## Third tutorial

1- For the following uniform-series amounts determine: $i$ ) the present value $i$ i) the future value in year 11 and iii) equivalent annual series (Annualized over 11 years) ( $i=\% 8$ ).

i) $\mathrm{P}=500(\mathrm{P} / \mathrm{A}, 8 \%, 4) *(\mathrm{P} / \mathrm{F}, 8 \%, 3)=500 * 3.3121 * 0.7938=\$ 1,314.57$
ii) First way: $\mathrm{F}=\mathrm{P}(\mathrm{F} / \mathrm{P}, 8 \%, 11)=1,314.57 * 2.3316=\$ 3,065$

Second way: $\mathrm{F}=500(\mathrm{~F} / \mathrm{A}, 8 \%, 4) *(\mathrm{~F} / \mathrm{P}, 8 \%, 4)=500 * 4.5061 * 1.3605=\$ 3,065$
iii) $\mathrm{A}=1,314.57(\mathrm{~A} / \mathrm{P}, 8 \%, 11)=3,065(\mathrm{~A} / \mathrm{F}, 8 \%, 11)=\$ 184.14$

2- Annualize the following cash flow over 11 years ( $\mathrm{i}=\boldsymbol{\%} 10$ ).

$$
\mathrm{i}=\% 10
$$



The present value of the Arithmetic Gradient will always be located two periods before the gradient starts (at year 4):
$\mathrm{P}_{4}=100(\mathrm{P} / \mathrm{A}, 10 \%, 4)+100(\mathrm{P} / \mathrm{G}, 10 \%, 4)=754.8$
$\mathrm{P}_{0}=754.8(\mathrm{P} / \mathrm{F}, 10 \%, 4)=515.5$
$\mathrm{A}=515.5(\mathrm{~A} / \mathrm{P}, 10 \%, 11)=79.4$

3- Annualize the following cash flow over 10 years ( $\mathrm{i}=\boldsymbol{\%} 15$ ).

$\mathrm{P}_{4}=\mathrm{D}(\mathrm{P} / \mathrm{A}, \mathrm{E} \%, \mathrm{i} \%, \mathrm{n})=1,000(\mathrm{P} / \mathrm{A}, 10 \%, 15 \%, 4)=1000 * 3.258=3,258$
$(\mathrm{P} / \mathrm{A}, 10 \%, 15 \%, 4)=\frac{1}{E-i} *\left[\frac{(1+E)^{n}}{(1+i)^{n}}-1\right]=\frac{1}{0.1-0.15} *\left[\frac{(1+0.1)^{4}}{(1+0.15)^{4}}-1\right]=3.258$
$\mathrm{P}_{0}=3,258(\mathrm{P} / \mathrm{F}, 15 \%, 4)=1,863$
$\mathrm{A}=1,863(\mathrm{~A} / \mathrm{P}, 15 \%, 10)=371$

4- Annualize the following cash flow over 10 years ( $i=\% 8$ ).
$\mathrm{i}=\% 8$


Year
$\mathrm{P}_{\mathrm{G}}=$ the present value of arithmetic gradient $=[3,000(\mathrm{P} / \mathrm{A}, 8 \%, 4)-500(\mathrm{P} / \mathrm{G}, 8 \%, 4)] *(\mathrm{P} / \mathrm{F}, 8 \%, 2)=6,525$
$\mathrm{P}_{\mathrm{A}}=$ the present value of uniform-series amounts $=1,500(\mathrm{P} / \mathrm{A}, 8 \%, 3) *(\mathrm{P} / \mathrm{F}, 8 \%, 6)=2,436$
$P_{T}=P_{G}+P_{A}=6,525+2,436=8,961$
$\mathrm{A}=8,961(\mathrm{~A} / \mathrm{P}, 8 \%, 10)=1,335$

