CMPE 321 Signals and Systems for Computer Engineers (Fall 2019)

Final Review Sheet

CHAPTER 2 SINUSOIDS

- Amplitude A, radian frequency ω_o, and phase shift ϕ of a sinusoidal signal $x(t) = A\cos(\omega_o t + \phi)$
- Basic relationship between radian frequency ω_o and cyclic frequency f_o ($\omega_o = 2\pi f_o$) and relationship between frequency and period T_o of sinusoids ($T_o = 1/f_o$)
- Relationship between phase shift and time shift of a sinusoid: $t_1 = -\frac{\phi}{\omega_o} = -\frac{\phi}{2\pi f_o}$
- Euler's formula: $e^{j\theta} = \cos\theta + j\sin\theta$
- Inverse Euler's formulas: $\cos\theta = \frac{1}{2}(e^{j\theta} + e^{-j\theta})$ and $\sin\theta = \frac{1}{2j}(e^{j\theta} - e^{-j\theta})$
- Sinusoids and complex exponential signals (rotating phasors):

$$A\cos(\omega_o t + \phi) = Re\{Ae^{j(\omega_o t + \phi)}\} = Re\{Ae^{j\phi}e^{j\omega_o t}\} = \frac{1}{2}Xe^{j\omega_o t} + \frac{1}{2}X^* e^{-j\omega_o t}$$

  with complex amplitude or phasor $X = Ae^{j\phi}$
- Positive and negative frequency concept of rotating phasors
- Phasor addition and phasor diagrams
- Solve problems: P-2.7, P-2.11, P-2.15, P-2.16

CHAPTER 3 SPECTRUM REPRESENTATION

- Spectrum of a sum of sinusoids

$$x(t) = A_o + \sum_{k=1}^{N} A_k \cos(2\pi f_k t + \phi_k)$$

  where A_o is referred to as the DC component
- Amplitude modulated (AM) sinusoidal signals or beat signals:
  $x(t) = \cos(2\pi f_1 t) + \cos(2\pi f_2 t) = 2\cos(2\pi f_\Delta t)\cos(2\pi f_c t)$
- Envelope of an AM signal, its carrier or center frequency $f_c = \frac{1}{2}(f_1 + f_2)$, and its deviation frequency $f_\Delta = \frac{1}{2}(f_2 - f_1)$
- Bandwidth of a signal, frequency-division multiplexing
- Operations on the spectrum; differentiation, time-shifting, frequency-shifting
- Frequency Modulation (FM): generating frequency variation by a time-varying angle function $\Psi(t)$:

$$x(t) = A\cos(\Psi(t)) = Re\{Ae^{j\Psi(t)}\}$$

- Instantaneous frequency of an FM signal: $\omega_i = \frac{d}{dt}\Psi(t)$ rad/s or $f_i = \frac{1}{2\pi}\frac{d}{dt}\Psi(t)$ Hz
- Linear FM signals or chirp signals: $f_i = 2\mu t + f_o$
- Spectrum of periodic signals

$$x(t) = A_o + \sum_{k=1}^{N} A_k \cos(2\pi k f_o t + \phi_k) = \sum_{k=-N}^{N} a_k e^{j2\pi k f_o t}$$

  with fundamental frequency $f_o = \gcd\{f_k\}$ (gcd: greatest common divisor), and harmonic frequencies

$f_k = k f_o$

- Properties of complex exponentials:

  $\int_0^{T_o} e^{j\left(\frac{2\pi}{T_o}\right)kt} dt = 0$ (Integration over one period)

  $\frac{1}{T_o}\int_0^{T_o} e^{j\left(\frac{2\pi}{T_o}\right)kt} e^{-j\left(\frac{2\pi}{T_o}\right)lt} dt = \begin{cases} 0 & \text{if } k \neq l \\ 1 & \text{if } k = l \end{cases}$ (Orthogonality)
- Fourier analysis of periodic signals: (a_k are referred to as the Fourier series coefficients)

$$a_k = \frac{1}{T_o}\int_0^{T_o} x(t)\, e^{-j2\pi k f_o t} dt$$

$$a_0 = \frac{1}{T_o}\int_0^{T_o} x(t)\, dt$$

- Finite Fourier synthesis formula: (approximate the signal using a_k)

$$x(t) = \sum_{k=-N}^{N} a_k e^{j2\pi k f_o t}$$

- Full-wave rectified sine and half-wave rectified sine signals
- The Gibbs' phenomenon
- Solve problems: P-3.1, P-3.9, P-3.11, P-3.21, P-3.22, P-3.24, P-3.27, P-C.1

