

# A User Friendly Intelligent Algebra Tutor

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**Abstract.** We present a new intelligent tutoring system, “Equation Guru” (EG), which is designed to help high school students at the grade 8 level with equation solving with one unknown. EG consists of two parts. The first is the “Tutorial” part where students are taught interactively, through the usage of an animated agent and visually stimulating graphics, how to solve equations with one unknown. In the second, “Drill and Practice” part, they practice their skills in equation solving by attempting to solve equations automatically generated by EG, which constantly monitors the student’s performance and adjusts the difficulty level of the equations accordingly.

## 1. Introduction

Using the computer as an educational tool for various subjects dates back more than 20 years. The first educational systems were in the category of “computer-aided instruction” (CAI). Although CAI systems were somewhat effective in helping learners, they were not individualized to the learners’ needs and could not provide the same kind of attention that a student receives from a human tutor. These deficiencies led to the development of “intelligent tutoring systems”, or ITSs for short.

An ITS is a versatile computer program which incorporates techniques of artificial intelligence (AI), psychology, education and cognitive science. The philosophy behind the ITS is to teach learners on a one-to-one basis. It simulates a human tutor and provides problem solving environments. This lets the student learn by doing. It achieves its “intelligence” by monitoring the learner, capturing information, and using it to make pedagogical decisions about how to teach, as well as to provide feedback and remedial material when appropriate.

Research shows that ITSs are highly effective at increasing students’ performance and motivation [1], [2], [3], [4]. In this paper, we present a new cognitive intelligent tutoring system, called Equation Guru (EG) that tutors in the domain of equation solving with one unknown. EG was designed for helping high school students (grade 8). We have chosen this domain because equation solving is basic to mathematics. Students are faced formally with equations and equation solving at grade 8 for the first time, and instructors make a great effort to teach this topic in a classroom setting. A tool such as EG, which tutors students on a one-to-one basis, can be very helpful in getting the student to understand the material in a better way and correct his/her misconceptions about the subject matter.

In EG there are two parts: the “Tutorial” part and the “Drill and Practice” part. The purpose of the “Tutorial” part is to teach the student the concept of equations, as well as the steps required to solve equations. It achieves this in an interactive manner with Microsoft Agents, which are animated characters that can talk and also understand speech. The dialogs that are used in this part are similar to the ones observed in a real classroom setting. The “Tutorial” part is further divided into sections. One of the sections includes a game, based on the “beam balance analogy.” In the other sections, step by step, the students are taught how to solve equations and are prepared for the “Drill and Practice” part. The “Drill and Practice” part, in turn, runs in three modes. The first (main) mode makes use of the student modeling and the pedagogical modules. It stores the student’s actions and then makes pedagogical decisions (like determining the next question) according to student model. The way pedagogical decisions are made is based on the suggestions of an experienced human tutor. In the second mode, the student can ask an equation to EG, view the step by step solution and hear the explanation. In the third mode, the student can choose a level, get an equation from that level and try to solve it. The student modeling module is not involved in the second and third modes, since the questions or their level are not determined by the system, but by the student himself/herself.

Microsoft Agents are used all throughout EG for the interaction of the system with the student. These animated characters can speak, make gestures, and understand spoken words, although we have not used speech understanding in EG due to the limited nature of the speech understanding capability of the agents. The agent both provides immediate feedback on the current problem at hand, and motivates of the student through encouraging words (e.g. “Well done, Jane!”), leading to a highly effective learning environment.

The rest of the paper is organized as follows. In section 2, we describe EG in detail. We compare EG to other equation solving ITSs in section 3. Finally, we conclude the paper in section 4.

## 2. Equation Guru in Detail

Two major software components were used in the implementation of Equation Guru: Visual Basic 6.0 for the user interface, and Amzi Prolog + Logic server 7.0.21 for the actual computations, such as monitoring the student’s performance, generation of equations etc. Microsoft Agents were used for most of the interactions of the students with the system.

## 2.1 The “Tutorial” Part

The aim of the “Tutorial” part is to teach the student how to solve linear equations with one unknown based on the “beam balance” analogy. This is the teaching strategy of an experienced human tutor. Figure 1 is a screenshot from the “Tutorial” part.

