CHAPTER 5 – ENTROPY GENERATION

Instructor: Prof. Dr.UGHur Atikol
Chapter 5
Entropy Generation (Exergy Destruction)

Outline
• Lost Available Work
• Cycles
  • Heat engine cycles
  • Refrigeration cycles
  • Heat pump cycles
• Nonflow Processes
• Steady-Flow Processes
• Exergy wheel diagrams
Lost Available Work

Atmospheric temperature and pressure reservoir at \((T_0, P_0)\)

\[
\dot{W} = P \frac{dV}{dt} + \dot{W}_{electrical} + \dot{W}_{shear} + \dot{W}_{magnetic}
\]

Work done against the atmosphere

\[
\dot{W} = \frac{P_0}{dt} \frac{dV}{dt}
\]

(All modes of work transfer)

\[
\dot{W}_u = \dot{W} - P_0 \frac{dV}{dt}
\]

\[
\dot{m}_u = \dot{m}_{in} - \dot{m}_{out}
\]

\[
\dot{Q}_0 \quad \dot{Q}_1 \quad \dot{Q}_2 \quad \dot{Q}_n
\]

System

Reservoir at \(T_1\)

Reservoir at \(T_2\)

Reservoir at \(T_n\)
Lost Available Work

**First law:**
\[
\frac{dE}{dt} = \sum_{i=0}^{n} \dot{Q}_i - \dot{W} + \sum_{\text{in}} \dot{m}h^o + \sum_{\text{out}} \dot{m}h^o
\]

Note: \( h^o \) is known as [methalpy](#), such that
\[
h^o = h + \frac{V^2}{2} + gz
\]

**Second law:**
\[
\dot{S}_{\text{gen}} = \frac{dS}{dt} - \sum_{i=0}^{n} \frac{\dot{Q}_i}{T_i} - \sum_{\text{in}} \dot{m}s + \sum_{\text{out}} \dot{m}s \geq 0
\]
Lost Available Work

\[
\frac{dE}{dt} = \sum_{i=0}^{n} \dot{Q}_i - \dot{W} + \sum_{in} \dot{m} h^o + \sum_{out} \dot{m} h^o
\]

\[
\dot{S}_{gen} = \frac{dS}{dt} - \sum_{i=0}^{n} \frac{\dot{Q}_i}{T_i} - \sum_{in} \dot{m} s + \sum_{out} \dot{m} s \geq 0
\]

Eliminate \(\dot{Q}_0\) between the two equations:

\[
\dot{W} = -\frac{d}{dt} (E - T_0 S) + \sum_{i=1}^{n} \left(1 - \frac{T_0}{T_i}\right) \dot{Q}_i + \sum_{in} \dot{m} (h^o - T_0 s) - \sum_{out} \dot{m} (h^o - T_0 s) - T_0 \dot{S}_{gen}
\]

When reversible \(\dot{S}_{gen}\) is zero, hence:

\[
\dot{W}_{rev} = -\frac{d}{dt} (E - T_0 S) + \sum_{i=1}^{n} \left(1 - \frac{T_0}{T_i}\right) \dot{Q}_i + \sum_{in} \dot{m} (h^o - T_0 s) - \sum_{out} \dot{m} (h^o - T_0 s)
\]

Therefore generally:

\[
\dot{W} = \dot{W}_{rev} - T_0 \dot{S}_{gen}
\]

However we know that \(\dot{W}_{lost} = \dot{W}_{rev} - \dot{W}\)

Hence

\[
\dot{W}_{lost} = T_0 \dot{S}_{gen}
\]

Also known as «exergy destruction \(X_{des}\)» or «Irreversibility»
Lost Available Work

$\dot{W}_{lost}$ is always positive although $\dot{W}$ and $\dot{W}_{rev}$ can be either positive or negative (remember $\dot{W}_{lost} = \dot{W}_{rev} - \dot{W}$)

The main purpose of studying the lost available work is to diagnose the areas where irreversibilities are taking place in a process so that thermodynamic improvements can be made.
Lost Available Work

