IENG/MANE112

Engineering Economy

1. Time Diagram of Investments: Shows the net cash in-flow/out-flow.

- **Salvage value (Future value):** The value of the equipment at the end of its use.
- **Costs in time diagram (cash flow diagram):** 1. Investment, 2. Yearly installment, 3. Operation, 4. Maintenance.

An example:

As president of a company, one of your major decisions is to decide which one if either of two new automated machines should be purchased to replace a manual method now in use. The product line on which all three methods are used will be discontinued entirely in 5 years. The manual method costs $10,000 per year. The new automated machine A costs $30,000 now (purchase price) and will cost $3,500 per year to operate over the next 5 years, at which time it will be worthless. Another new automated machine B is available that costs $40,000 new and will cost $3,000 per year to operate for the next 5 years, at which time it will be worth $9,000.
The company, requires a 10% return on all investments.

2. Interest Factors:

The purpose of the interest factors developed here is to reflect the cost of tying money up in one place when the money could have been invested in another place. This cost is sometimes called opportunity cost. The first factor is called the *single payment compound amount factor.*

Suppose that you have $1,000 today and could invest that at 10 % interest per year for 5 years. What would the $1,000 be worth then? Obviously, at the end of the first year it would be worth $1,100 ($1,000 + $100 interest). For the second year, there is $1,100 at the beginning of the year; so there would be $1,210 at the end of the year [$1,100 + 10% (1100), or $1,210]. This could be continued as shown Table.

A *Year*	*B* *Investment at Beginning of Year*	*C* *Interest Earned 10% (Col. B)*	*D* *Amount at End of Year*
1	$1,000	1,000 (.10) = 100	1,000 (1.10) = 1,100
2	$1,100	1,100 (.10) = 110	$1,000 (1.10)^2 = 1,210$
3	$1,210	1,210 (.10) = 121	$1,000 (1.10)^3 = 1,331$
4	$1,331	1,331 (.10) = 133	$1,000 (1.10)^4 = 1,464$
5	$1,464	1,464 (.10) = 146	$1,000 (1.10)^5 = 1,610$

2.1 Important symbols:

i **=** the annual interest rate;
n **=** the number of annual interest periods;
P **=** a present principal sum (e.g., \$1,500 at the end of year 0, *p* = 1,500);
A **=** a single payment in a series of *n* equal payments made at the end of each annual interest period (e.g., \$100 at the end of each year for 5 years, *A* = \$100);
F **=** a future sum, *n* annual period hence, equal to the compound amount of a present principal sum *p*, or equal to the sum of the compound amounts of payments *A* in a series (e.g., \$2,000 10 years from now, *F* = \$2,000).

In the previous Table , \$1,000 is a present principal sum (*P*), \$1,610 is a future sum (*F*), 10% is the annual interest rate (*i*), and 5 is the number of annual interest periods (*n*).

may be rewritten, substituting in the symbols and extending out to n years, as shown in Table below.

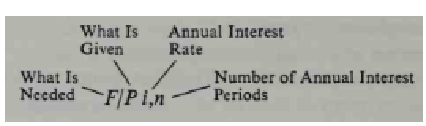
INTEREST FACTOR DERIVATION—*single payment compound amount factor*.

A Year	*B* Investment at Beginning of Year	*C* Interest Earned 10% (Col. B)	*D* Amount at End of Year
1	P	P_i	$P(1+i)$
2	$P(1+i)$	$P(1+i)i$	$P(1+i)^2$
3	$P(1+i)^2$	$P(1+i)^2 i$	$P(1+i)^3$
4	$P(1+i)^3$	$P(1+i)^3 i$	$P(1+i)^4$
5	$P(1+i)^4$	$P(1+i)^4 i$	$P(1+i)^5$
⋮	⋮	⋮	⋮
n	$P(1+i)^{n-1}$	$P(1+i)^{n-1} i$	$P(1+i)^n = F$

Therefore, for any amount *p*, any annual interest rate *i*, and any number of years *n*, the amount *F* that it will be equal to at the end of *n* year is given by Equation:

$$F = P(1+i)^n$$

The factor $(1+i)^n$ is called the single payment compound amount factor. Since this is a lengthy title, perhaps a shorthand designation would be more appropriate. The designation used is *F/P i,n*, which is interpreted as follows:



since the factor $(1+i)^n$ is independent of the amount (or *F*), the factor $(1+i)^n$ can be tabulated for various interest rates *i* and annual interest periods *n*.

2.2 Other Interest Factors:

As might be expected, factors can be derived in a similar manner for all combinations of *P*, *F*, and *A*. Those not covered so far are

(1) $(^{P/F\ i,n})$ Find *P* given *F*.
(2) $(^{F/A\ i,n})$ Find *F* given *A*.
(3) $(^{A/F\ i,n})$ Find *A* given *F*.
(4) $(^{P/A\ i,n})$ Find *P* given *A*.
(5) $(^{A/P\ i,n})$ Find *A* given *P*.

All these factors can be tabulated for arbitrary annual interest rates and annual interest periods.

3. Examples for Interest Factors

In the previous section $1,000 was invested at 10% for 5 years. The amount F at the end of the fifth year should be given by

$$F = (^{F/P\ i,n})$$
$$= 1{,}000\ (^{F/P\ 10,5})$$
$$= 1{,}000 * 1.611$$
$$= \$1{,}611$$

Following are other examples:

3.1. How much will $150 be worth 10 years from now at an annual interest rate of 10%?

3.2. How much would have to be set aside now to provide $10,000 fifteen years from now at an annual interest rate of 10 % ?

3.3. If $100 is set aside at the end of each year for 8 years at an annual interest rate of 10%, what would it be worth at the end of the eighth year?

3.4. How much would be required at the end of each year for 10 years to repay a loan of $1,500 now if the interest rate is 10% per year?

3.5. How much would be required at the end of each year for 9 years to accumulate $2,000 at the end of the ninth year if the annual interest rate is 10%?

3.6. How much can be borrowed now, if it can be repaid by five equal end of year payments of $100 each? The annual interest rate is 10%.

4.Back to the Company— Present Worth Calculation

In the machine determination problem for the company, the problem was in deciding which of three alternatives was best, given varying cash flows. One way of doing this is to find the amount at the end of year that is equal to the cash flow when interest is considered and choosing the best. This amount is called the present worth amount. The alternatives are:

Alternative 1. No investment, yearly *nominal* cost is 10,000 USD.
Nominal value is the value which is not adjusted according to the inflation.

Alternative 2. 30,000 USD investment and yearly nominal cost 3,500 USD. No salvage value.

Alternative 3. 40,000 USD investment and yearly nominal cost 3,000 USD. The salvage value at the end of year 5 is 6,000 USD.

We can find all values above using the following equations:

$$F = P(1+i)^n$$

$$F = A\frac{(1+i)^n - 1}{i}$$

So,

$$P = A\frac{(1+i)^n - 1}{i\,(1+i)^n}$$