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EMU- COMPUTER ENGINEERING DEPARTMENT
CMPE461/CMSE461 ARTIFICIAL INTELLIGENCE
SECOND MIDTERM EXAMINATION

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Duration: 90 Minutes Ahmet Ünveren

Q.1. i) Determine if each statement below is TRUE (T) or FALSE (F), and explain your reasoning in one sentence.

(a) If the statement q is true, then, for any statement p, the statement $p \Rightarrow q$ is true.

TRUE because $P \Rightarrow TRUE = TRUE$

(b) There are truth values for P and Q such that $P \Rightarrow$ Q and $Q \Rightarrow P$ are both false.

FALSE because when $P \Rightarrow Q$ is FALSE, $Q \Rightarrow P$ is TRUE and vice versa.

(c) If the statement P is a contradiction, then, for any statement Q, the statement $P \Rightarrow Q$ is a tautology.

TRUE because $FALSE \Rightarrow Q$ is always TRUE

(f) If two statements are logically equivalent, then so are their negations.

TRUE because if $P \Leftrightarrow Q$ is TRUE, Then either $P, Q = FALSE$ or $P, Q = TRUE$. Hence $\sim P \Leftrightarrow \sim Q$.

ii) Convert the following PL statement to CNF:

A <==> (BVC)

$$A \Leftrightarrow B \vee C \equiv (A \Rightarrow (B \vee C)) \wedge ((B \vee C) \Rightarrow A)$$

$$\equiv (\sim A \vee B \vee C) \wedge ((\sim B \wedge \sim C) \vee A)$$

$$\equiv \left.\begin{array}{l} \sim A \vee B \vee C \\ \sim B \vee A \\ \sim C \vee A \end{array}\right\} \text{AND elimination}$$

Q.2. Assume that a and b are integers. Consider the statements:

A = If c is a prime number such c divides a*b, then c divides a or c divides b.

B = If c is a prime number such c divides a*b, and if c does not divide b, then c divides a.

Write the statements A and B in symbolic form and then show that they are logically equivalent.

let.
P = c is a prime number
Q = c divides a×b
R = c divides a
S = c divides b

$A = (P \wedge Q) \Rightarrow (R \vee S)$

$B = (P \wedge Q \wedge \sim S) \Rightarrow R$

$A \equiv \sim P \vee \sim Q \vee R \vee S$

$B \equiv \sim P \vee \sim Q \vee S \vee R$

$A \Leftrightarrow B$ is a tautology

Since A and B are the same logical expressions.

Q.3. Show that P is a logical consequence of the statements:

$U \Rightarrow R$
$(R \wedge S) \Rightarrow (P \vee T)$
$Q \Rightarrow (U \wedge S)$
$\sim T$

$Q \Rightarrow P$

in CNF:

1. $\sim U \vee R$
2. $\sim R \vee \sim S \vee P \vee T$
3. $\sim Q \vee U$ } AND elimination
4. $\sim Q \vee S$
5. $\sim T$
6. Q } Negation of conclusion
7. $\sim P$

8. S 4, 6 DS
9. U 3, 6 DS
10. R 1, 9 DS
11. $P \vee T$ 2, 8, 10 DS
12. P 5, 11 DS

Q.4. Check the validity of the given conclusion using propositional logic infeence rules.

If it does not rain or it is not foggy then the sailing race will be held and life saving demonstrations will go on. If the sailing race is held, then the trophy will be awarded. The trophy was not awarded.