When the system is doing work against the atmosphere that has pressure $P_0$ then the atmosphere consumes a work rate of $P_0 \frac{dV}{dt}$ such that:

$$\dot{X}_W = \dot{W} - P_0 \frac{dV}{dt}$$

Rate of available work

In most flow systems $P_0 \frac{dV}{dt} = 0$, therefore $\dot{X}_W = \dot{W}$ (i.e., exergy transfer by work is simply the work itself)

$$= -\frac{d}{dt} (E + P_0 V - T_0 S) + \sum_{i=1}^{n} \left(1 - \frac{T_0}{T_i}\right) \dot{Q}_i$$

$$+ \sum_{in} \dot{m} (h^o - T_0 s) - \sum_{out} \dot{m} (h^o - T_0 s) - T_0 \dot{S}_{gen}$$
Lost Available Work

In the reversible limit:

\[
\left( \dot{X}_W \right)_{rev} = \dot{W}_{rev} - P_0 \frac{dV}{dt}
\]

\[
\left( \dot{X}_W \right)_{rev} = -\frac{d}{dt} (E + P_0 V - T_0 S) + \sum_{i=1}^{n} \left( 1 - \frac{T_0}{T_i} \right) \dot{Q}_i + \sum_{in} \dot{m}(h^o - T_0 s) - \sum_{out} \dot{m}(h^o - T_0 s)
\]

Maximum delivery of available power

Accumulation of nonflow exergy \( d\Phi/dt \)

Exergy transfer with heat transfer

Intake of flow exergy with mass flow \( \sum \dot{m} \psi \)

Release of flow exergy with mass flow \( \sum \dot{m} \psi \)

\[ \dot{X}_{Q_1} \]
\[ \dot{X}_{Q_2} \]
\[ \dot{X}_{Q_n} \]

\[ T_1 \]
\[ T_2 \]
\[ T_n \]

Environment \( (T_0, P_0) \)

Accumulation of nonflow exergy \( \frac{d\Phi}{dt} \)

Intake of flow exergy with mass flow \( \sum_{in} \dot{m} \psi \)

Release of flow exergy with mass flow \( \sum_{out} \dot{m} \psi \)

Maximum delivery of available mechanical power \( \left( \dot{X}_W \right)_{rev} \)
Lost Available Work

**Lost available work** is defined as the difference between the maximum available work $W_{rev}$ and the actual work $W$. Alternatively it can be defined as:

\[ \dot{X}_{lost} = (\dot{X}_W)_{rev} - \dot{X}_W \]

Same as $\dot{W}_{lost}$

Same as $\dot{X}_{des}$

**Diagram:**
- **Environment** $(T_0, P_0)$
- **Actual work** $\dot{X}_W$
- **Lost available work** $(\dot{X}_W)_{lost}$
- **Lost exergy**
- **Exergy destruction**
- **Irreversibilities**

**Equations:**
\[
\sum_{in} \dot{m} \psi = \dot{X}_Q_1 + \dot{X}_Q_2 + \cdots + \dot{X}_Q_n
\]
\[
\frac{d\Phi}{dt}
\]
\[
(\dot{X}_W)_{rev} - (\dot{X}_W)_{lost}
\]
Lost Available Work

Exergy balance of the open system discussed can be shown on a flow diagram as follows:
Lost Exergy in Cycles

Consider as closed systems that operate in an integral number of cycles. The ceiling value for available power (maximum available power) is

\[(\dot{X}_W)_{rev} = \sum_{i=1}^{n} \left( 1 - \frac{T_0}{T_i} \right) \dot{Q}_i\]

Exergy content of heat transfer \((\dot{Q}, T, T_0)\) can be expressed as

\[\dot{X}_Q = \dot{Q} \left( 1 - \frac{T_0}{T} \right)\]

Therefore the lost available work for closed systems operating in cycles:

\[\dot{W}_{lost} = \sum_{i=1}^{n} \left( (\dot{X}_Q)_i - \dot{X}_W \right)\]
Heat Engine Cycles

First and second laws state that:

\[ Q_H - Q_L - W = 0 \]

\[ S_{\text{gen}} = \frac{Q_L}{T_L} + \frac{Q_H}{T_H} \geq 0 \]

\( W_{\text{lost}} \) can be expressed as follows if temperature \( T_L \) is assumed to be \( T_0 \):

\[ W_{\text{lost}} = X_{Q_H} - X_W = Q_H \left( 1 - \frac{T_L}{T_H} \right) - W \]

Also can be expressed as:

\[ W_{\text{lost}} = T_L S_{\text{gen}} \]
Heat Engine Cycles

Temperature -energy diagram for a heat engine cycle proposed by Adrian Bejan

Reversible

Irreversible

\[ \tan \alpha = \frac{W_{\text{lost}}}{T_L} \]

since \( W_{\text{lost}} = T_L S_{\text{gen}} \),

\[ \tan \alpha = S_{\text{gen}} \quad \text{or} \quad \alpha = \tan^{-1} S_{\text{gen}} \]
Heat Engine Cycles

