## EENG 224, Quiz\#1 Solution, 2022-23 FALL

Q. 1 For the circuit shown below determine the average power absorbed by $40 \Omega$ resistor.


## Solution\#1

By applying Nodal analysis to the following circuit


At node 1;

$$
6=\frac{\mathbf{V}_{1}}{j 10}+\frac{\mathbf{V}_{1}-\mathbf{V}_{2}}{-j 20}
$$

Multiply both sides by 20 j and rearrange the above equation;

$$
\begin{equation*}
\mathbf{V}_{1}=j 120-\mathbf{V}_{2} \tag{1}
\end{equation*}
$$

## At node 2;

$$
\begin{array}{ll} 
& 0.5 \mathbf{I}_{\mathrm{o}}+\mathbf{I}_{\mathrm{o}}=\frac{\mathbf{V}_{2}}{40} \\
\text { But, } & \mathbf{I}_{\mathrm{o}}=\frac{\mathbf{V}_{1}-\mathbf{V}_{2}}{-\mathrm{j} 20} \\
\text { Hence, } & \frac{1.5\left(\mathbf{V}_{1}-\mathbf{V}_{2}\right)}{-\mathrm{j} 20}=\frac{\mathbf{V}_{2}}{40}
\end{array}
$$

Multiply both sides by 40 j and rearrange

$$
\begin{equation*}
3 \mathbf{V}_{1}=(3-j) \mathbf{V}_{2} \tag{2}
\end{equation*}
$$

Substituting (1) into (2),

$$
\begin{gathered}
\mathrm{j} 360-3 \mathbf{V}_{2}-3 \mathbf{V}_{2}+\mathrm{j} \mathbf{V}_{2}=0 \\
\mathbf{V}_{2}=\frac{\mathrm{j} 360}{6-\mathrm{j}}=\frac{360}{37}(-1+\mathrm{j} 6) \\
P=\frac{1}{2}\left|\mathbf{I}_{2}\right|^{2} \mathrm{R}=\frac{1}{40}\left(\frac{9}{\sqrt{37}}\right)^{2}(40)=43.78 \mathbf{W}
\end{gathered}
$$

## Q. 2

By using principles of superposition find $i_{0}$ in the circuit shown below.


## Solution\#2

Let lo = lo1 + lo2, where lo1 is due to the dc source and lo2 is due to the ac source. For lo1, consider the circuit shown below.


Clearly,

$$
\mathrm{I}_{\mathrm{ol}}=8 / 2=4 \mathrm{~A}
$$

For lo2, consider the circuit shown below.


If we transform the voltage source, we have the circuit shown below, where $(4 \Omega|\mid 2 \Omega)=4 / 3$ $\Omega$.


By the current division principle,

$$
\mathbf{I}_{o 2}=\frac{4 / 3}{4 / 3+j 4}\left(2.5 \angle 0^{\circ}\right)=0.25-j 0.75=0.79 \angle-71.56^{\circ}
$$

Thus

$$
i_{0}=i_{01}+i_{02}=\left[4+0.79 \operatorname{Cos}\left(4 \mathrm{t}-71.56^{\circ}\right)\right] \mathrm{A}
$$

## Q. 3

If the voltage $v_{0}$ across the $2 \Omega$ resistor is $10 \operatorname{Cos} 2 \mathrm{t} \mathrm{V}$, find $i_{S}$.


## Solution\#3

$$
\begin{aligned}
& 0.1 \mathrm{~F} \longrightarrow \frac{1}{\mathrm{j} \omega \mathrm{C}}=\frac{1}{\mathrm{j}(2)(0.1)}=-\mathrm{j} 5 \\
& 0.5 \mathrm{H} \longrightarrow \mathrm{j} \omega \mathrm{~L}=\mathrm{j}(2)(0.5)=\mathrm{j}
\end{aligned}
$$

where $\mathrm{V}_{0}=10 \quad 0^{0}$


The current I through the $2-\Omega$ resistor is

$$
\mathbf{I}=\frac{1}{1-\mathrm{j} 5+\mathrm{j}+2} \mathbf{I}_{\mathrm{s}}=\frac{\mathbf{I}_{\mathrm{s}}}{3-\mathrm{j} 4}, \quad \text { where } \quad \mathbf{I}=\frac{10}{2} \angle 0^{\circ}=5
$$

Therefore

$$
\mathbf{I}_{\mathrm{s}}=(5)(3-\mathrm{j} 4)=25 \angle-53.13^{\circ}
$$

Hence

$$
i_{S}=25 \operatorname{Cos}\left(2 t-53.13^{\circ}\right) \mathrm{A}
$$

