Example 1 $H(\omega) = \dfrac{200\, j\omega}{(j\omega+2)(j\omega+10)}$ Sketch the Bode plot for the given transfer function

Solution

$$H(\omega) = \frac{10\, j\omega}{\left(1+\frac{j\omega}{2}\right)\left(1+\frac{j\omega}{10}\right)}$$

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$$H_{dB} = 20\log 10 + 20\log|j\omega| - 20\log\left|1+\frac{j\omega}{2}\right| - 20\log\left|1+\frac{j\omega}{10}\right|$$

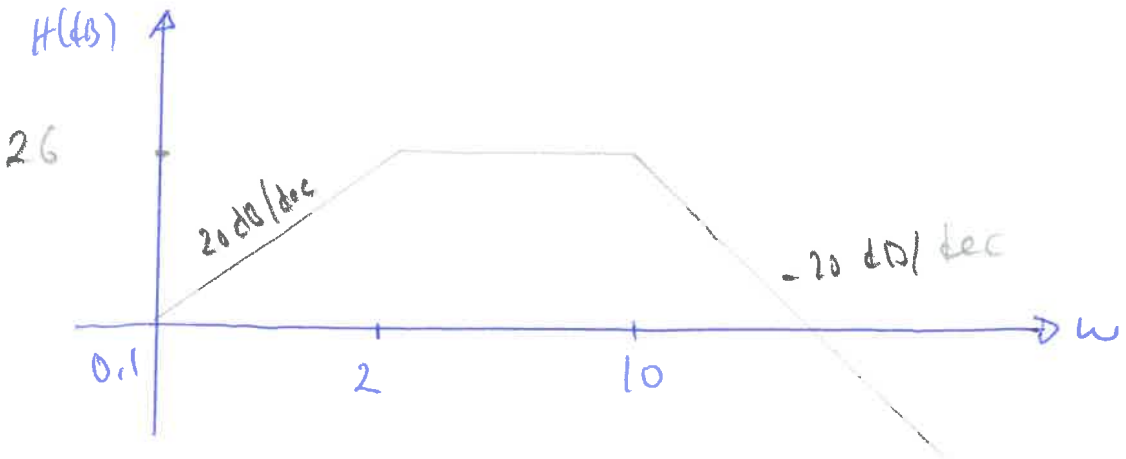
$\phi = 90° - \tan^{-1}\frac{\omega}{2} - \tan^{-1}\frac{\omega}{10}$. There are two corner frequencies $\omega = 2, 10$.

Magnitude	2	10	ω (rad/s)
$j\omega$	20	20	20
$\left(1+\frac{j\omega}{2}\right)^{-1}$	0	-20	-20
$\left(1+\frac{j\omega}{10}\right)^{-1}$	0	0	-20
	20	0	-20