Comparison between the first- and second-law efficiency of a heat-engine cycle

High Temperature Reservoir at $T_H$

Heat Engine

$Q_H$ \quad $W = \eta_I \times Q_H$ \quad $Q_L$

Low Temperature Reservoir at $T_L$

Exergy transfer by heat transfer

$X_{QH}$

$\eta_{II} \times X_{QH}$

$\eta_{II} = \frac{X_W}{X_{Q_H}}$

where $X_{Q_H} = (X_W)_{rev} = Q_H \left(1 - \frac{T_L}{T_H}\right)$
Heat Engine Cycles

Second-law efficiency of a heat-engine cycle can also be expressed as follows:

$$\eta_{II} = \frac{X_W}{(X_W)_{rev}} = \frac{(X_W)_{rev} - W_{lost}}{(X_W)_{rev}} = 1 - \frac{T_L S_{gen}}{(X_W)_{rev}}$$

Relationship between first and second law efficiencies:

$$\eta_I = \frac{W}{Q_H} \quad \text{and} \quad \eta_{II} = \frac{X_W}{X_{Q_H}}$$

We know that work transfer is the same as the exergy transfer associated with it (i.e., $W = X_W$) Therefore,

$$\eta_I = \frac{\eta_{II} X_{Q_H}}{Q_H} = \frac{\eta_{II} \times Q_H (1 - T_L/T_H)}{Q_H}$$

$$\eta_I = \eta_{II} \left(1 - \frac{T_L}{T_H}\right)$$
Refrigeration Cycles

- They are closed systems in communication with two heat reservoirs
- (1) the cold space (at $T_L$) from which refrigeration load $Q_L$ is extracted
- (2) the ambient (at $T_H$) to which heat $Q_H$ is rejected

First and second laws state that:

$$Q_L - Q_H + W = 0$$

$$S_{gen} = \frac{Q_L}{T_L} + \frac{Q_H}{T_H} \geq 0$$

Here dead state - temperature $T_0$ is the temperature of the ambient, which is $T_H$. $W_{lost}$ can be expressed as follows:

$$W_{lost} = \frac{Q_L}{T_L} - \frac{X}{Q_L} \left(1 - \frac{T_H}{T_L}\right) = Q_L \left(1 - \frac{T_H}{T_L}\right) - (-W)$$

Rearranging \hspace{1em} \Rightarrow \hspace{1em} W = Q_L \left(1 - \frac{T_H}{T_L}\right) + W_{lost}$$
Refrigeration Cycles

Temperature-energy diagram for a refrigeration cycle proposed by Adrian Bejan

Reversible

Irreversible

\[ \tan \beta = \frac{W_{\text{lost}}}{T_L} \]

since \( W_{\text{lost}} = T_H S_{\text{gen}} \),

\[ \tan \beta = S_{\text{gen}} \quad \text{or} \quad \beta = \tan^{-1} S_{\text{gen}} \]
Refrigeration Cycles

Energy conversion vs exergy destruction during a refrigeration cycle

\[ \text{COP} = \frac{Q_L}{W} \]

High Temperature Reservoir at \( T_H \)

Low Temperature Reservoir at \( T_L \)

\[ \eta_{il} = \frac{(-X_W)_{rev}}{-X_Q_L} = \frac{-X_Q_L}{(-X_Q_L) + T_H S_{gen}} \]

\[ \text{COP} = \frac{Q_L}{W} \quad \text{and} \quad \eta_{il} = \frac{\text{COP}}{\text{COP}_{rev}} \]

Noting that \( \text{COP}_{rev} = \frac{1}{T_H/T_L - 1} \)

\[ \eta_{il} = \text{COP} \left( \frac{T_H}{T_L} - 1 \right) \quad \text{or} \quad \text{COP} = \frac{\eta_{il}}{T_H/T_L - 1} \]
Heat-Pump Cycles

Energy conversion vs exergy destruction during a heat-pump cycle

\[ \text{COP} = \frac{Q_H}{W} \]

High Temperature Reservoir at \( T_H \)

Low Temperature Reservoir at \( T_L \)

\[ W_{\text{lost}} = \left( 1 - \frac{T_L}{T_H} \right) (-Q_H) - (-W) \]

