**EASTERN MEDITERRANEAN UNIVERSITY**

 **Department of Industrial Engineering**

**IENG514 Stochastic Processes and Applications**

**HOMEWORK 5 Fall 2019-20**

1. Show that an Erlang distribution with shape parameter *k* and rate parameter *λ* has the same distribution as the random variable *X1+ X2+ …+Xk*, where each *Xi* is an exponential random variable with parameter *kλ*, and the *Xi* ‘s are independent random variables.
2. Suppose that defects in a sheet of material follow the Poisson model with an average of 3 defects per 3 square meters. Consider a 6 square meter sheet of material.
3. Find the probability that there will a. be at least 4 defects.
4. Find the mean and standard deviation of the number of defects.
5. Suppose that raisins in a cake follow the Poisson model with an average of 2 raisins per cubic inch. Consider a slab of cake that measures 3 by 4 by 1 inches.

a) Find the probability that there will be at no more than 20 raisins.

b) Find the mean and standard deviation of the number of raisins.

1. The number of accidents in a town follows a Poisson process with mean of 4 per day and the number *Xi* of people involved in the *i*th accident has the distribution ( independent) *Pr{Xi=k}=(0.75)(0.25)k (k>0)*. Find the mean and the variance of the number of people involved in accidents per week. If the probability for involving a child in each accident be 0.05, what is the probability that there are 5 children in one week?
2. A person enlists subscriptions to a magazine; the number enlisted being given by Poisson process with mean rate 10 per day. Subscribers may subscribe for 1, 2 or 3 years two by two independently, with respective probabilities,  and. If he received commission $2, $5 and $12 for 1, 2 and 3 year subscriptions respectively, compute expected value and variance of the total commission earned in 30 months.

1. In a Poisson process with mean *λ*, show that the waiting time for change of a state same as *i* has exponential distribution with parameter *λ*.
2. For a non–homogenous Poisson process, the intensity function is given by



Asuume that we know number of events in first 3 minutes is 7. Calculate the probability that next event occur after more than first 4minutes.

1. Suppose that number of defects in each 5by3 sheet of carpet follow the non hemogeneous Poisson model with the following intensity function



1. What is the probability that exactly 5 defects exists in first 3 m2 of this sheet?()
2. What is the mean of the defects in rest of the sheet?
3. Customers arrive at a restaurant in groups consisting 2,3 or 4 individuals and the arrival of groups in accordance with a Poisson process with mean rate *3* . The arrival probability of group with 3 individuals is two times of arrival probability of other groups.
4. Find the mean of customers arriving in 4 hours.
5. Find the probability that the total arrival customers in one hour exactly be 5 persons.
6. With which probability this restaurant must prepare 4 tables for groups with 4 individuals in first 2 hours.
7. Assume that the arrival process to a store is governed by exponential distribution with mean rate 0.1 of a minute. This store advertises 4 types of a new brand perfume by distributing the small sample testers of this brand with very low prices. Customers may select one of these types randomly and pay 2,7,5 and 6 cents respectively to each one of testers. If just 60% of the customers joint to this advertising program,
8. What is the expected collected money after 8 hours?
9. What is the probability that 3$ collected after 2 hours?