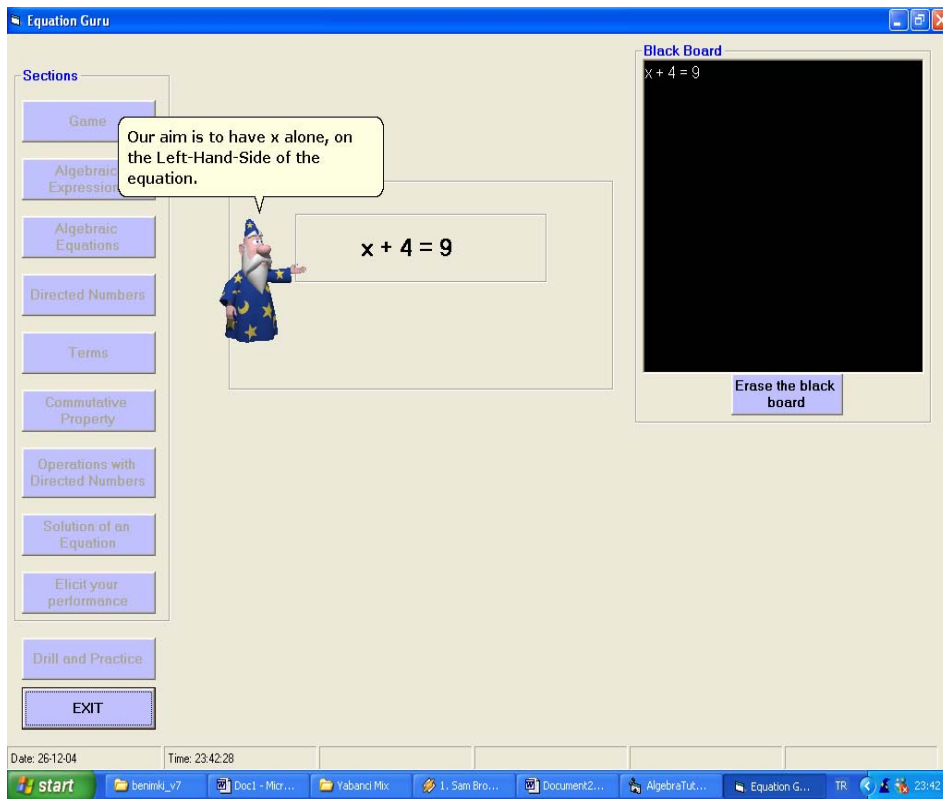


Figure 1: “Solution of an Equation” section in the “Tutorial” Part

The “Tutorial” part is divided into 9 sections. These are: “Game”, “Algebraic Expressions”, “Algebraic Equations”, “Directed Numbers”, “Terms”, “Commutative Property”, “Operations with Directed Numbers”, “Solution of an Equation” and “Elicit your Performance”. Each section is explained to the student through the use of MS Agents in an interactive manner. Figure 1 depicts how the guru (a Microsoft agent – we use the term “guru” to mean both the system itself as a whole, and the animated character that the student sees as his/her tutor) teaches interactively. During the solution of an equation, a black-board appears and all the solution steps are written on to it, just like in a real classroom setting. We briefly introduce each section of the “Tutorial” part below.

- The purpose of the “Game” section is to teach the “beam balance” analogy to the student. In the game, there is a beam balance, and a weight box. The students are asked to drag-and-drop the weights from the weight box to balance the beam to teach them the concept that “whatever you add/subtract on one side of the equation, you should add/subtract the same on the other side of the equation.”
- Algebraic expressions and algebraic equations are defined and illustrated in “Algebraic Expressions” and “Algebraic Equations” sections respectively.
- In the “Directed Numbers” section, directed numbers are presented and some examples of such numbers are requested from the students in order to be sure that they got the idea.
- Terms and the commutative property of the addition operation are explained in the “Terms” and “Commutative Property” sections respectively.
- Simple arithmetic operations with directed numbers are illustrated in the “Operations with Directed Numbers” section.
- In the “Solution of an Equation” section, an equation is solved for the student. If the student feels uncomfortable with equation solving, then another window appears and more examples are solved for better understanding.
- In the “Elicit your Performance” section, students can elicit their performance. An equation is shown to the students and system helps them solve it. At the end of this section, the students are directed to the “Drill and Practice” part.