Exergy destruction \( T_L^* \) or \( -X_{Q_H} \)

\[ W = \left( 1 - \frac{T_L}{T_H} \right) Q_H + W_{\text{lost}} \]

or re-arranging

\[ -X_{Q_H} = \left( 1 - \frac{T_L}{T_H} \right) Q_H \]

\[ W = -X_W \]
Heat-Pump Cycles

The second-law efficiency of the heat-pump cycle is calculated by dividing the minimum work requirement by the actual work:

\[ \eta_\text{II} = \frac{(-X_W)_{\text{rev}}}{-X_W} = \frac{-X_{Q_H}}{(-X_{Q_H}) + T_L S_{\text{gen}}} \]

Heat-pump

\[ -X_{Q_H} = \left(1 - \frac{T_L}{T_H}\right)Q_H \]

\[ W = -X_W \]

\[ COP = \frac{Q_H}{W} \quad \text{and} \quad \eta_\text{II} = \frac{COP}{COP_{\text{rev}}} \]

Noting that \( COP_{\text{rev}} = \frac{1}{1 - T_L/T_H} \)

\[ \eta_\text{II} = COP\left(1 - \frac{T_L}{T_H}\right) \quad \text{or} \quad COP = \frac{\eta_\text{II}}{1 - T_L/T_H} \]
Nonflow Processes

General equation for available work:

\[ \dot{X}_W = W - P_0 \frac{dV}{dt} \]

Rate of available work

\[ = -\frac{d}{dt}(E + P_0 V - T_0 S) + \sum_{i=1}^{n} \left(1 - \frac{T_0}{T_i}\right) \dot{Q}_i + \sum_{in} m(h^o - T_0 s) - \sum_{out} m(h^o - T_0 s) - T_0 \dot{S}_{gen} \]

For the closed system shown consider a process \( 1 \rightarrow 2 \) and integrate the above equation from \( t = t_1 \) to \( t = t_2 \):

\[ X_W = A_1 - A_2 + \sum_{i=1}^{n} (X_Q)_i - T_0 S_{gen} \]

where \( A = E - T_0 S + P_0 V \) \( \quad \text{or} \quad A = e - T_0 s + P_0 v \) Nonflow availability

A is a thermodynamic property of the system as long as \( T_0 \) and \( P_0 \) are fixed.
Nonflow Processes

\[ X_W = A_1 - A_2 + \sum_{i=1}^{n} (X_Q)_i - T_0 S_{gen} \]

When the atmosphere is the only reservoir, the max work a closed system delivers can be expressed as:

\[ (X_W)_{rev} = A - A_0 \]

This is known as the nonflow exergy.

Note that the last two terms in the original equation drop out. The nonflow exergy in full:

\[ \Phi = A - A_0 = E - E_0 - T_0 (S - S_0) + P_0 (V - V_0) \]

\[ \phi = a - a_0 = e - e_0 - T_0 (s - s_0) + P_0 (v - v_0) \]

The nonflow exergy is the reversible work delivered by a fixed-mass system during a process in which the atmosphere is the only reservoir.
Steady-flow Processes

General equation for available work:

\[ \dot{X}_W = \dot{W} - P_0 \frac{dV}{dt} \]

Rate of available work

\[ = -\frac{d}{dt} (E + P_0 V - T_0 S) + \sum_{i=1}^{n} \left( 1 - \frac{T_0}{T_i} \right) \dot{Q}_i + \sum_{i=1}^{n} \dot{m} (h^o - T_0 s) - \sum_{i=1}^{n} \dot{m} (h^o - T_0 s) - T_0 \dot{S}_{gen} \]

\[ = \sum_{i=1}^{n} (\dot{X}_{Q})_i + \sum_{in} \dot{m} b - \sum_{out} \dot{m} b - T_0 \dot{S}_{gen} \]

The flow availability at each port is defined as:

\[ B = H^o - T_0 S \]

\[ b = h^o - T_0 s \]
Steady-flow Processes

Consider multi-stream flow through devices where the streams do not mix. The equation obtained in the previous slide

\[
\dot{X}_w = \sum_{i=1}^{n} (\dot{X}_Q)_i + \sum_{in} \dot{m}b - \sum_{out} \dot{m}b - T_0 \dot{S}_{gen}
\]

can be written as

\[
\dot{X}_w = \sum_{i=1}^{n} (\dot{X}_Q)_i + \sum_{k=1}^{r} \left[ (\dot{m}b)_{in} - (\dot{m}b)_{out} \right]_k - T_0 \dot{S}_{gen}
\]

where \( k \) is the number of streams between 1 and \( r \)

Most popular examples would be single-stream devices and two-stream heat exchangers.