$H(0.1) = 20\log 10 + 20\log 0.1 = 0$ dB

$H(2) = 20\log 10 + 20\log 2 = 26$ dB

$H(10) = 20\log 10 + 20\log 10 - 20\log\sqrt{26} = 26$ dB

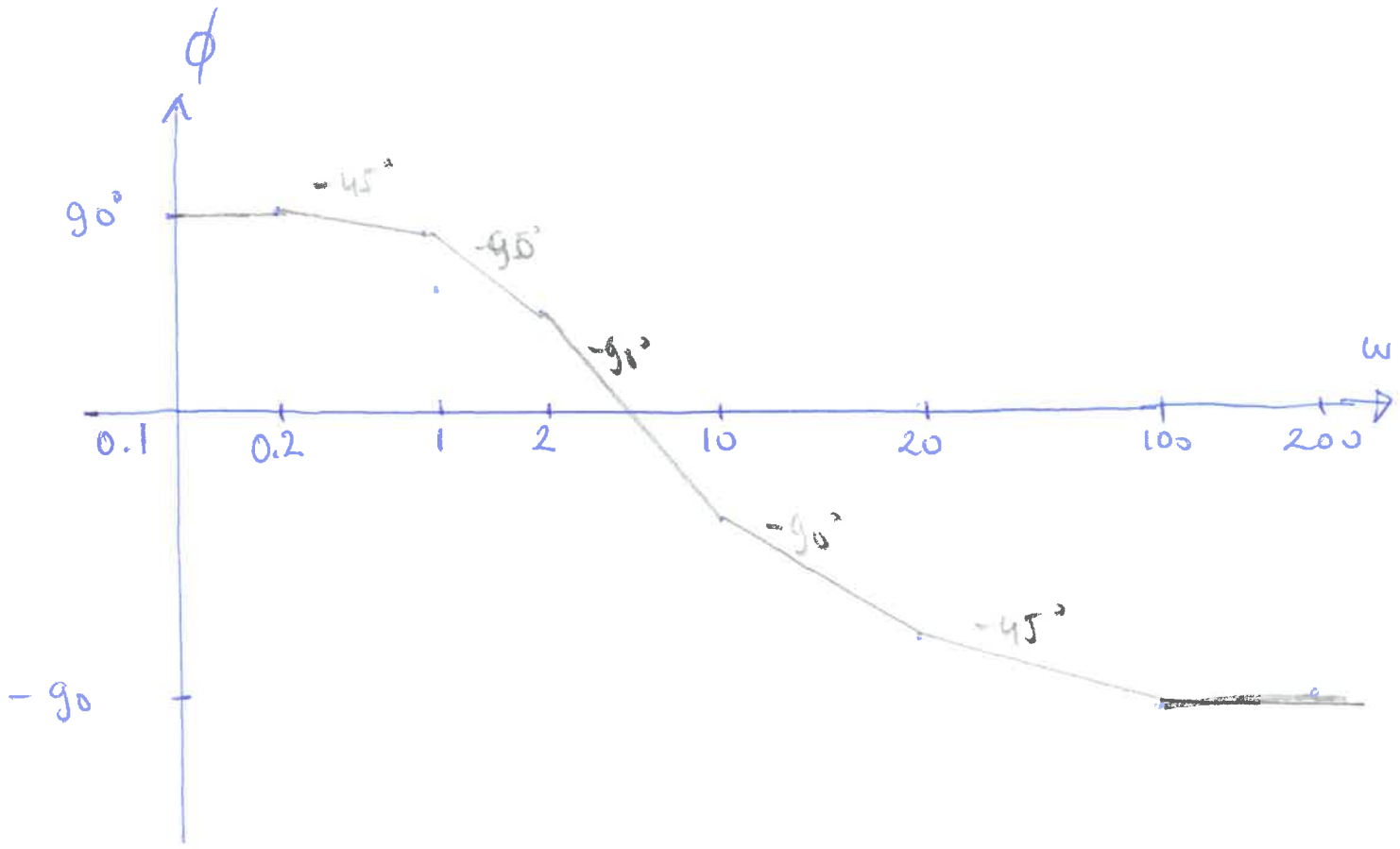


$$\phi = 90^\circ - \tan^{-1}\frac{\omega}{2} - \tan^{-1}\frac{\omega}{10}$$

Phase	0.2	1	2	10	20	100
$(1+j\omega/2)^{-1}$	0	-45°	-45°	-45°	-45°	0	0
$(1+j\omega/10)^{-1}$	0	0	-45°	-45°	-45°	-45°	0
	0	-45°	-90°	-90°	-90°	-45°	0

$\angle H(0.2) = 90^\circ$ $\quad \angle H(2) = 45^\circ$ $\quad \angle H(20) = -58^\circ$

$\angle H(1) = 69^\circ$ $\quad \angle H(10) = -34^\circ$ $\quad \angle H(100) = -83^\circ$



Example 2 $H(\omega) = 5(2+j\omega) / j\omega(10+j\omega)$

Solution: $H(\omega) = \dfrac{(1+j\omega/2)}{j\omega\left(1+\dfrac{j\omega}{10}\right)}$ We have two corner frequencies 2 and 10