In order to make the tutoring as realistic as possible, we interviewed high school students from Eastern Mediterranean College (Famagusta, Cyprus), and produced dialogs in light of these interviews. An example of these dialogs (from the “Algebraic Equations” section) is provided in Figure 2 below.

(Tutor gestures left and shows the equation  $x+4=9$ )  
**Tutor:** "We call this expression a mathematical sentence or equation."  
**Tutor:** "An equation is a mathematical sentence which uses the equal sign to state that the two expressions on the Left-Hand-Side and the Right-Hand-Side of the equal sign are equal."  
**Tutor:** "Now, can you translate this number phrase into a word phrase?"  
**Tutor:** "In other words can you put this algebraic equation into words?"  
**Student:** "x plus four is equal to nine"  
(Tutor congratulates)  
**Tutor:** "Well done!"  
**Tutor:** "Now, our task is to find the value of the unknown x which will make the Left-Hand-Side equal to Right-Hand-Side in the equation  $x + 4 = 9$ ."  
**Tutor:** "Now click on the [Directed Numbers] button to continue or if you want you can repeat this part."

Figure 2: Sample dialog in EG

## 2.2 The “Drill and Practice” Part

The aim of this part is to enable students to drill and practice based on what they have learned in the “Tutorial” part. The architecture of this part is given in Figure 3, which in fact is common to most intelligent tutoring systems. There are four components in this architecture: “Domain Expert”, “Pedagogical”, “Interface” and “Student Diagnostic” modules. We briefly explain each one of these below.

The “Domain Expert” module is the “technical expert” of EG. It acts as the lexical analyzer, problem solver and evaluator. Its job is to solve equations with one unknown. It uses the “Domain Knowledge-Base” which stores knowledge about solving linear equations with one unknown in the form of rules in the Prolog programming language.

The “Pedagogical” module is the one that runs the show, making use of the services of the “Domain Expert”, “Student Diagnostic” and “Interface” modules. It applies pedagogical strategies, stored in the “Pedagogical Knowledge” component, to the current situation at hand to decide on the next action. Whenever the student submits a solution step in the form a revised equation, it asks the “Domain Expert” whether the submitted equation is equivalent to the previous one (i.e. if it has the same solution as the previous one). If indeed it preserves the solution, this time the “Domain Expert” checks whether the student’s response represents a move in the right direction toward solving the equation or not. If the student has moved away from finding a solution by making the equation more complex, s/he is warned, but allowed to continue trying to solve the equation. However, if the equation entered by the student does not preserve the solution to the original equation, then the “Student Diagnostic” module is notified, which determines the error and updates the “Student Model” accordingly. Furthermore, the services of the Interface module are used to notify the student of the error.

The “Student Diagnostic” module maintains a model of the student. This involves keeping track of the student’s current level of knowledge, his/her progress as well as the kinds of mistakes s/he has been making.

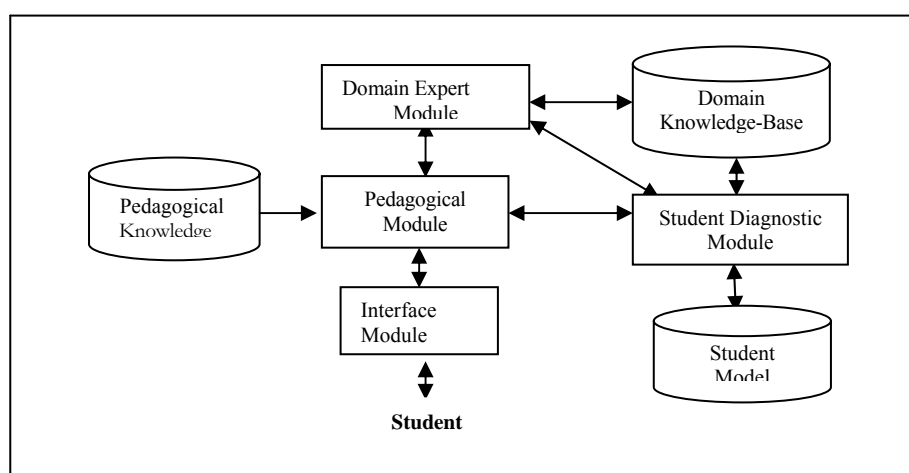


Figure 3: Overall architecture of the “Drill and Practice part”

The job of the “Interface Module” is to interact with the student in a friendly manner, presenting him/her with equations to be solved, reading the answers, guiding the student etc. It achieves this through dialog boxes, pushdown buttons, checkboxes, but most importantly through animated agents which talk and make gestures.

