Supply Chain Management

Part 6

Demand Forecasting in the Supply Chain

The role of demand forecasting in supply chain

Demand forecasting in supply chain management refers to the process of planning or predicting the demand of materials to ensure you can deliver the right products and in the right quantities to satisfy customer demand without creating a surplus.

Demand forecasting is the basis for all strategic and planning decisions in a supply chain.

Accurate demand forecasting results in: customer satisfaction, better allocation of resource, streamlining inventory, better sales strategies, and better suppliers and purchase terms.

Characteristics of forecasts

1- Forecasts are always inaccurate.

2- Long-term forecasts are usually less accurate than short-term forecast.

3- Aggregate forecasts are usually more accurate than disaggregate forecasts.

Forecasting methods

Qualitative: primarily subjective; rely on judgment and opinion.

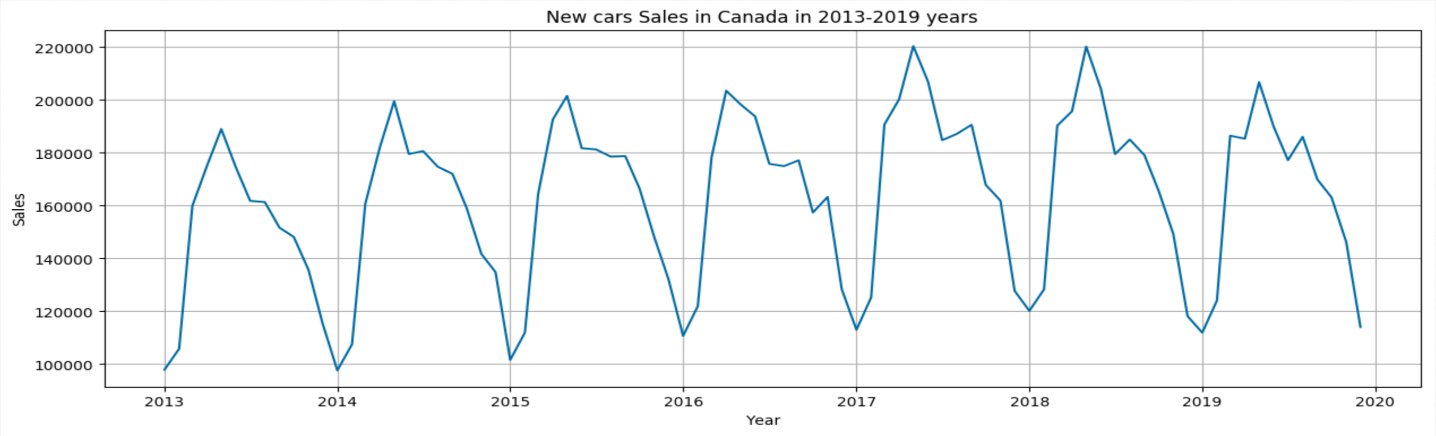
Time Series: use historical demand only.

Causal: use the relationship between demand and some other factor to develop forecast.

Simulation: imitate consumer choices that give rise to demand. Can combine time series and causal methods.

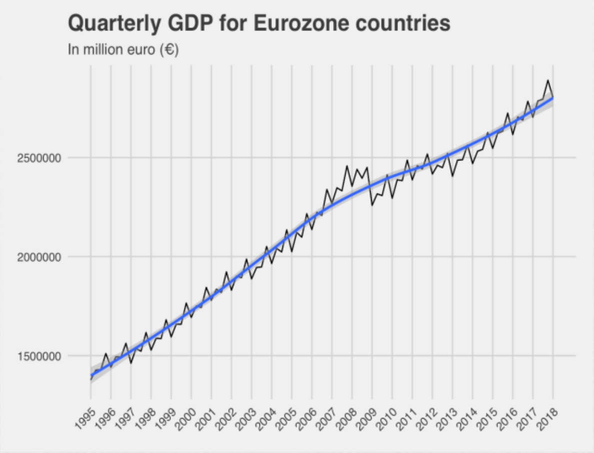
Time series forecasting

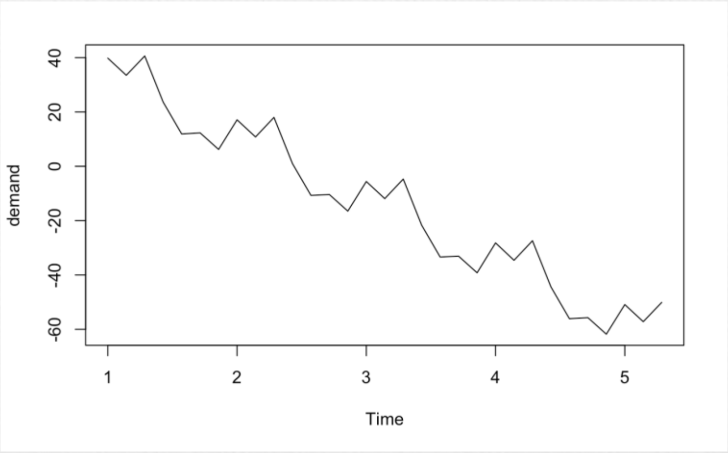
- A time series is simply a series of data points ordered in time. In a time series, time is often the independent variable and the goal is usually to make a forecast for the future. (example: historical data on sales, inventory, costs, etc.)