If the flow availability evaluated at standard environmental conditions \( (T_0, P_0) \) is \( b_0 \), such that

\[
b_0 = h_0 - T_0 s_0
\]

then we can define flow exergy \( x_f \) as:

\[
x_f = b - b_0
\]

Hence

\[
\dot{X}_w = \sum_{i=1}^{n} (\dot{X}_Q)_i + \sum_{k=1}^{r} \left[ (\dot{m}x_f)_{in} - (\dot{m}x_f)_{out} \right]_k - T_0 \dot{S}_{gen}
\]

Remember the flow exergy from Chp 4

The flow work is done against the fluid upstream in excess of the boundary work against the atmosphere such that exergy associated with this flow work:

\[
x_{flow} = P\nu - P_0\nu = (P - P_0)\nu
\]
Steady-flow Processes

Consider a Rankine cycle operating between a high temperature $T_H$ and the atmospheric reservoir temperature $T_0$. Using the equation

$$\dot{X}_w = \sum_{i=1}^{n} (\dot{X}_Q)_i + \sum_{in} \dot{m}b - \sum_{out} \dot{m}b - T_0 \dot{S}_{gen}$$

it is possible to derive the following equation:

$$\dot{X}_w = \sum_{i=1}^{n} (\dot{X}_Q)_i + \sum_{in} \dot{m}x_f - \sum_{out} \dot{m}x_f - T_0 \dot{S}_{gen}$$
Steady-flow Processes

\[
\dot{X}_W = \sum_{i=1}^{n} (\dot{X}_Q)_i + \sum_{in} \dot{m}x_f - \sum_{out} \dot{m}x_f - T_0 \dot{S}_{gen}
\]

In the case of the boiler:

\[
0 = \dot{X}_{Q_H} + \dot{m}(x_f)_2 - \dot{m}(x_f)_3 - T_0 S_{gen, boiler}
\]

or

\[
\dot{X}_{Q_H} + \dot{m}(x_f)_2 = \dot{m}(x_f)_3 + T_0 S_{gen, boiler}
\]

Exergy inflow \hspace{1cm} Exergy outflow \hspace{1cm} Exergy destroyed

In the case of the turbine \((\dot{X}_{W_t} = \dot{W}_t)\):

\[
\dot{X}_{W_t} = 0 + \dot{m}(x_f)_3 - \dot{m}(x_f)_4 - T_0 S_{gen, turb}
\]

or

\[
\dot{m}(x_f)_3 = \dot{X}_{W_t} + \dot{m}(x_f)_4 + T_0 S_{gen, turb}
\]

Exergy inflow \hspace{1cm} Exergy outflow \hspace{1cm} Exergy destroyed
Steady-flow Processes

\[ \dot{X}_w = \sum_{i=1}^{n} (\dot{X}_Q)_i + \sum_{in} \dot{m}x_f - \sum_{out} \dot{m}x_f - T_0 \dot{S}_{gen} \]

In the case of the condenser:

\[ 0 = \dot{m}(x_f)_4 - \dot{m}(x_f)_1 - T_0 S_{gen,condenser} \]

A significant portion of stream exergy is destroyed due to heat transfer from condenser to the ambient.

The exit temperature of the condenser is \( T_1 \), which is greater than \( T_0 \) and hence the exit exergy \( (x_f)_1 \) is finite.

In the case of the pump \( (-\dot{X}_{wp} = -\dot{W}_p) \):

\[ \dot{X}_{wp} = 0 + \dot{m}(x_f)_1 - \dot{m}(x_f)_2 - T_0 S_{gen,pump} \]

\[ \rightarrow \dot{m}(x_f)_2 + T_0 S_{gen,pump} = \dot{X}_{wp} + \dot{m}(x_f) \]

It is so small, it is not shown on the diagram.
Determine (by drawing an *exergy wheel* diagram) the exergy flow with the associated exergy destruction components of each component of a simple vapor-compression refrigeration cycle. Write down the exergy balance equations for each component and state any assumptions made.
Mechanisms of Entropy Generation or Exergy Destruction

- Heat Transfer across a Finite Temperature Difference
- Flow with Friction
- Mixing