Math 104 – Mathematics for Business and Economics II *QUIZ III* *Duration 50 minutes*			*A*
Name		*Student No*	
Surname		*Group*	

For the following questions ***show all your work clearly*** to find the answer.

Question 1. (*60 pts.*) Given the total cost function $C = q^2 + 2q + 500$ and the demand function $q = 100 - 0.5p$.

a) Find the total revenue function in quantity q; $R(q)$.

Solution:

$q = 100 - 0.5p$, $\quad p = -2q + 200$

$R(q) = (-2q + 200)q = -2q^2 + 200q$

b) Find the output level q for minimum average cost.
Solution:

The average cost is $\bar{C} = \frac{C}{q} = \frac{q^2 + 2q + 500}{q} = q + 2 + \frac{500}{q}$

For minimum average cost, $\bar{C}'(q) = 1 - \frac{500}{q^2} = 0$, $\quad q^2 - 500 = 0$, $\quad q = \sqrt{500}$ units

From the second derivative test, $\bar{C}''(q) = \frac{1000}{q^3}$,

$\bar{C}''\left(\sqrt{500}\right) = \frac{1000}{\left(\sqrt{500}\right)^3} = \frac{1000}{500\sqrt{500}} = \frac{2}{\sqrt{500}} > 0$, Relative minimum

c) Is the 21^{st} unit going to be profitable for the company?

Solution:

$MR = R'(q) = -4q + 200$, $\quad MR = R'(20) = -4(20) + 200 = 120$

$MC = C'(q) = 2q + 2$, $\quad MC = C'(20) = 2(20) + 2 = 42$

So, $MR(20) > MC(20)$ means profitable.

d) Find the profit maximizing capacity of output q.
Solution:

For profit maximizing, $MR = MC$, $-4q + 200 = 2q + 2$, $6q = 198$, $\boxed{q = 33}$ units

e) What will be the selling price at maximum profit?

Solution:

$$p = 200 - 2(33) = 200 - 66 = 134\$$$

Question 2. (*40 pts.*) Given the demand function $q = -5p^2 - 2p + 1000$,

a) Find and explain the elasticity of demand if $p = \$10$

Solution:

The elasticity is $\eta = \frac{p}{q}\frac{dq}{dp}$

$$q = -5(10)^2 - 2(10) + 1000 = -500 - 20 + 1000 = 480 , \quad p = 10$$

$$\frac{dq}{dp} = -10p - 2 , \quad \frac{dq}{dp}(10) = -10(10) - 2 = -100 - 2 = -102$$

$$\eta = \frac{p}{q}\frac{dq}{dp} = \frac{10}{480}(-102) = -2.125$$

$|\eta| = |-2.125| = 2.125 > 1$, Demand is **elastic**.

%1 increase in price will result %2.125 decrease in demand.

b) Show that when demand has unit elasticity total revenue is at its maximum.

Solution:

$$R(p) = (-5p^2 - 2p + 1000)p = -5p^3 - 2p^2 + 1000p$$

$\boxed{R'(p) = -15p^2 - 4p + 1000 = 0}$ for maximizing revenue.

$$\frac{dq}{dp} = -10p - 2$$

$$\eta = \frac{p}{q}\frac{dq}{dp} = \frac{p}{-5p^2 - 2p + 1000}(-10p - 2) = -1$$

$$\frac{-10p^2 - 2p}{-5p^2 - 2p + 1000} = -1 , \quad -10p^2 - 2p = 5p^2 + 2p - 1000$$

$15p^2 + 4p - 1000 = 0$ or $\boxed{-15p^2 - 4p + 1000 = 0}$

Math 104 – Mathematics for Business and Economics II *QUIZ III* *Duration 50 minutes*			*B*
Name		***Student No***	
Surname		***Group***	

For the following questions ***show all your work clearly*** to find the answer.