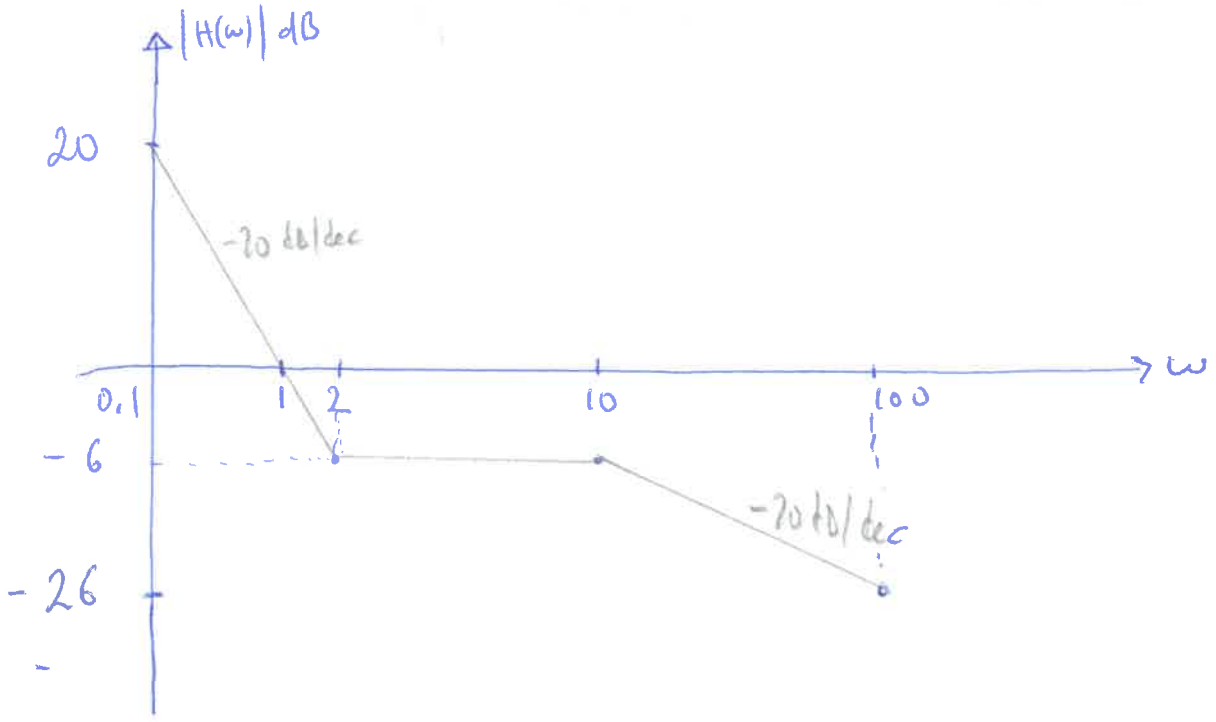
$$H_{dB} = |H(\omega)| = 20\log|1+j\omega/2| - 20\log|j\omega| - 20\log\left|1+\frac{j\omega}{10}\right|$$

$$\angle H(j\omega) = \phi = \tan^{-1}\frac{\omega}{2} - 90^\circ - \tan^{-1}\frac{\omega}{10}$$

Magnitude	2	10	ω (rad/sec)
$j\omega^{-1}$	-20	-20	-20
$(1+j\omega/2)$	0	20	20
$\left(1+\frac{j\omega}{10}\right)^{-1}$	0	0	-20
	-20	0	-20

