Math 104 – Mathematics for Business and Economics II
QUIZ II
Duration 50 minutes

Name		***Student No***	
Surname		***Group***	

For the following questions ***show all your work clearly*** to find the answer.

Question 1. (*25 pts.*) Find the following **derivative**.

$$y = \sqrt{2x+x}\ln\left(x^3+2x+5\right)+e^{3x^2+2x+1}+x^e$$

$$y = \frac{3}{2}\left(2x+x\right)^{-\frac{1}{2}}\ln\left(x^3+2x+5\right)+\frac{3x^2+2}{x^3+2x+5}\sqrt{2x+x}+\left(6x+2\right)e^{3x^2+2x+1}+ex^{e-1}$$

Question 2. (*25 pts.*) Find the equation of the **tangent line** touching to $y=\frac{1}{x^2}$ at $x=1$.

$y'=\frac{-2x}{x^4}=-\frac{2}{x^3}$, $m=\frac{-2}{1^3}=-2$, when $x=1$ then $y=\frac{1}{1^2}=1$

The equation of the tangent line of this curve is

$y-y_1=m\left(x-x_1\right)$, $y-1=-2\left(x-1\right)$

$y=-2x+3$

Question 3. (*25 pts.*) Find all x on $y=f\left(x\right)=3x^4-4x^3+2$ where the tangent line to y at x is horizontal.

The slope of the horizontal tangent lines is zero. $m=0$

$$m=y'=f'\left(x\right)=12x^3-12x^2=0$$
$$12x^2(x-1)=0,\quad x=0 \quad \text{and} \quad x=1$$

Question 4. (*25 pts.*) Find all **criticals (min / max)** on $y=3x^4-4x^3+2$.

$y'=12x^3-12x^2=12x^2(x-1)=0$. The critical points are $x=0$ and $x=1$. We can use first derivative test ,

x	0	1	
$12x^2$	+	+	+
$x-1$	-	-	+
y'	-	-	+
y	decreasing	decreasing	increasing

If $x=0$ then $y=2$. If $x=1$ then $y=1$.

So relative minimum point is $\left(1,1\right)$.

$\left(0,2\right)$ is an inflection point from the second derivative of y.

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For the following questions ***show all your work clearly*** to find the answer.

Question 1. (*25 pts.*) Find the following **derivative**.

$$y = \ln\left(\frac{2x+3}{3x-4}\right) + e^{\left(x^3+x-1\right)^2} + x^{\ln(2)}$$

$$y' = \frac{2(3x-4)-3(2x+3)}{(3x-4)^2\left(\frac{2x+3}{3x-4}\right)} + 2\left(x^3+x-1\right)\left(3x^2+1\right)e^{\left(x^3+x-1\right)^2} + \ln(2)x^{\ln(2)-1}$$

Question 2. (*25 pts.*) Find the equation of the **tangent line** touching to $y = \frac{1}{x-1}$ at $x = 2$.

$y' = \frac{-1}{(x-1)^2}$, $m = \frac{-1}{1^2} = -1$, when $x = 2$ then $y = \frac{1}{1} = 1$

The equation of the tangent line of this curve is

$y - y_1 = m(x - x_1)$, $y - 1 = -(x-2)$, $\boxed{y = -x+3}$

Question 3. (*25 pts.*) Find all x on $y = f(x) = 2x^3 - x^4$ where the tangent line to y at x is horizontal.
The slope of the horizontal tangent lines is zero. $m = 0$

$$m = y' = f'(x) = 6x^2 - 4x^3 = 0$$

$$2x^2(3-x) = 0, \quad x = 0 \quad \text{and} \quad x = 3$$

Question 4. (*25 pts.*) Find all **criticals (min / max)** on $y = 2x^3 - x^4$.
$y' = 6x^2 - 4x^3 = 2x^2(3-x) = 0$. The critical points are $x = 0$ and $x = 3$. We can use first derivative test ,

x		0	3
$2x^2$	+	+	+
$3-x$	+	+	-
y'	+	+	-
y	increasing	increasing	decreasing

If $x = 0$ then $y = 0$. If $x = 3$ then $y = -27$.
So, relative maximum points are $\boxed{(3,-27)}$.

$(0,0)$ is an inflection point from the second derivative of y.

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For the following questions ***show all your work clearly*** to find the answer.

Question 1. (*20 pts.*) Find all x on $f(x)$ where the tangent line to $f(x)$ at x is horizontal

$$f(x) = \frac{1}{4}x^4 - \frac{1}{2}x^2 + 3$$

The slope of horizontal tangent lines is zero.

$m = f'(x) = x^3 - x = x(x-1)(x+1) = 0$

So , $\boxed{x = 0, x = -1, x = 1}$

Question 2. (*60 pts.*) Find the absolute extrema of the given function on the given interval.

$$f(x) = \frac{1}{4}x^4 - \frac{1}{2}x^2 + 3, \quad [-2,3]$$

$f'(x) = x^3 - x = x(x-1)(x+1) = 0 \quad , \quad x = 0, -1, 1 \in [-2,3]$

x		-1	0	1
$x+1$	-	+	+	+
x	-	-	+	+
$x-1$	-	-	-	+
y'	-	+	-	+
y	decreasing	increasing	decreasing	increasing

If $x = 0$ then $y = 3$ .If $x = -1$ then $y = \frac{1}{4} - \frac{1}{2} + 3 = \frac{11}{4} = 2.75$. If $x = 1$ then $y = \frac{1}{4} - \frac{1}{2} + 3 = \frac{11}{4} = 2.75$.

If $x = -2$ then $y = 4 - 2 + 3 = 5$. If $x = 3$ then $y = \frac{81}{4} - \frac{9}{2} + 3 = \frac{75}{4} = 18.5$.

So, relative maximum points are $\boxed{(0,3)}$ .Relative minimum points are $\boxed{\left(-1, \frac{11}{4}\right) \text{ and } \left(1, \frac{11}{4}\right)}$.

Absolute maximum point is $\boxed{\left(3, \frac{75}{4}\right)}$ and Absolute minimum points are $\boxed{\left(-1, \frac{11}{4}\right) \text{ and } \left(1, \frac{11}{4}\right)}$.

Question 3. (*20 pts.*) Find an equation of the **tangent line** to the curve of $y = \frac{x+4}{5-x}$ at the point $x = 1$.

$y' = \frac{(5-x)+(x+4)}{(5-x)^2}$, when $x = 1$, $m = y' = \frac{(4)+(5)}{(4)^2} = \frac{9}{16}$ and $y = \frac{5}{4}$, The equation of tangent line is $y - y_1 = m(x - x_1)$, $\quad y - \frac{5}{4} = \frac{9}{16}(x-1)$, $\quad \boxed{y = \frac{9}{16}x + \frac{11}{16}}$