IENG/MANE 112

Out-Door Transportation and Traveling Salesman Problem (TSP)

1. Main transportation ways:

1. **Road**
   a. Flexible, can achieve any point.
   b. Track is paid mainly by the state.
   c. Its importance and market share are increasing in spite of the efforts of the governments.

2. **Railways**
   a. Very large mass can be transported.
   b. The track is paid by itself and it is contained in the tariff.
   c. The tendency is in Europe that the track should be separated from the transportation companies to make market competition more effective.
   d. It loses importance and market share.
   e. Sensitive items need good packing.

3. **Sea and river**
   a. It is cheap, however it is slow.
   b. It is not applicable everywhere.
   c. Very large mass can be transported.
   d. The importance of container transportation is growing even in the case of bulk product like wheat.

4. **Air**
   a. Expensive and fast.

5. **Tube**
   a. It can transport special products only like gas, oil, water, and powder.
   b. It needs large investment, however the operation is cheap.
   c. Turkey is interested in such transportation.
   d. TRNC will get sweet water from Turkey via tube.

2. The logic of railways transportation and private postal services

Railways

Local customer → collection wagons → marshaling yard (shunting yard) → trains to the other marshaling yard → wagon distribution → local customer

Postal services

Local agent → region/country collection center → continental center → transportation to the other continent → region/country distribution center (same as region/country collection center) → local agent

3. The Traveling Salesman Problem (TSP)

A salesman must visit must visit a lot of other cities to make business starting from his home city and at the end he most return home. Find the tour having minimal total length.

Origin of TSP: German handbook 1832.

First mathematical formulation: Dantzig-Fulkerson-Johnson 1954. It is the solution of the problem on the 49 capitals of USA.

3.1 Applications of TSP in IE
3.1.1 A Lorry Serves Several Shops
- home city: depot
- other cities: shops
- distance:
  - geographic distance: in the case of large distances or early hours like in the case of newspaper or
  - Time: in the city during rush hours.

3.1.2 Chip Production
- The material of integrated circuits consists of layers.
- The electric properties depend on that how many layers are cut off.
- Cutting-off is made by a laser ray. Burning time is technologically determined.
- The most effective way of the production is when the total move of the laser ray is minimized.
- Cities: points on the surface
- Distance: the angle what the laser ray must rotate. It has the good geometric properties: symmetry, triangle inequality.
- The largest solved TSP problems are of this kind because of the regular positions of the "cities". The size is industrial secret.

3.1.3 Automatic Drill Machine
- The most effective way of the production is again when the total move of the tool is minimized.
- The part may have a complex surface.
- The move of the machine from one point to another point consists of three parts: (i) pull out and lift the tool into a safety height, (ii) move above the next point, and (iii) go down.
- The triangle inequality may not be satisfied.
- Computationally difficult problems.

3.1.4 The Minimization of the Total Set-Up Time

Set-up is the adjustment of a machine to make it able to produce the next product. During set-up time, there is no production, i.e. it is a kind of loss, although it is necessary.

- Cities: products; distances: set-up times.
- Non-symmetric, no triangle inequality.

3.1.5 The Minimization of the Total Length of the Route of an Assembly Robot

- The move of the robot is "bipartite": picks up a part, <u>goes to the assembly point</u>, fixes it, and <u>goes for the next part</u>.
- The two classes of cities: cells of parts and assembly points.
- The most effective way of the production is again when the total move of the robot is minimized.

3.1.6 The Minimization of the Total Delay in Chemical Industry

- In chemical industry the products cannot wait before the machines.
- Therefore products can start after one another with delay only.
- The length of the necessary delay depends on the two products.
- Cities: products; distances: delays.

3.1.7 Collecting Children and Transporting Them to School

- The start and the end is not the same (Hamiltonian Path instead of Hamiltonian circuit)
- Segments

3.2 Mathematical Formulation of TSP

- **Decision Variables** $x_{ij} = \begin{cases} 1 \text{ if city j is visited just after city i} \\ 0 \text{ if another city is visited after i} \end{cases}$

- **Objective function**

$$Min = \sum_{i=1}^{n} \sum_{j=1, i \neq j}^{n} c_{ij} x_{ij}$$

- **1-st constraint set:** Every city must be left

$$\text{for all i: } \sum_{j=1}^{n} x_{ij} = 1$$

- **2-nd constraint set:** The salesman must arrived to every city

$$\text{for all j: } \sum_{i=1}^{n} x_{ij} = 1$$

- These constraints still allow subtours: $1 \rightarrow 2 \rightarrow 3 \rightarrow 1$, $4 \rightarrow 5 \rightarrow 6 \rightarrow 4$
- Subtour elimination constraints: in every subset of cities the number on travels is at most the number of cities minus 1: S has at least 2 and at most n-1 elements

$$\sum_{i,j \in S} x_{ij} \leq |S| - 1$$

3.3 Difficulty of the Problem

Home city is the first one. All other cities can be in any order. Therefore the number of all cases is (n-1)(n-2)x…x3x2x1 = (n-1)! = "n-1 factorial".

n	$n!$	n	$n!$
0	1	8	40,320
1	1	9	362,880
2	2	10	3628,800
3	6	11	39916,800
4	24	12	479,001,600
5	120	13	6,227,020,800
6	720	14	87,178,291,200
7	5040	15	1,307,674,368,000

Exact model helps to solve problems with much larger number of cities. Iterative method by adding violated subtour elimination constraints works for many industrial problems. For example, printed circuit board assembly can be solved up to 100+ parts.

4. Timetable for public transport
- Trains, busses need cyclic schedule.

- A train on the track has an ID number. It goes from A to B first. Then the same train gets a new ID number and goes from B back to either A or somewhere else. At the end of the time period it must get back to the initial position.
- Slow and fast trains must be distinguished.
- Time window solutions.

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In-Class Problems

1. Determine the distances of points in a Manhattan like city. The points are in the left-lower corner.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1												A						
2																		
3																		
4																C		
5				B														
6																		
7												F						
8																		
9																		
10																		
11																		
12																	E	
13								D										
14																		

2. A workcenter processes several products. Each product needs different tools as follows:

Product	Tools
ST101	G3, F5
LA98	A4, G3
BG50	A4, F5
K23	F5, G3, H5
ST105	G3, H4

The tool magazine of the workcenter has 3 positions. A move is either to put a tool into the tool magazine, or to take out a tool. Determine the number of moves for every pair of products.