$|H(0.1)| = -20\log 0.1 = 20$ dB $\quad |H(2)| = -20\log 2 = -6$ dB

$|H(1)| = 0$ dB $\quad |H(10)| = 20\log|1+j5| - 20\log|10j| \cong -6$ dB

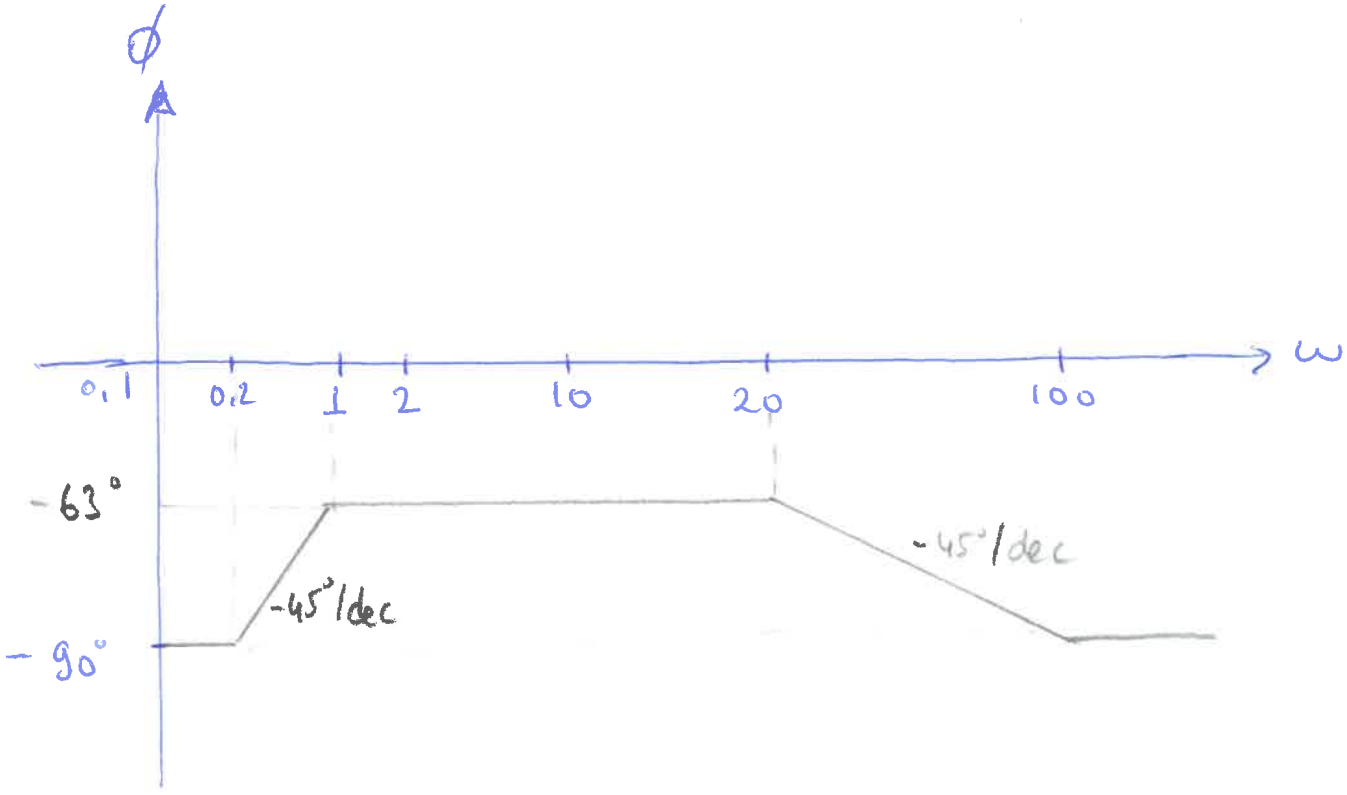


$$\phi = \tan^{-1}\frac{\omega}{2} - 90^\circ - \tan^{-1}\frac{\omega}{10}$$

Phase	0	0.2	1	2	10	20	100
$(1+\frac{j\omega}{2})$	0	45°	45°	45°	45°	0	0
$(1+\frac{j\omega}{10})^{-1}$	0	0	-45°	-45°	-45°	-45°	0
	0	45°	0°	0°	0°	-45°	0°

$\angle H(0.1) = -90^\circ = \angle H(0.2)$ $\quad\quad$ $\angle H(100) = -90^\circ$

$\angle H(1) \cong -63^\circ$



Example 3: $H(\omega) = \dfrac{j\omega + 10}{j\omega\,(j\omega+5)^2}$ Obtain the Bode plot for the transfer function $H(\omega)$.