## 2.3 The Three Modes of the “Drill and Practice” Part

The first mode, named “Let my Guru find my level” is fully automated. It includes all the modules depicted in Figure 3. The equation to be solved and the action to be taken next (such as whether hints will be generated, or whether some encouragement will

be given) is determined by consulting the student model and making use of pedagogical knowledge about the tutoring process. Figure 4 is the screenshot of this mode.

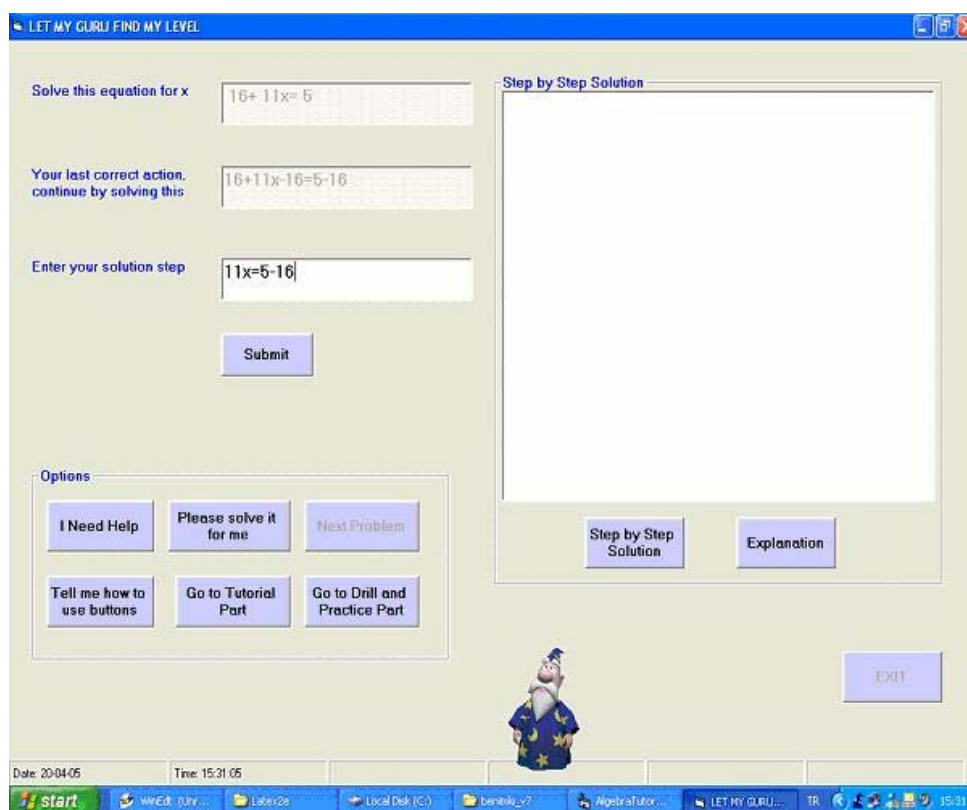


Figure 4: Mode 1 of the “Drill and Practice” part - “Let my Guru find my level”

In the second mode, “I will specify the equation,” the student poses an equation to be solved, and watches the system solve the equation in a step-by-step fashion and hears the explanation of each step.

In the third mode, “Let me specify my level,” the student can choose the level of difficulty of the equation to be generated, and the equation is generated at the specified level of difficulty for the student to solve.

The three modes of the “Drill and practice” part provide a very flexible environment in which students can improve their equation solving skills.

### 3. Comparison of EG with Other Equation Solving Tutors

This section compares Equation Guru with other equation solving tutors. For this comparison only equation solving tutors in the domain of linear equations are considered.