CONCLUSION: Therefore, it rained.

let
P = It rains
Q = It is foggy
R = Sailing race is held
S = Life saving demonstrations will go on.
T = Trophy is awarded

1. $(\sim P \vee \sim Q) \Rightarrow (R \wedge S)$
2. $R \Rightarrow T$
3. $\sim T$

Conclusion: P

In CNF:

1. $P \vee R$
2. $Q \vee R$
3. $P \vee S$
4. $Q \vee S$

from 1, AND elimination

5. $\sim R \vee T$
6. $\sim T$
7. $\sim P$ Neg. conclusion

8. $\sim R$ 5,6 DS
9. R 1,7 DS

Contradiction, hence Conclusion is TRUE

Q.5. Convert the following natural language sentences into firts order logic sentences.

a) Everyone walks or talks.
b) Every student walks or talks.
c) Every student who walks talks.
d) Every student who loves Mary is happy.
e) Every boy who loves Mary hates every boy who Mary loves.

a) $\forall x$ WALKS(x) $\vee$ TALKS(x)

b) $\forall x$ STUDENT(x) $\Rightarrow$ (WALKS(x) $\vee$ TALKS(x))

c) $\forall x$ (STUDENT(x) $\wedge$ WALKS(x)) $\Rightarrow$ TALKS(x)

d) $\forall x$ (STUDENT(x) $\wedge$ LOVES(x, Mary)) $\Rightarrow$ HAPPY(x)

e) $\forall x$ BOY(x) $\wedge$ LOVES(X, Marry) $\Rightarrow$ ($\forall y$ (BOY(y) $\wedge$ LOVES(Mary, y)) $\Rightarrow$ HATES(x, y))

or $\sim$ LOVES(x, y)

Q.6. Prove or disprove the given conclusion using first order logic inference rules:

All doctors are college graduates. Some doctors are not golfers.

CONCLUSION: Some golfers are not college graduates.

Let $D(x)$: x is a doctor

$C(x)$: x is college graduate

$G(x)$: x is golfer

1. $\forall x \; D(x) \Rightarrow C(x)$
2. $\exists x \; D(x) \land \sim G(x)$

Conc. $\exists x \; G(x) \land \sim C(x)$

Neg. Conclusion:

$\forall x \; \sim G(x) \lor C(x)$

In CNF:

1. $\sim D(x) \lor C(x)$
2. $D(a)$ } Existential instantiation
3. $\sim G(a)$ }
4. $\sim G(x) \lor C(x)$ Neg. Conc.

5. $C(a)$ 1, 2 $\theta = \{x/a\}$

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Cannot be proven.

Q.7. Given the following KB in first order logic. Use predicates
CSCuorse(x): x is a CS Course, Test(x,y): x is a test of y
Pass(x,y): x passes from y Fail(x,y): x fails from y
Easy(x) : x is easy

"For every test in a CS course, at least one person fails."

$\forall x \forall y \; CSCourse(x) \land Test(y,x) \Rightarrow \exists z \; Fail(z,y)$

"Everyone passes an easy test in a course."

$\forall y \; ((\exists x \; Test(y,x) \land Easy(y)) \Rightarrow \forall z \; Pass(z,y)$

"No one can both pass and fail the same test."

$\sim \exists x \exists y (Pass(x,y) \land Fail(x,y))$

"Class1 had an easy test."

$Test(Exam1, class1)$

$Easy(Exam1)$

Use resolution to prove:

"Class1 is not a CS course."

$\sim CSCourse(class1)$

PNF and then CNF:

1. $\sim CSCourse(x) \lor \sim Test(y,x) \lor Fail(a,y)$
2. $\sim Test(y,x) \lor \sim Easy(y) \lor Pass(z,y)$
3. $\sim Pass(x,y) \lor \sim Fail(x,y)$
4. $Test(Exam1, Class1)$
5. $Easy(Exam1)$
6. $CSCourse(Class1)$, Neg. Conc.

7. $\sim Test(Exam1, x) \lor Pass(z, Exam1)$, 2, 5 $\theta = \{y/Exam1\}$
8. $Pass(z, Exam1)$ 4, 7 $\theta = \{x/Class1\}$
9. $\sim Fail(y, Exam1)$ 2, 8 $\theta = \{z/x, y/Exam1\}$
10. $Fail(a, Exam1)$ 1, 5, 6 $\theta = \{x/class1, y/Exam1\}$
11. $\sim Fail(a, Exam1)$ 9 $\theta = \{y/a\}$

Contradiction, Hence conclusion is TRUE