Sol.: Try to put $H(\omega)$ in standard form.

$$H(\omega) = \frac{0.4\,(1 + j\omega/10)}{j\omega\,(1 + j\omega/5)^2}$$

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$$H_{dB} = 20\log 0.4 + 20\log\left|1+\frac{j\omega}{10}\right| + 20\log|j\omega| - 40\log\left|1+\frac{j\omega}{5}\right|$$

$$\phi = \tan^{-1}\frac{\omega}{10} - 90^\circ - 2\tan^{-1}\frac{\omega}{5}$$

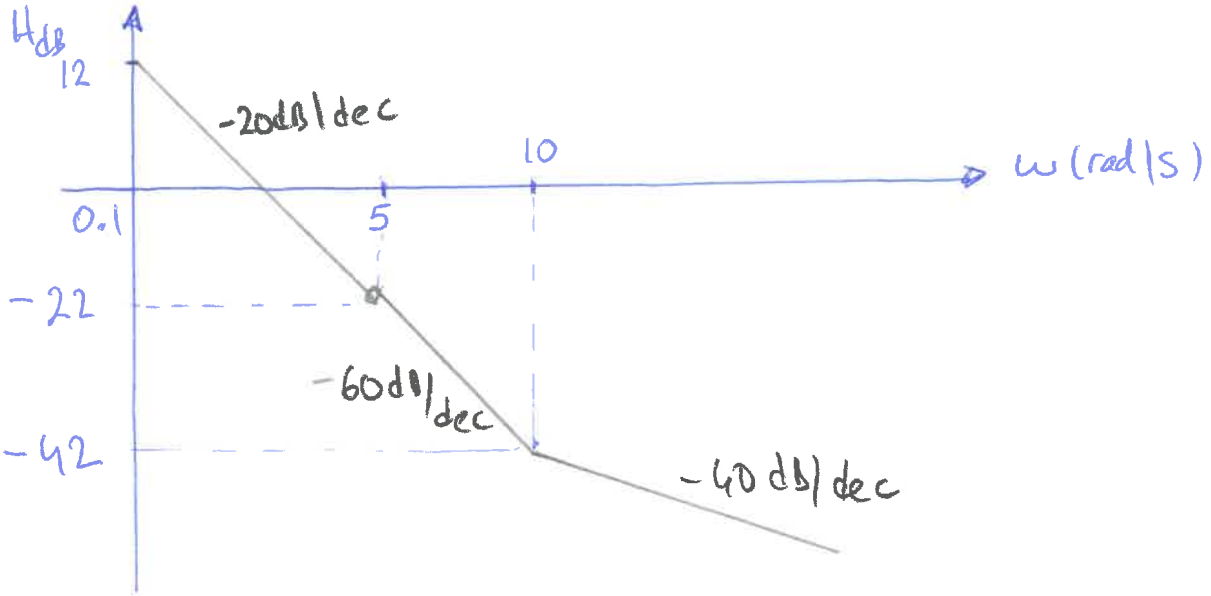
Magnitude	5	10	ω (rad/s)
$(1+j\omega/10)$	0	0	20
$(j\omega)^{-1}$	-20	-20	-20
$(1+\frac{j\omega}{5})^{-2}$	0	-40	-40
	-20	-60	-40

There are two corner frequencies at $\omega = 5, 10$ rad/sec.

$H(0.1) = 20\log 0.4 - 20\log 0.1 = 12$ dB

$H(5) = 20\log 0.4 - 20\log 5 = -22$ dB

$H(10) = 20\log 0.4 - 20\log 10 - 40\log\sqrt{5} \cong -42$ dB



$$\phi = \tan^{-1}\frac{\omega}{10} - 90^\circ - 2\tan^{-1}\frac{\omega}{5}$$

Phase	0,5	1	5	10	50	100	ω (rad/s)
$(1+j\omega/10)$	0	0	45°	45°	45°	45°	0
$(1+j\frac{\omega}{5})^{-2}$	0	-90°	-90°	-90°	-90°	0	0
	0	-90°	-45°	-45°	-45	45°/dec	0

$\angle H(0.1) = -90^\circ$ $\quad \angle H(1) = \tan^{-1}0.1 - 90^\circ - 2\tan^{-1}0,2 = -107^\circ$

$\angle H(50) = \tan^{-1}5 - 90^\circ - 2\tan^{-1}10 \cong -190^\circ$