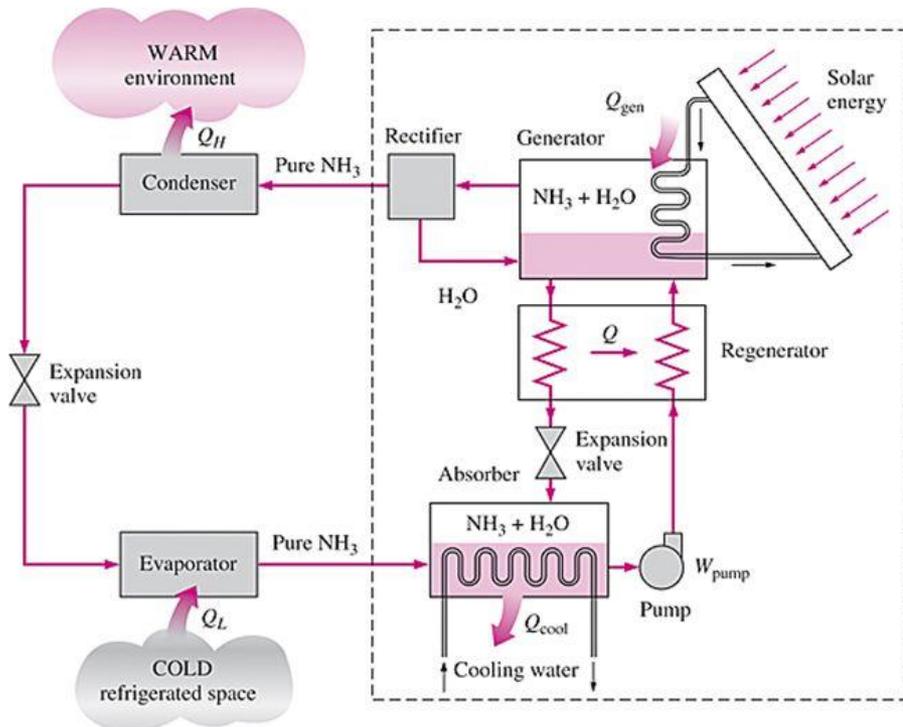
$$\dot{Q}_L = \dot{Q}_H - \dot{W}_{net,in} = (75,000 - 30,000)$$

$$\dot{Q}_L = 45,000 \text{ kJ/h}$$



# COP of Absorption Refrigeration Systems

## ABSORPTION REFRIGERATION SYSTEMS



When there is a source of inexpensive thermal energy at a temperature of 100 to 200°C is **absorption refrigeration**.

Some examples include **geothermal energy, solar energy, and waste heat from cogeneration or process steam plants, and even natural gas** when it is at a relatively low price.



$$COP_{\text{absorption}} = \frac{\text{Desired Output}}{\text{Required Input}}$$

$$= \frac{Q_L}{Q_{\text{gen}} + W_{\text{pump, in}}} \approx \frac{Q_L}{Q_{\text{gen}}}$$

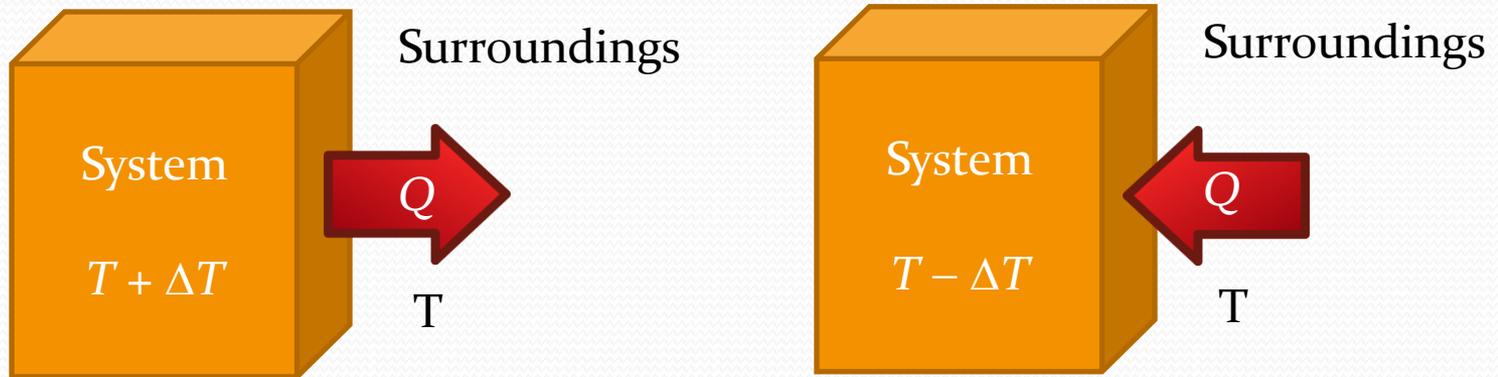
Very small compared to  $Q_{\text{gen}}$

Ammonia absorption refrigeration cycle.