Question 1. (*60 pts.*) Given the average cost $\bar{C} = q + 2 + \frac{500}{q}$, demand function $q = \frac{200-p}{2}$

a) Find the total revenue function in quantity q ; $R(q)$.

Solution:

$$q = \frac{200-p}{2}, \quad p = -2q + 200$$

$$R(q) = \left(-2q + 200\right)q = -2q^2 + 200q$$

b) Find the output level q for minimum average cost.

Solution:

$$q = \frac{200-p}{2}, \quad p = -2q + 200$$

$\bar{C}'(q) = 1 - \frac{500}{q^2} = 0$, $q = \sqrt{500}$ units

From the second derivative test, $\bar{C}''(q) = \frac{1000}{q^3}$,

$\bar{C}''\left(\sqrt{500}\right) = \frac{1000}{\left(\sqrt{500}\right)^3} = \frac{1000}{500\sqrt{500}} = \frac{2}{\sqrt{500}} > 0$, Relative minimum

c) Find the approximated additional cost of producing the 21^{st} unit.

Solution:

$$C = \bar{C}q = q^2 + 2q + 500$$

$$MC = C'\left(q\right) = 2q + 2$$

The approximated additional cost is $MC = C'\left(20\right) = 2(20) + 2 = 42$

d) Is the 21^{st} unit going to be profitable for the company?

Solution:

$MR = R'(q) = -4q + 200$, $MR = R'(20) = -4(20) + 200 = 120$

$MC = C'(q) = 2q + 2$, $MC = C'(20) = 2(20) + 2 = 42$

So, $MR(20) > MC(20)$ means profitable.

e) What will be the selling price at maximum profit?

Solution:

For profit maximizing, $MR = MC$, $-4q + 200 = 2q + 2$, $6q = 198$, $\boxed{q = 33}$ units

$p = 200 - 2(33) = 200 - 66 = 134\$$

Question 2. (*40 pts.*) Given the demand function $q = \frac{130}{p} - 0.2p + 5$,

a) Find and explain the elasticity of demand if $p = \$10$

Solution:

The elasticity is $\eta = \frac{p}{q}\frac{dq}{dp}$

$q = \frac{130}{10} - 0.2(10) + 5 = 13 - 2 + 5 = 16$, $p = 10$

$\frac{dq}{dp} = -\frac{130}{p^2} - 0.2$, $\frac{dq}{dp}(10) = -\frac{130}{100} - 0.2 = -1.5$

$\eta = \frac{p}{q}\frac{dq}{dp} = \frac{10}{16}(-1.5) = -0.937$

$|\eta| = |-0.937| = 0.937 < 1$, Demand is **inelastic**.

%1 increase in price will result $\%\,0.937$ decrease in demand.

b) Find p at which demand is unit elastic.

Solution:

$$|\eta| = \left|\frac{p}{q}\frac{dq}{dp}\right| = \left|\frac{p}{\frac{130}{p} - 0.2p + 5}\left(-\frac{130}{p^2} - 0.2\right)\right| = 1$$

$$\frac{p}{\frac{130}{p} - 0.2p + 5}\left(-\frac{130}{p^2} - 0.2\right) = 1 \quad \text{or} \quad \frac{p}{\frac{130}{p} - 0.2p + 5}\left(-\frac{130}{p^2} - 0.2\right) = -1$$

$\dfrac{-130 - 0.2p^2}{130 - 0.2p^2 + 5p} = 1$, $p = -52$ which is not possible.

$\dfrac{p}{\frac{130}{p} - 0.2p + 5}\left(-\dfrac{130}{p^2} - 0.2\right) = -1$, $\dfrac{-130 - 0.2p^2}{130 - 0.2p^2 + 5p} = -1$, $0.4p^2 - 5p = 0$,

$p = 0, p = 12.5$, but $p = 0$ is not possible because when $p = 0$, $q = \dfrac{130}{p} - 0.2p + 5$ is not defined.

So, $\boxed{p = 12.5\$}$.