Elements of time series

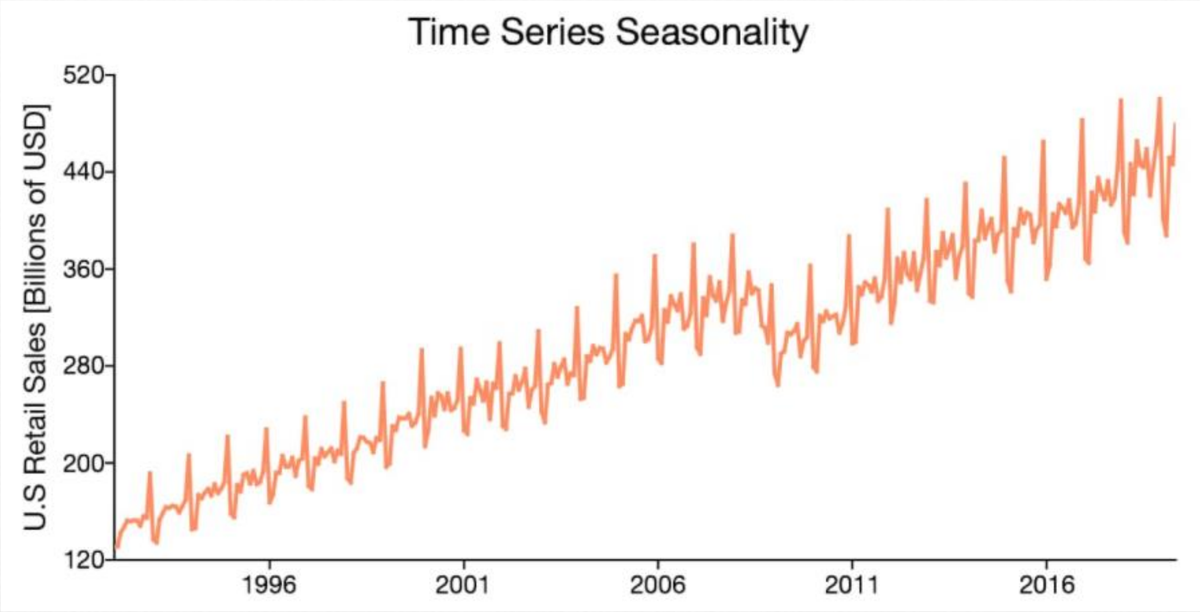
- ***Trend:*** trend is the long term pattern of a time series. A trend can be positive or negative (The growth or decline in Sales over time).





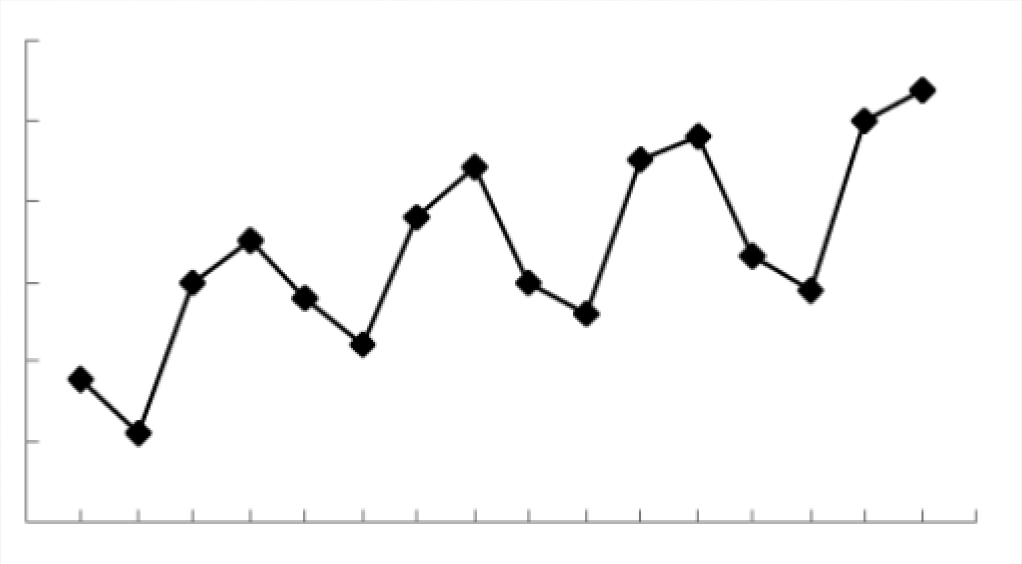
Elements of time series

- ***Seasonality:*** the repeating short-term cycle in the series. (Example. You may have strong sales in the summer but weak sales in the spring and fall).



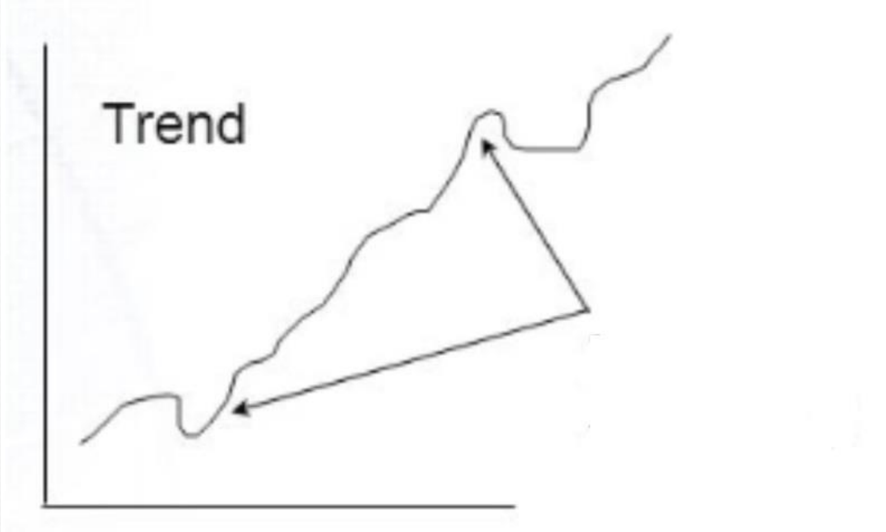
Elements of time series

- ***Cyclic variation:*** a cyclic pattern exists when data exhibit rises and falls that are not of fixed period.