Source: Y. A. Çengel and M. A. Boles, Thermodynamics: An Engineering Approach, McGraw-Hill (Book)

# Reversible and Irreversible Processes

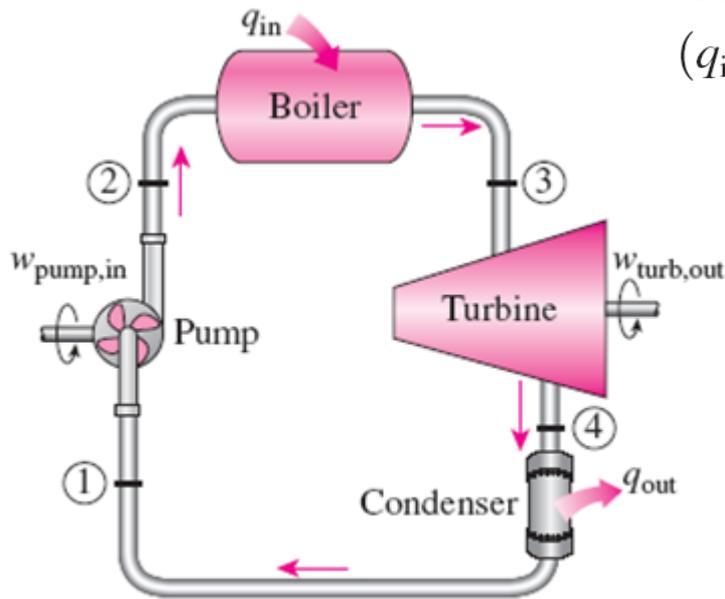
In a reversible process the system changes in such a way that when the process is reversed the system and the surroundings are restored to their original states. Changes are infinitesimally small in a reversible process.



# Vapor Cycles

Steady-flow energy equation for each component:

$$(q_{in} - q_{out}) + (w_{in} - w_{out}) = h_e - h_i \quad (\text{kJ/kg})$$



*Pump* ( $q = 0$ ):

$$w_{pump,in} = h_2 - h_1$$

$$w_{pump,in} = v(P_2 - P_1)$$

$$h_1 = h_f @ P_1 \quad \text{and} \quad v \cong v_1 = v_f @ P_1$$

*Boiler* ( $w = 0$ ):

$$q_{in} = h_3 - h_2$$

*Turbine* ( $q = 0$ ):

$$w_{turb,out} = h_3 - h_4$$

*Condenser* ( $w = 0$ ):

