IENG/MANE112

Scheduling Theory

Scheduling is a decision making process that is used to on a regular basis in many manufacturing and services industries. It deals with the allocation resources to task over given time periods and its goal to optimize one or more objectives.

1. **Gantt chart:** It is the visualization of scheduling production. Every machine has its time axis. Production and idle periods are showed on the axis.

   Necessary conditions

   a. The machine must finish the previous operation.

   b. The task must be finished on the previous machine.

   c. Example: F2

	Task 1	**Task 2**
Machine 1	**2**	**1**
Machine 2	**2**	**1**

Orders: (Task 1, Task 2); (Task 2, Task 1)

2. **Parameters of scheduling problems:**

Parameter	**Notation**
Processing time: The time of an operation	p_j or p_{ij}
Completion time: the time when the production of a job is finished	C_j
Arrival time of job j	r_j
Due-date: the time when the production of a job should be finished	d_j
Set-up time between job j and k	set-up$_{jk}$ or s_{jk}
Lateness: C_j - d_j	L_j
Tardiness: max{0, C_j - d_j}	T_j
Unit penalty: max{0, sgn(T_j)}	U_j
Earliness: max{0, d_j - C_j}	E_j

3. Compact description of scheduling problems: α | β | γ

a. Field α: Machine environment (m = # of Machines)

Notation	Description
1	Single machine problem
Any number	The number of machines
P	Identical parallel machines
R	Unrelated parallel machines
F	Flow shop: jobs visit the machines in the same machine order
J	Job shop: jobs visit the machines in the different machine order

b. Field β: Job environment (n = # of jobs)

Notation	Description	Default
prmp	Preemption: a process can be stopped and be continued later without any loss.	Not allowed.
overlap	Several machines may work on a job at the same time.	Not allowed.
idle	The machine may not work although at least one job waits for processing.	Not allowed.
r_j	The jobs have release times, i.e. they become at different times available for processing.	All jobs are available for processing at time 0.
prec	There is precedence relation among jobs.	No precedence relation.
tree	The structure of the precedence relation is tree-like.	No special structure.
res	There are resource constraints.	No resource constraints.
res1	There is one resource constraints.	No resource constraints.
set-up	There is set-up time between two jobs.	No set-up.
no-wait (nwt)	The jobs cannot wait before the machines.	The jobs can wait before the machines.
p_j= 1, 2	Special properties of the processing times. In this example each processing time is either 1, or 2.	No special property. All processing times are positive.
d_j=d	Special properties of due-dates. In this example all due-dates are equal.	No special property. All due-dates are positive.

c. **Field γ: Objective function**

Created functions		
C_{max}	ΣC_j	$\Sigma w_j C_j$
L_{max}	ΣL_j	$\Sigma w_j L_j$
T_{max}	ΣT_j	$\Sigma w_j T_j$
U_{max}	ΣU_j	$\Sigma w_j U_j$
	$\Sigma(T_j+E_j)$	

Remarks. All weights w_j are positive. All objective functions are to be minimized.

The engineering meaning of the objective functions:

- C_{max} is the *makespan*: the completion time of the last job.
- ΣC_j is the total *Work-In-Process* quantity.
- $\Sigma w_j C_j$ is the total WIP taking into account the different values of the items.
- L_{max} is the maximal lateness.
- U_{max} is 0 if all items are completed in time, and 1 if at least one tardy job.
- ΣT_j is the number of tardy jobs.

$$L_j = C_j - d_j.$$

$$T_j = \max(C_j - d_j, 0) = \max(L_j, 0).$$

$$U_j = \begin{cases} 1 \text{ if } C_j > d_j \\ 0 \text{ otherwise} \end{cases}, \qquad U_j \neq T_j$$

Relations of objective functions:

- The optimal value of L_{max} gives the optimal value of T_{max}. If $L_{max} < 0$ then the optimal value T_{max} is 0, otherwise the two optimal values are equal.
- The optimal value of L_{max} gives the optimal value of U_{max}. If $L_{max} \leq 0$ then the optimal value U_{max} is 0, otherwise the optimal value of U_{max} is 1.

4. Examples

- **$1||\sum C_j$**

  - Single machine problem.
  - No preemption
  - No overlap (As a matter of fact it has no meaning, because it needs at least two machines.)
  - The jobs are available for processing at time=0.
  - No precedence relation among jobs.
  - No resource constraint.
  - There is no set-up between two jobs.
  - Jobs can wait before machines.
  - No special properties of properties of processing times.
  - No due-dates.
  - The objective is to minimize the WIP.
- **$P|r_j$, set-up $|C_{max}$**
  - Identical parallel machines
  - No preemption
  - No overlap (Overlap makes sense here as one operation could be divided among several machines.)
  - The jobs arrive at different times.
  - No precedence relation among jobs.
  - No resource constraint.
  - There is set-up between two jobs.
  - Jobs can wait before machines.
  - No special properties of properties of processing times.
  - No due-dates.
  - The objective is to minimize the makespan.

5. Important orders of jobs

- **Shortest Processing Time (SPT) order:** is the increasing order of the processing times.
- **Longest Processing Time (LPT) order:** is the decreasing order of the processing times.
- **Earliest Due-Date (EDD) order:** is the increasing order of the due-dates.

6. Exactly solved cases

- **The optimal solution of the problem $1||\sum C_j$ is given by the SPT order.**

- **The optimal solution of the problem $1||L_{max}$, $1||T_{max}$ is given by the EDD order.**

7. **List scheduling for P||C**max

- **The algorithm.**
    - **STEP 1:** Prepare the list of jobs, e.g. LPT order.
    - **STEP 2:** Assign the jobs in the order of the list to machines. Each job must be assigned to the least loaded machine.

8. **Flow shop**
    - In general it is difficult.

9. **Job shop**

    - **Dispatching rules are applied:** SPT, LPT, EDD, FIFO, LIFO, etc.