Elements of time series

- ***Irregular variation:*** Irregular variations are fluctuations in time series that are short in duration, erratic in nature and follow no regularity in the occurrence pattern.



Naive forecast

- ***Naive forecast:*** nothing will change.

Future value = current value

Example: Last month you sold 250 computers, so you predict that this month you'll sell 250 computers again.

Good in very steady environment

Moving average

- ***Moving average:*** is used when demand has no observable trend and seasonality.

$$F_t = \frac{D_{t\_1}+D_{t\_2}+\cdots+D_{t\_n}}{n} = \frac{1}{n}\sum_{i=1}^{n} D_{t\ \_\ i}$$

Where:

$F_t =$ forecast demand of period t

n = number of periods in moving average

D_{t-i} = demand in period t-i

Moving average

Example: a supermarket has experienced weekly demand of milk of D1=120, D2=127, D3=114, and D4=122 gallons over the past four weeks. Forecasts demand for period 5 using a four-period moving average. What is the forecast error if demand in period 5 turns out to be 125 gallons?

$$F_5 = \frac{D_{5\_1}+D_{5\_2}+D_{5\_3}+D_{5\_4}}{4} = \frac{122+114+127+120}{4} = 120.75$$

$$E_5 = F_5 – D_5 = 125\text{-}120.75 = 4.25$$

Moving average

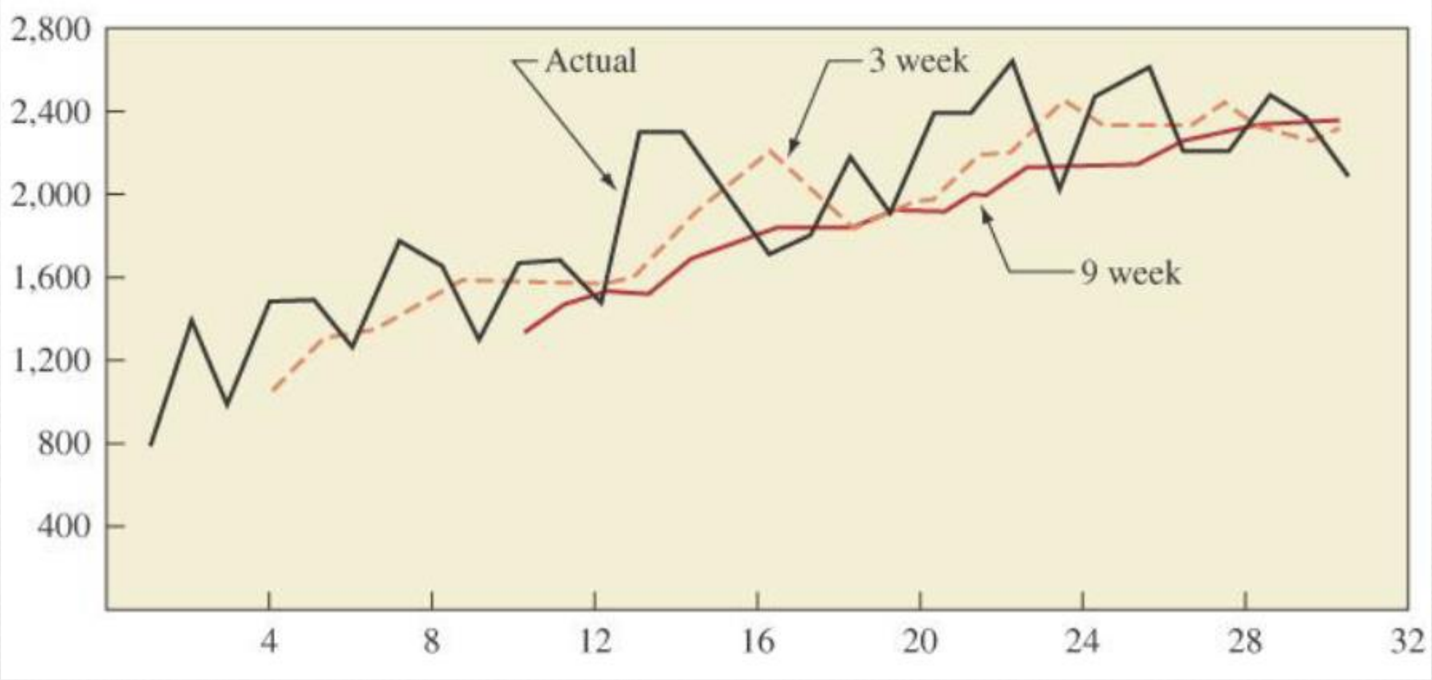
- Question: What are the 3 week and 9 week moving average forecasts for the following demand?