$$q_{out} = h_4 - h_1$$

$$w_{net} = q_{in} - q_{out} = w_{turb,out} - w_{pump,in}$$

$$\eta_{th} = \frac{w_{net}}{q_{in}} = 1 - \frac{q_{out}}{q_{in}}$$

# Brayton Cycles

$$(q_{in} - q_{out}) + (w_{in} - w_{out}) = h_{exit} - h_{inlet}$$

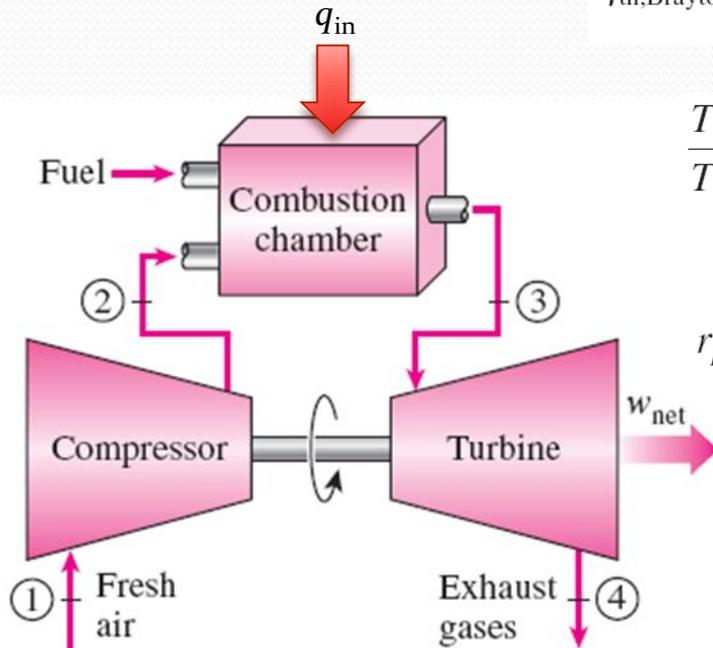
$$q_{in} = h_3 - h_2 = c_p(T_3 - T_2)$$

$$q_{out} = h_4 - h_1 = c_p(T_4 - T_1)$$

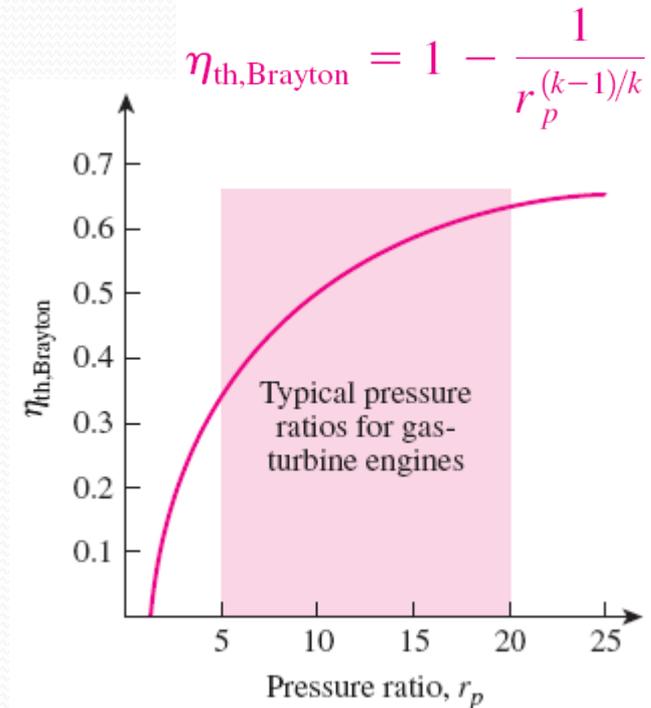
$$\eta_{th,Brayton} = \frac{w_{net}}{q_{in}} = 1 - \frac{q_{out}}{q_{in}} = 1 - \frac{c_p(T_4 - T_1)}{c_p(T_3 - T_2)} = 1 - \frac{T_1(T_4/T_1 - 1)}{T_2(T_3/T_2 - 1)}$$

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{(k-1)/k} = \left(\frac{P_3}{P_4}\right)^{(k-1)/k} = \frac{T_3}{T_4}$$

$$r_p = \frac{P_2}{P_1} \text{ Pressure ratio}$$



Thermal efficiency of the ideal Brayton cycle as a function of the pressure ratio.



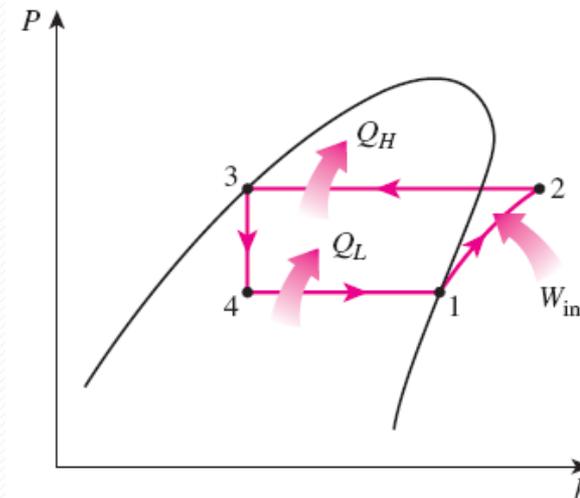
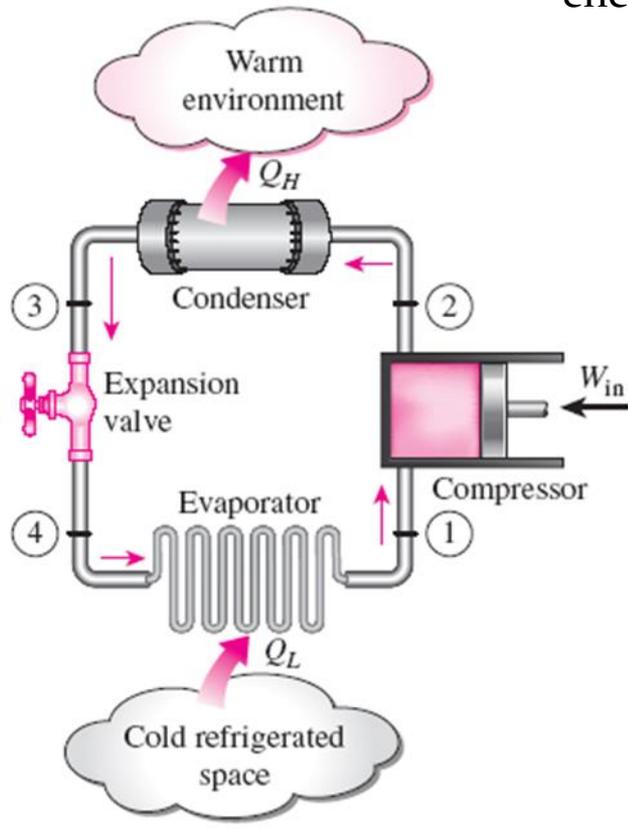
# Refrigeration Cycles

Steady-flow energy balance  $(q_{in} - q_{out}) + (w_{in} - w_{out}) = h_e - h_i$

$$COP_R = \frac{q_L}{w_{net,in}} = \frac{h_1 - h_4}{h_2 - h_1}$$

$$COP_{HP} = \frac{q_H}{w_{net,in}} = \frac{h_2 - h_3}{h_2 - h_1}$$

$h_1 = h_g @ P_1$  and  $h_3 = h_f @ P_3$  for the ideal case



The  $P$ - $h$  diagram of an ideal vapor-compression refrigeration cycle.