Eastern Mediterranean University
Department of Mathematics
Math 106– Linear algebra
2. Quiz
Duration: 45 minutes

27 April 2011

— Solution Manual

Name		No:	
Surname		Signature:	

Q1	Q2	Q3	Total
/25	/30	/55	/110

Question 1..Show that if $\underline{u}$ and $\underline{v}$ are orthogonal vectors in R^n such that $\|\underline{u}\|=1, \|\underline{v}\|=1$ then $d(\underline{u},\underline{v})=\sqrt{2}$

$$d(\underline{u},\underline{v}) = \|\underline{u}-\underline{v}\| = \sqrt{(\underline{u}-\underline{v})\cdot(\underline{u}-\underline{v})} = \sqrt{\underline{u}\cdot\underline{u} - 2\underline{u}\cdot\underline{v} + \underline{v}\cdot\underline{v}}$$

5, 5, 5

$$\|\underline{u}\|^2 - 2\underline{u}\cdot\underline{v} + \|\underline{v}\|^2$$

$$= \sqrt{\underbrace{\|\underline{u}\|^2}_{1} - 2\overbrace{\underline{u}\cdot\underline{v}}^{0} + \underbrace{\|\underline{v}\|^2}_{1}} \quad 5,$$

$\sqrt{2}$ 5,

Question 2. Show that $\begin{vmatrix} a & a & a & a \\ b & b & b & -b \\ c & c & -c & -c \\ d & -d & -d & -d \end{vmatrix} = 8abcd$

5.

$$= abcd \begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & -1 \end{vmatrix} = abcd \begin{vmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 2 & 0 \\ 0 & 2 & 0 & 0 \\ 1 & -1 & -1 & -1 \end{vmatrix} \xrightarrow{} C_3 \to -C_2 + C_3$$

5.

$R_3 \to -R_4 + R_3$
$R_2 \to -R_4 + R_2$
$R_1 \to R_4 + R_1$ 15

$$= abcd \begin{vmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 2 & -2 & 0 \\ 1 & -1 & 0 & -1 \end{vmatrix} = 8\,abcd$$

5.

Q3) Use Cramer's Rule to solve for y. Do not solve for x, z, w.

$4x + y + z + w = 6$
$3x + 7y - z + w = 1$
$7x + 3y - 5z + 8w = -3$
$x + y + z + 2w = 3$

$$A = \begin{bmatrix} 4 & 1 & 1 & 1 \\ 3 & 7 & -1 & 1 \\ 7 & 3 & -5 & 8 \\ 1 & 1 & 1 & 2 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 4 & 6 & 1 & 1 \\ 3 & 1 & -1 & 1 \\ 7 & -3 & -5 & 8 \\ 1 & 3 & 1 & 2 \end{bmatrix}$$

$\det(A) \Rightarrow$ $R_1 \leftrightarrow R_4$ $(-)$ $\begin{vmatrix} 1 & 1 & 1 & 2 \\ 3 & 7 & -1 & 1 \\ 7 & 3 & -5 & 8 \\ 4 & 1 & 1 & 1 \end{vmatrix}$ $R_2 \to (-3)R_1 + R_2$, $R_3 \to (-7)R_1 + R_3$, $R_4 \to (-4)R_1 + R_4$ no change

$$(-)\begin{vmatrix} 1 & 1 & 1 & 2 \\ 0 & 4 & -4 & -5 \\ 0 & -4 & -12 & -6 \\ 0 & -3 & -3 & -7 \end{vmatrix} = (-1)(1)\cdot(-1)^{1+1} \begin{vmatrix} 4 & -4 & -5 \\ -4 & -12 & -6 \\ -3 & -3 & -7 \end{vmatrix}$$

$$= (-1)\left[4 \begin{vmatrix} -12 & -6 \\ -3 & -7 \end{vmatrix} + 4 \begin{vmatrix} -4 & -6 \\ -3 & -7 \end{vmatrix} - 5 \begin{vmatrix} -4 & -12 \\ -3 & -3 \end{vmatrix} \right]$$

$$= -1 \left[4(84-18) + 4(28-18) - 5(12-36) \right]$$

$$= -1 \left[264 + 40 + 120 \right] = -424$$

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$\det A_2 \Rightarrow \begin{vmatrix} 4 & 6 & 1 & 1 \\ 3 & 1 & -1 & 1 \\ 7 & -3 & -5 & 8 \\ 1 & 3 & 1 & 2 \end{vmatrix}$ $R_1 \leftrightarrow R_4$ $\Rightarrow (-1) \begin{vmatrix} 1 & 3 & 1 & 2 \\ 3 & 1 & -1 & 1 \\ 7 & -3 & -5 & 8 \\ 4 & 6 & 1 & 1 \end{vmatrix}$

$R_2 \to (-3)R_1 + R_2$, $R_3 \to (-7)R_1 + R_3$, $R_4 \to (-4)R_1 + R_4$ no change

$$(-1)\begin{vmatrix} 1 & 3 & 1 & 2 \\ 0 & -8 & -4 & -5 \\ 0 & -24 & -12 & -6 \\ 0 & -6 & -3 & -7 \end{vmatrix} = -1(1)^{1+1}\left[\det \begin{vmatrix} -8 & -4 & -5 \\ -24 & -12 & -6 \\ -6 & -3 & -7 \end{vmatrix}\right]$$

$$= -1\left[-8 \begin{vmatrix} -12 & -6 \\ -3 & -7 \end{vmatrix} + 4 \begin{vmatrix} -24 & -6 \\ -6 & -7 \end{vmatrix} - 5 \begin{vmatrix} -24 & -12 \\ -6 & -3 \end{vmatrix} \right]$$

$$= -\left[-8(84-18) + 4(168-36) - 5(\underbrace{\cancel{72}-72}_{0})\right]$$

$$= -\left[-528 + 528\right] = 0$$

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$$y = \frac{0}{-424} = 0$$