Week	Demand	Week	Demand
1	650	7	850
2	678	8	758
3	720	9	892
4	785	10	920
5	859	11	789
6	920	12	844

Moving average

Week	Demand	3 Week	9 Week	Week	Demand	3 Week	9 Week
1	800			16	1,700	2,200	1,811
2	1,400			17	1,800	2,000	1,800
3	1,000			18	2,200	1,833	1,811
4	1,500	1,067		19	1,900	1,900	1,911
5	1,500	1,300		20	2,400	1,967	1,933
6	1,300	1,333		21	2,400	2,167	2,011
7	1,800	1,433		22	2,600	2,233	2,111
8	1,700	1,533		23	2,000	2,467	2,144
9	1,300	1,600		24	2,500	2,333	2,111
10	1,700	1,600	1,367	25	2,600	2,367	2,167
11	1,700	1,567	1,467	26	2,200	2,367	2,267
12	1,500	1,567	1,500	27	2,200	2,433	2,311
13	2,300	1,633	1,556	28	2,500	2,333	2,311
14	2,300	1,833	1,644	29	2,400	2,300	2,378
15	2,000	2,033	1,733	30	2,100	2,367	2,378

Moving average



Weighted moving average

- ***Weighted moving average:*** a weighted moving average puts more weight on recent data and less on pass data.

$$F_t = W_1 D_{t\_1} + W_2 D_{t\_2} + \ldots + W_n A_{t\_n}$$

Where:

W_i = the weight of i previous period

$0 \leq W_i \leq 1$ and $\sum_{i=1}^{n} W_i$

Weighted moving average

- Example. What is the forecast value for November?

MONTH	WEIGHT	DATA
August	17%	130
September	33%	110
October	50%	90

$$F_4 = W_1D_3 + W_2D_2 + W_3A_1 =$$
$$[(0.5*90)+(0.33*110)+(0.17*130)] = 103.4$$

Simple exponential smoothing

- ***Simple exponential smoothing:*** based on the premise that the most recent observations might have the highest predictive value. Therefore, we should give more weight to the more recent time periods when forecasting.

$F_{t+1} = \alpha D_t + (1 - \alpha) F_t$

where:

F_{t+1} forecast for next period

D_t actual demand for present period

F_t previously determined forecast for present period

α weighting factor, smoothing constant

Simple exponential smoothing

- $0.0 \leq \alpha \leq 1.0$

If $\alpha = 0.20$, then $F_{t+1} = 0.20\, D_t + 0.80\, F_t$

If $\alpha = 0$, then $F_{t+1} = 0\, D_t + 1\, F_t = F_t$. Forecast does not reflect recent data

If $\alpha = 1$, then $F_{t+1} = 1\, D_t + 0\, F_t = D_t$. Forecast based only on most recent data

Simple exponential smoothing

Choosing appropriate values for α

- If actual demands are stable, use a small α to lessen effects of short term changes in demand.
- If actual demands rapidly increase or decrease, use a large α so forecasts keep pace with the changes in demand.
- α is usually determined by trial and error, with several values tested on existing data or a portion of data (aim is to minimize the average forecasting error.

Simple exponential smoothing

Example. demands for an electrical component for the past 6 months are as follows.

Period	1	2	3	4	5	6
Demand	59	65	60	71	65	68

Obtain the forecast value of the demand for this product for the seventh period by using the simple exponential smoothing method and by selecting the appropriate alpha from the numbers 0.2, 0.3, and 0.4. Assume the forecasted demand for this product in the first period is 55.

Simple exponential smoothing

		α = 0.2		α = 0.3		α = 0.4	
Period	Demand	Forecast	error	Forecast	error	Forecast	error
1	59	55	4	55	4	55	4
2	65	(0.2*59)+(0.8*55)= 55.8	9.2	56.2	8.8	56.6	8.4
3	60	=(0.2*65)+(0.8*55.8)=57.6	2.4	58.8	1.2	60	0
4	71	=(0.2*60)+(0.8*57.6)=58.1	12.9	59.2	11.8	60	11
5	65	=(0.2*71)+(0.8*58.1)=60.7	4.3	62.7	2.3	64.4	0.6
6	68	=(0.2*65)+(0.8*60.7)=61.6	6.4	63.4	4.6	64.4	3.4

Simple exponential smoothing

$$E_\alpha = \frac{\sum_{i=1}^{N} |D_i - F_i|}{N}$$

$$E_{0.2} = \frac{39.2}{6} = 6.53$$

$$E_{0.3} = \frac{32.7}{6} = 5.45$$

$$E_{0.4} = \frac{27.4}{6} = 4.57$$

Min $E_{0.4}$

F7 = 0.4*D6 + 0.6*F6 = 0.4*68 + 0.6*64.4 ≈ 66

Trend-adjusted Exponential Smoothing (Holt's Model)

- ***The trend-adjusted exponential smoothing (Holt's model)*** is appropriate when demand is assumed to have a trend in, but no seasonality.

In this case, in Period t, given estimates of level L_t and trend T_t, the forecast for future periods is expressed as:

$$F_t = L_{t-1} + T_{t-1}$$

$$L_t = \alpha D_t + (1- \alpha)(L_{t-1} + T_{t-1})$$

$$T_t = \beta(L_t - L_{t-1}) + (1-\beta)T_{t-1}$$

where:

L_t: is estimates of level in period t

T_t: is estimates of trend in period t

α and β are smoothing constants for level and trend, respectively ($0< \alpha$ and $\beta<1$)

Trend-adjusted Exponential Smoothing (Holt's Model)