CHAPTER 4 SAMPLING AND ALIASING

- Discrete-time sinusoidal signals $x[n] = A\cos(\hat{\omega}\, n + \phi)$ where $\hat{\omega} = \omega T_s = \omega / f_s$ is called normalized radian frequency or discrete-time frequency or digital frequency (f_s is the sampling rate)
- Aliases of a sinusoid with frequency $\hat{\omega}_o$: $\hat{\omega}_o$, $\hat{\omega}_o + 2\pi l$, $-\hat{\omega}_o + 2\pi l$ where l is an integer
- Spectrum of a discrete-time signal
- Sampling theorem and the Nyquist rate ($2f_{max}$)
- Over-sampling and under-sampling
- Aliasing due to under-sampling and folding due to under-sampling
- Ideal C-to-D and D-to-C converters and A-to-D and D-to-A converters
- Reconstruction and interpolation
- Solve problems: P-4.1, P-4.3, P-4.4, P-4.7, P-4.14, P-4.26

CHAPTER 5 FIR FILTERS

- FIR systems, difference equations, FIR filter coefficients b_k
- Impulse response
- Convolution operation and convolution of finite-length signals using synthetic polynomial multiplication
- Linear Time-Invariant (LTI) systems, causality, superposition
- Cascaded LTI systems, block diagrams
- Solve problems: P-5.1, P-5.10, P-5.18

CHAPTER 6 FREQUENCY RESPONSE OF FIR FILTERS

- Sinusoidal response of FIR systems
- Frequency response of an FIR system: $H(e^{j\hat{\omega}}) = \sum_{k=0}^{M} b_k\, e^{-j\hat{\omega}k} = \sum_{k=0}^{M} h[k]\, e^{-j\hat{\omega}k}$
- Magnitude and phase response, gain of a system
- $y[n] = |H(e^{j\hat{\omega}})| A\cos(\hat{\omega}n + \phi + \angle H(e^{j\hat{\omega}}))$ when $x[n] = A\cos(\hat{\omega}n + \phi)$
- Superposition and frequency response
- Relation of frequency response to impulse response and difference equation
- Periodicity and symmetry properties of frequency response
- Simple low-pass filters
- Cascaded LTI systems and the frequency response
- Solve problems: P-6.1, P-6.3, P-6.4, P-6.5(a),(b), P-6.8(a),(c), P-6.13, P-6.18, P-6.19, P-6.20(a)

CHAPTER 7 DISCRETE-TIME FOURIER TRANSFORM

- DTFT: $X(e^{j\hat{\omega}}) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\hat{\omega}n}$
- Inverse DTFT: $x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\hat{\omega}})\, e^{j\hat{\omega}n} d\hat{\omega}$
- Basic DTFT pairs (Table 7-1)
- DTFT properties: Periodicity, linearity, conjugate symmetry, time-delay, frequency-shift, modulation, convolution (Table 7-2)
- Energy of a signal: $E = \sum_{n=-\infty}^{\infty} |x[n]|^2$
- Parseval's Theorem: $\sum_{n=-\infty}^{\infty} |x[n]|^2 = \frac{1}{2\pi}\int_{-\pi}^{\pi} |X(e^{j\hat{\omega}})|^2\, d\hat{\omega}$
- Autocorrelation function: $c_{xx}[n] = x[-n] * x[n] = \sum_{k=-\infty}^{\infty} x[-k]x[n-k] = \sum_{m=-\infty}^{\infty} x[m]x[n+m]$
- Frequency-selective digital filters: Ideal lowpass, highpass, and bandpass filters
- Windowing: Rectangular and Hamming Windows
- Solve problems: P-7.7(a),(b), P-7.14, P-7.15

CHAPTER 9 *Z*-TRANSFORMS

- Three domains of representation of signals and systems: time domain, frequency domain, *z*-domain
- *z*-transform: $X(z) = \sum_{k=0}^{N} x[k] z^{-k}$
- *z*-transform of an FIR filter
- System function: $H(z) = \sum_{k=0}^{M} b_k\, z^{-k} = \sum_{k=0}^{M} h[k]\, z^{-k}$
- Superposition and time-delay properties
- Convolution and *z*-transform
- Cascaded LTI systems
- $H(e^{j\hat{\omega}}) = H(z)|_{z=e^{j\hat{\omega}}}$
- The *z*-plane, poles and zeros of $H(z)$, pole-zero plots
- Nulling filters
- Solve problems: P-9.1, P-9.2, P-9.3, P-9.4, P-9.12

CHAPTER 10 IIR FILTERS

- IIR systems and difference equations involving feedback terms
- Time-domain response and initial rest conditions
- Linearity and time-invariance of IIR filters

- Impulse response of a first-order IIR system
- System function of an IIR filter, poles and zeros, pole-zero plots
- Stability
- Frequency response of IIR filters
- Pole locations and the shape of the frequency response
- Solve problems: P-10.6, P-10.7, P-10.21, P-10.26