# Lab 2. MODELING OF RLC CIRCUIT RESPONSE ON HARMONIC INPUT (31.03.2020-20.04.2020)

# Lab 2 is a continuation of Lab 1. Now, a 2nd order differential equation is related to RLC circuit, and harmonic inputs-outputs are to be considered.

# 1. Theoretical part

Consider an RLC circuit on Figure 1.



Figure 1. RLC circuit

The problem is to study dependency of  on , where are input and output voltages, and  is a frequency of the input and output signals . Thus far, we assume that

, (1)

and output signal finally is in the form

,

where  is the phase shift of the output signal relative to the input signal.

Let’s consider the mathematical model of the circuit.

Because of the sum of the electric potential changes around any closed circuit must add up to zero (Kirchhoff’s second rule on potentials), definitions of inductance, Ohm’s law, definition of capacitance, and Kirchhoff’s first rule on currents (sum of currents in a junction is zero), we get

 (2)

where are the currents in inductance, *L*, resistance, *R*, and capacitance, *C*, respectively. For more details on the aforementioned laws, see, e.g., a textbook of Douglas C. Giancoli, Physics for Scientists and Engineers, 3rd Ed., Prentice Hall, 2000, ISBN 0-13-243106-8. From (2), we get

 (3)

Substituting the last expression from (3) into the first equation of (2), we get

 (4)

Substitute (1) in (5):

 (6)

From (6), we can get so called transfer function:

, (7)

which is also given in “RLC Circuit Response Demo” in Matlab (Help/Demos/Toolboxes/Control System/Interactive Demos/RLC Circuit Response).

Let

 (8)

in (2). Then, from (6) and (7), we get

. (9)

The value of is complex, and can be represented as

, (10)

where and are the magnitude and phase of the transfer function ,

and theycan be obtained by multiplication of the numerator and denominator in (9) by the conjugate to the denominator value:





where

, (11)

. (12)

From (1), (2), (8)-(10), we get

, (13)

if we assume that the input signal, , meets (1), (2), and (8).

Solving equation

 (14)

regarding , we can find from (11) and (14) a frequency where maximum of  takes place. In Fig. 2, we see (upper pane) that has one maximum, and from (11) and (14), it can be found that

 (15)

which is close to

 . (16)

From (12) and (16), we also get that



that complies with the lower pane of Fig. 2. We see, actually in Figure 2, that maximum of  and 90 degrees (or,  radians) of take place at the practically same frequency  (about 0.5 in Fig. 2).

# 2. What to do?

1. Program an interface for getting parameters of the system (1) and (4) - . Assume that.

2. Program the computation of the system output, , by solving equation (4) using Euler and Runge-Kutta methods (display  as a function of time, *t*). Use time step T=0.1\* (10 steps inside one period of oscillations), or less.

3. Get graphics of the output signal, , for frequencies,  in [0.01.. 10], for time periods sufficient to see established steady oscillations (may be, about eight oscillatory periods).

4. Check that the frequency of the steady oscillations of the output signal is equal to the frequency  of the input signal. Frequency of the output signal can be estimated by

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where is the estimated period of the output signal (it can be measured by considering two consecutive points on the graphic having the same values).

5. Define phase  of the output signal by finding by what time value the output signal is shifted relative to the input signal. It can be done as follows. Find time interval  between two closest points, , when the input and output signal both change sign from negative to positive. Then

,

and

,

and

.

6. Plot graphics of the magnitude, , and phase, , as functions of the frequency,  (have at least 4 points on the graphic for frequencies 0.01, 0.1, 1, 10). Compare obtained dependences of the magnitude, , and phase, , on the frequency, , with the Bode diagram, Fig. 2, from Matlab R2017a, Help/Examples/Control System Toolbox/ Analyzing the Response of an RLC Circuit.



Figure 2. Bode diagram from “Analyzing the Response of an RLC Circuit” in Matlab.

**You may use any programming language and tool available in CMPE137 for task implementation and next demonstration.**

# 3. Defense of the work

You are to show developed working product to evaluator and to be able to explain

* what was done,
* what for it was done,
* how it was done.

**You are to prepare and printout report on the work, in which**

1. Show the title of the work and your name(s)
2. Formulate task of the work (requirements) – 1-2 pages
3. Describe your design (algorithm, interfaces) – 1-3 pages
4. Describe program implementation (functions, their goals, interfaces) – 1-2 pages
5. Give snapshots of the working program showing its proper work for testing examples - 5-6 snapshots
6. Conclusion – brief summary of the work done - 1 page
7. Sources of the program codes

Given above pagination of report’s parts is approximate – write as short as possible, but your ideas should be understandable for the reader (evaluator).

Provide also CD with all materials related to the laboratory work (report, sources, executables, etc.)

**Work shall be done in teams (1 report/team).**

**Due date for the report submission – 20.04.2020, Monday, 16.30, CMPE137.**