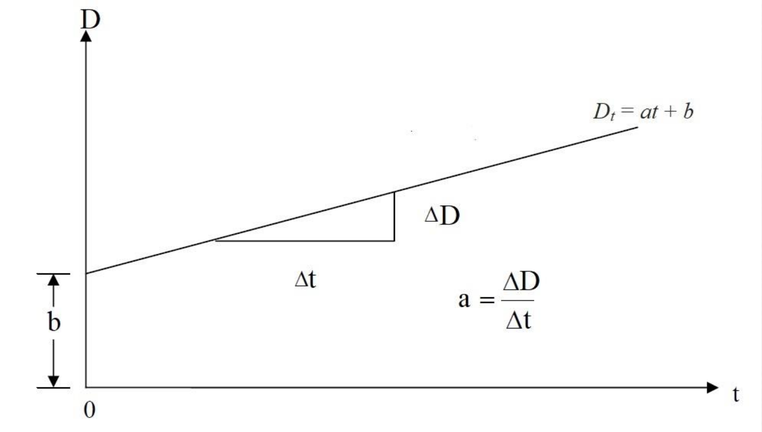
We obtain an initial estimate of level and trend by running a linear regression between demand, D_t, and time, Period t, of the form

$D_t = at + b$

In this case, running a linear regression between demand and time periods is appropriate because we have assumed that demand has a trend but no seasonality.

The constant b measures the estimate of demand at Period $t = 0$ and is our estimate of the initial level L_0. The slope a measures the rate of change in demand per period and is our initial estimate of the trend T_0.

Trend-adjusted Exponential Smoothing (Holt's Model)

- 

$$a = \frac{n\sum tD - \sum t \sum D}{n\sum t^2 - (\sum t)^2} \qquad b = \frac{\sum D - a\sum t}{n}$$

Trend-adjusted Exponential Smoothing (Holt's Model)

Example. An electronics manufacturer has seen demand for its latest MP3 player increase over the past six months. Observed demand (in thousands) has been $D1$ = 8415, $D2$ = 8,732, $D3$ = 9014, $D4$ = 9,808, $D5$ =10416, and $D6$ = 11,961. Forecast demand for Period 7 using trend-adjusted exponential smoothing with $\alpha = 0.1$, $\beta = 0.2$.

The first step is to obtain initial estimates of level and trend using linear regression.

Trend-adjusted Exponential Smoothing (Holt's Model)

t	D	t^2	tD
1	8415	1	8415
2	8732	4	17464
3	9014	9	27042
4	9808	16	39232
5	10413	25	52065
6	11961	36	71766
Sum	58343	91	215984

- $$a = \frac{n\sum tD - \sum t \sum D}{n\sum t^2 - (\sum t)^2} = \frac{(6*215984)-(21*58343)}{(6*91)+(21)^\wedge 2} = 673$$

- $$b = \frac{\sum D - a\sum t}{n} = \frac{58343-(673*21)}{6} = 7367$$

So, $L_0 = 7367$ and $T_0 = 673$

$F_1 = L_0 + T_0 = 7367 + 673 = 8040$

Trend-adjusted Exponential Smoothing (Holt's Model)

$L_1 = \alpha D_1 + (1- \alpha)(L_0 + T_0) = (0.1*8415) + 0.9*(7367 + 673) = 8078$

$T_1 = \beta(L_1 – L_0) + (1-\beta)T_0 = [0.2 * (8078 – 7367)] + (0.8*673) = 681$

$F_2 = L_1 + T_1 = 8078 + 681 = 8759$

Continuing in this manner, we obtain $L_2 = 8755$, $T_2 = 680$, $L_3 = 9393$, $T_3 = 672$, $L_4 = 10039$, $T_4 = 666$, $L_5 = 10676$, $T_5 = 661$, $L_6 = 11399$, $T_6 = 673$.

This gives us a forecast for Period 7 of

$F_7 = L_6 + T_6 = 11399 + 673 = 12072$

Trend- and seasonality-adjusted Exponential Smoothing (Winter's Model)

- ***Trend- and seasonality-adjusted exponential smoothing (winter's model):*** This method is appropriate when the systematic component of demand has a level, a trend, and a seasonal factor.

In Period t, given estimates of level, L_t, trend, T_t, and seasonal factors, S_t, the forecast for future periods is given by

$F_t = (L_{t-1} + T_{t-1})S_t$
$L_t = \alpha(D_t/S_t) + (1- \alpha)(L_{t-1} + T_{t-1})$
$T_t = \beta(L_t – L_{t-1}) + (1-\beta)T_{t-1}$
$S_{t+p} = \gamma (D_t/L_t) + (1- \gamma)S_t$

where:

α, β, are γ smoothing constants for level, trend, and seasonal factor, respectively. (0< <1)

Trend- and seasonality-adjusted Exponential Smoothing (Winter's Model)

Steps of Winter's model:

1- Deseasonalizing the demand data. Deseasonalized demand represents the demand that would have been observed in the absence of seasonal fluctuations.

2- Run linear regression based on deseasonalized demand to estimate initial level and trend (same as Holt's model).

3- Estimate seasonal factors.