Some equation solving tutors’ domain is restricted to equations in some specific form. For instance, Equation Solving Tutor (EST) [5] helps students in solving linear equations only in the form of  $ax+b=c$ , while in E-Sit [6], the equations are only in the form of  $ax+bx=c$ . Another tutor, E-Tutor [7] supports cross multiplication and expansion in the domain of linear equations while AlgeBrain [8] has these features in the domain of linear and quadratic equations. Different than these tutors, EG supports all forms of linear equations plus equations with fractions. This means that, an equation can require fraction addition and finding the common denominator in order to be solved. Furthermore, EG is not restricted to the kinds of equations generated by the system. Its approach to equation solving is general.

Motivation and getting the attention of the learner are important aspects in the learning process. In order to motivate the student, E-Sit uses a game. The game window appears automatically whenever the student gets a specific mark from the posed questions and the duration of the game depends on the student’s success. AlgeBrain uses a character in its tutoring process. Animation features of this character are limited and it has no speech capability. EG uses MS Agents for the motivation and getting the attention of the student which provides a more effective learning environment than the other equation solving tutors. These characters have a wide range of animations and they can speak and easily engage the student to the learning process.

E-tutor tutors in a dialog-based manner. Similarly, the “Tutorial” part of EG is designed in this manner.

In all of the above equation solving tutor, including EG, next problem selection is based on the information available in the student model while in E-Sit, the next problem selection is based on a utility function which does not support exactly student dependent tutoring.

AlgeBrain is a web-based and collaborative equation solving tutor while other tutors, including EG are standalone applications.

In E-Sit, the next expected action (next step) from the student is specific. For example, for the equation  $3x-5=15$ , the next expected action is  $3x=15+5$ . Also, the name of the solution step (like transformation, addition, etc) must be submitted to the system

by the student. The correctness analysis of the student's action is based on this assumption. In such a system, if the student submits a correct solution step ahead of the expected one ( $x=20/3$ , for the above example), then the ITS will consider this unexpected step as wrong solution step. Therefore, the wrong evaluation will yield a wrong student model. Furthermore, wrong modeling will yield wrong pedagogical decisions and strategies. In AlgeBrain, the next expected action is a set of possible actions that can be applied to simplify the considered equation.

Different from EG, the hint messages in Cognitive Tutor [9] are generated sequentially to the student. It always provides a strong hint, by telling exactly what to do at the end of the sequence. But this approach contradicts with the principles of effective teaching identified in [10]. That is, the tutor should not provide strong hints to problems when students need them. If so, then they may miss the opportunity to learn how to solve problems when they are provided an answer and not allowed to reason for themselves.

We should emphasize and important feature in EG, which is missing in all the other tutoring systems. In EG, the expected solution step from the student is not specific. As long as the student's action results in an equation with the same solution as the equation provided by the system, it is assumed to be correct move, and the student is allowed to carry on. However, the student is warned if his/her solution step takes him away from obtaining an equation in solved form. Also, there is no need for the student to provide the step name to the system. Furthermore, EG supports linear equations in a variety of forms including those with fraction additions, a feature missing in many other tutoring systems.

#### 4. Conclusion and Future Directions

In this paper, we reported a new intelligent tutoring system, called Equation Guru (EG), which is designed to help high school (grade 8) students with algebra. EG is a cognitive ITS in the domain of linear equation solving with one unknown. This subject is chosen as the domain of our EG since algebra is the foundation of mathematics and students face generally major difficulties at this level. Therefore, the students will require extra effort and time in order to learn this subject.

There are two parts in EG. The first is the "Tutorial" part. In this part, the students are taught how to solve equations with one unknown. The students are then directed to the "Drill and Practice" which provides three modes for students to practice their skills in equation solving.

Equation Guru tutors in a truly interactive manner. This is achieved by the use of animated characters, MS Agents, which provide a unique, entertaining experience to the users of the system.

Future work on EG includes adapting it to be Web enabled in order to support distance education, and adding robust natural language processing and understanding capability to it in order to make the experience of using it even more realistic and enjoyable. Furthermore, in order to measure its effectiveness, EG will be evaluated on a real classroom setting

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