4- Forecast

Trend- and seasonality-adjusted Exponential Smoothing (Winter's Model)

- ***Deseasonalizing the demand data.***

$$\bar{D}_t = \begin{cases} \left[D_{t-(p/2)} + D_{t+(p/2)} + \sum_{i=t+1-(p/2)}^{t-1+(p/2)} 2D_i \right] \Big/ (2p) & \text{for } p \text{ even} \\ \sum_{i=t-[(p-1)/2]}^{t+[(p-1)/2]} D_i/p & \text{for } p \text{ odd} \end{cases}$$

Where:

p (*periodicity) is the number of periods after which the seasonal cycle repeats itself*

$\bar{D}_t$ the deseasonalized demand for Period t

Trend- and seasonality-adjusted Exponential Smoothing (Winter's Model)

- ***Estimate seasonal factors.***

The seasonal factor $\bar{S}_t$ for period t is the ratio of actual demand D_t to deseasonalized demand $\bar{D}_t$ and is given as

$$\bar{S}_t = D_t / \bar{D}_t$$

The overall seasonal factor S_i, for a "season" is then obtained by averaging of the factors for a season. If there are *r* seasonal cycles, we obtain the overall seasonal factor as:

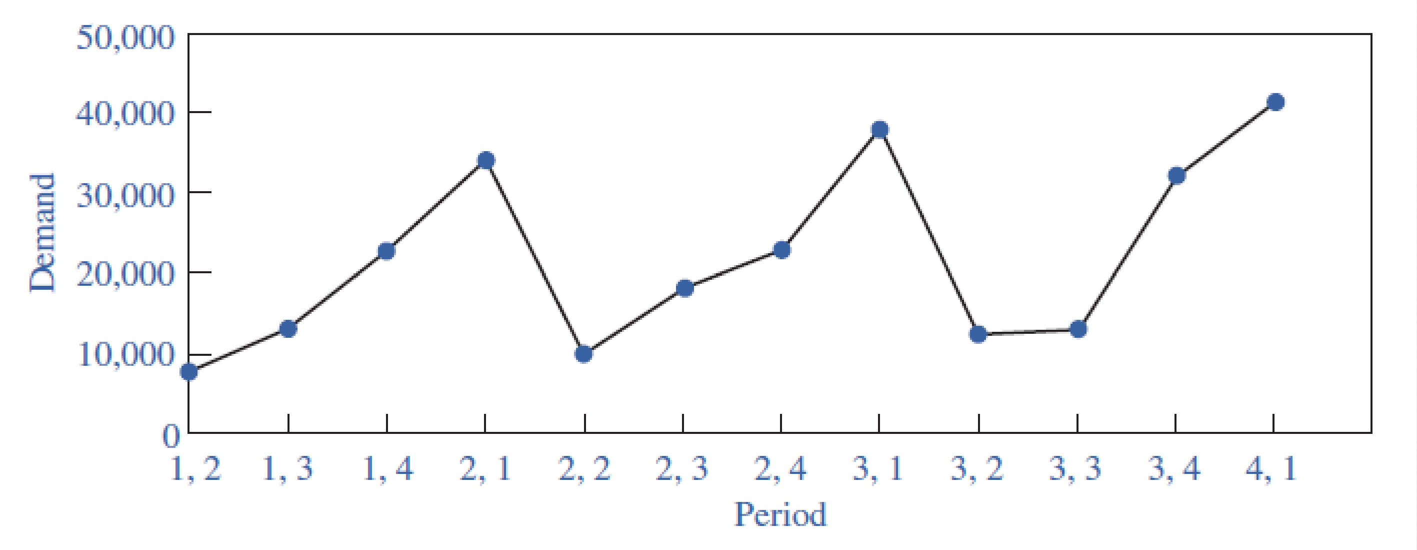
$$S_i = \frac{\sum_{j=0}^{r-1} \bar{S}_{jp+i}}{r} \quad \text{and } 1 \le i \le p$$

Trend- and seasonality-adjusted Exponential Smoothing (Winter's Model)

- Example. Quarterly retail demand data for the past three years are shown in bellow table. Using Winter's model, forecast demand for period 1 and 2 with α=0.1 , β=0.2, γ=0.1

Year	Quarter	Period, t	Demand, D_t
1	2	1	8,000
1	3	2	13,000
1	4	3	23,000
2	1	4	34,000
2	2	5	10,000
2	3	6	18,000
2	4	7	23,000
3	1	8	38,000
3	2	9	12,000
3	3	10	13,000
3	4	11	32,000
4	1	12	41,000

Trend- and seasonality-adjusted Exponential Smoothing (Winter's Model)



Given that we are measuring demand on a quarterly basis, the periodicity for the demand is $p = 4$

Trend- and seasonality-adjusted Exponential Smoothing (Winter's Model)

- Step 1. Deseasonalizing the demand data.

p = 4 is even,

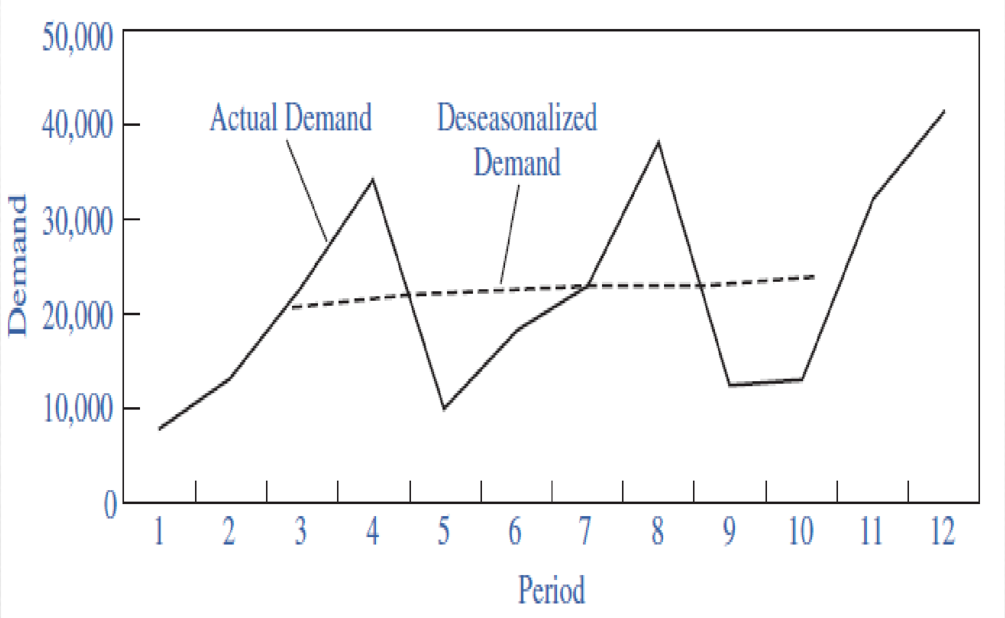
$\bar{D}_3 = (D_1+D_5+2D_2+2D_3+2D_4)/8$
=8000+10000+[(2)(13000)+(2)(23000)+(2)(34000)]/8 = 19750

$\bar{D}_4 = (D_2+D_6+2D_3+2D_4+2D_5)/8=$
13000+18000+[(2)(23000)+(2)(34000)+(2)(10000)]/8= 20625

With same procedure, we can obtain deseasonalized demand for other periods.

Trend- and seasonality-adjusted Exponential Smoothing (Winter's Model)

	A	B	C
1	*Period* t	*Demand* D_t	*Deseasonalized Demand*
2	1	8,000	
3	2	13,000	
4	3	23,000	19,750
5	4	34,000	20,625
6	5	10,000	21,250
7	6	18,000	21,750
8	7	23,000	22,500
9	8	38,000	22,125
10	9	12,000	22,625
11	10	13,000	24,125
12	11	32,000	
13	12	41,000	



Trend- and seasonality-adjusted Exponential Smoothing (Winter's Model)

- Step 2. Run linear regression based on deseasonalized demand to estimate initial level and trend.

$\overline{D}_t$ = L + tT

By linear regression explained in Hold's model, we obtain L=18439 and T=524.

$\overline{D}_t$ = 18349 + 524t

Step3. Estimate seasonal factors.

$\overline{S}_t = D_t/\overline{D}_t$

For t = 5 for example $\overline{S}_5 = D_5/\overline{D}_5$= 10000/21059 = 0.47

Trend- and seasonality-adjusted Exponential Smoothing (Winter's Model)

Period t	*Demand* D_t	*Deseasonalized Demand (Eqn 7.4)* $\overline{D}_t$	*Seasonal Factor (Eqn 7.5)* $\overline{S}_t$
1	8,000	18,963	0.42
2	13,000	19,487	0.67
3	23,000	20,011	1.15
4	34,000	20,535	1.66
5	10,000	21,059	0.47
6	18,000	21,583	0.83
7	23,000	22,107	1.04
8	38,000	22,631	1.68
9	12,000	23,155	0.52
10	13,000	23,679	0.55
11	32,000	24,203	1.32
12	41,000	24,727	1.66

Trend- and seasonality-adjusted Exponential Smoothing (Winter's Model)

- We should calculate the overall seasonal S_i, for each "season" by using this formula

$$S_i = \frac{\sum_{j=o}^{r-1} \bar{S}_{jp+i}}{r}$$

A total of 12 periods and a periodicity of p=4 imply that there are r = 3 seasonal cycles in the data.

$S_1 = (\bar{S}_1 + \bar{S}_5 + \bar{S}_9)/3 = (0.42+0.47+0.52)/3 = 0.47$

$S_2 = (\bar{S}_2 + \bar{S}_6 + \bar{S}_{10})/3 = (0.67+0.83+0.55)/3 = 0.68$

$S_3 = (\bar{S}_3 + \bar{S}_7 + \bar{S}_{11})/3 = (1.15+1.04+1.32)/3 = 1.17$

$S_4 = (\bar{S}_4 + \bar{S}_8 + \bar{S}_{12})/3 = (1.66+1.68+1.66)/3 = 1.67$

Trend- and seasonality-adjusted Exponential Smoothing (Winter's Model)

So far we obtained initial estimates of level, trend and seasonal factors as:

$L_0 = 18439$, $T_0 = 524$, $S_1 = 0.47$, $S_2 = 0.68$, $S_3 = 1.17$, $S_4 = 1.67$

$F_t = (L_{t-1} + T_{t-1})S_t$

$F_1 = (L_0 + T_0)S_1 = (18439 + 524) * 0.47 = 8913$

$L_t = \alpha(D_t/S_t) + (1-\alpha)(L_{t-1} + T_{t-1})$

$L_1 = \alpha(D_1/S_1) + (1-\alpha)(L_0 + T_0) = [0.1 * (8000/0.47)] + [0.9 * (18439 + 524] = 18769$

$T_t = \beta(L_t - L_{t-1}) + (1-\beta)T_{t-1}$

$T_1 = \beta(L_1 - L_0) + (1-\beta)T_0 = [0.2 *(18769 - 18439)] + (0.8 * 524) = 485$

Trend- and seasonality-adjusted Exponential Smoothing (Winter's Model)

So far we obtained initial estimates of level, trend and seasonal factors as:

$S_{t+p} = \gamma (D_t/L_t) + (1- \gamma)S_t$

$S_5 = \gamma (D_1/L_1) + (1- \gamma)S_1 = [0.1 * (8000/18769)] + (0.9 * .47) = 0.47$

$F_2 = (L_1 + T_1)S_2 = (18769 + 485) * 0.68 = 13093$

Static Method

A static method assumes that the estimates of level, trend, and seasonality do not vary as new demand is observed. In this case, we estimate each of these parameters based on historical data and then use the same values for all future forecasts.

$$F_t = (L_0 + tT_0)\ S_t$$

Note: L_0 (estimate of level at t=0), T_0 (estimate of trend at t=0), and St (overall seasonal factor at period t) are calculated same as Winter's model.

Static Method

Example. Consider the previous example and forecast for the next four periods using the static forecasting method.

we obtained initial estimates of level, trend and seasonal factors as in the previous example:

$L_0 = 18439$, $T_0 = 524$, $S_1 = 0.47$, $S_2 = 0.68$, $S_3 = 1.17$, $S_4 = 1.67$

$F_{13} = (L_0 + 13T_0)S_{13} = (18439 + 13 * 524)\ 0.47 = 11868$

$F_{14} = (L_0 + 14T_0)S_{14} = (18439 + 14 * 524)\ 0.68 = 17527$

$F_{15} = (L_0 + 15T_0)S_{15} = (18439 + 15 * 524)\ 1.17 = 30770$

$F_{16} = (L_0 + 16T_0)S_{16} = (18439 + 16 * 524)\ 1.67 = 44794$

Linear Regression

- Regression analysis is a mathematical method used to understand the relationship between a dependent variable (y) and one or more dependent variables (x_1, x_2, … x_n).

$y = a_1x_1 + a_2x_2 + \ldots a_nx_n + b$

There are multiple different types of regression analysis, but the most basic and common form is simple linear regression that uses the following equation: $y = ax + b$

$$a = \frac{n\sum xy - \sum x \sum y}{n\sum x^2 - (\sum x)^2} \qquad b = \frac{\sum y - b\sum x}{n}$$

Linear Regression

- In regression model, the most commonly known evaluation metric is R-squared (R2). R-squared measures the strength of the relationship between your model and the dependent variable.

$$R^2 = 1 - \frac{\sum(y_i - \hat{y}_i)^2}{\sum(y_i - \bar{y})^2}$$

Where:

$\hat{y}_i$ = predicted y value

y_i = actual y value

R-squared values range from 0 to 1 and are commonly stated as percentages from 0% to 100%.

Values near to 1 indicate a better fit

Linear Regression

Example. A company based on its research, has concluded that the amount of product sales is directly related to the amount of advertising. The data of the last 4 years are as follows:

Advertising cost (x)	**Sales amount (y)**
32	130
52	151
50	150
55	158

If the company allocates $ 53 million for advertising for the next year, forecast the amount of sales by using linear regression method. What is your idea about the accuracy of this model?

Linear Regression

	Advertising cost (x)	Sales amount (y)	x^2	xy
	32	130	1024	4160
	52	151	2704	7852
	50	150	2500	7500
	55	158	3025	8690
Sum	189	589	9253	28202

$$a = \frac{4(28202) - 189(589)}{4(9253) - (189)^2} = 1.15 \qquad b = \frac{589 - 1.15(189)}{4} = 92.91$$

y = 1.15x + 92.91

y = 1.15(53) + 92.91 = 153.86

Linear Regression

For point (32, 130)

$\hat{y}_1 = 1.15(32) + 92.1 = 129.71$ $\quad$ $y_1 - \hat{y}_1 = 130 – 129.71 = 0.29$

For point (52, 151)

$\hat{y}_2 = 1.15(52) + 92.1 = 152.71$ $\quad$ $y_2 - \hat{y}_2 = 151 – 152.71 = -1.71$

For point (50, 150)

$\hat{y}_3 = 1.15(50) + 92.1 = 150.41$ $\quad$ $y_3 - \hat{y}_3 = 150 – 150.41 = -0.41$

For point (55, 158)

$\hat{y}_4 = 1.15(55) + 92.1 = 156.16$ $\quad$ $y_4 - \hat{y}_4 = 158 – 156.16 = 1.84$

$\sum(y_i - \hat{y}_i)^2 = 6.56$

Linear Regression

- $y_1 - \bar{y} = 130 – 147.25 = -17.25$

$y_2 - \bar{y} = 151 – 147.25 = 3.75$

$y_3 - \bar{y} = 150 – 147.25 = 2.75$

$y_4 - \bar{y} = 158 – 147.25 = 10.75$

$\sum(y_i - \bar{y})^2 = 434.75$

$R^2 = 1 - \frac{6.56}{434.75} = 98.49\%$ (what is your conclusion?)

Measures of forecast error

- Forecast error for period t is given by E_t, where the following holds:

$E_t = F_t – D_t$

One measure of forecast error is the Mean Squared Error (MSE) where the following holds:

$$MSE = \frac{1}{n}\sum_{t=1}^{n} {E_t}^2$$

The MSE penalizes large errors much more significantly than small errors because all errors are squared.

Measures of forecast error

- Absolute deviation in period t, A_t, is the absolute value of the error in period t,

$$A_t = |E_t|$$

The Mean Absolute Deviation (MAD) is the average of the absolute deviation over all periods,

$$MAD = \frac{1}{n}\sum_{t=1}^{n} A_t$$

MAD is an appropriate choice when selecting forecasting methods if the cost of a forecast error is proportional to the size of the error.

Measures of forecast error

- The Mean Absolute Percentage Error (MAPE) is the average absolute error as a percentage of demand,

$$MAPE_n = \frac{\sum_{t=1}^{n}\left[\frac{E_t}{D_t}\right] * 100}{n}$$

MAPE is a good measure of forecast error when the underlying forecast has significant seasonality and demand varies considerably from one period to the next.

Forecasting Process