**EASTERN MEDITERRANEAN UNIVERSITY**



**Department of Industrial Engineering**

**IENG514 Stochastic Processes and Applications**

**HOMEWORK 3 Fall 2019-20**

1. Each American family is classified as living in an urban, rural, or suburban location. During a given year, 15% of all urban families move to a suburban location, and 5% move to a rural location; also, 6% of all suburban families move to an urban location, and 4% move to a rural location; finally, 4% of all rural families move to an urban location, and 6% move to a suburban location.
2. If a family now lives in an urban location, what is the probability that it will live in an urban area two years from now? A suburban area? A rural area?
3. Suppose that at present, 40% of all families live in an urban area, 35% live in a suburban area, and 25% live in a rural area. Two years from now, what percentage of American families will live in an urban area?
4. What problems might occur if this model were used to predict the future population distribution of the United States?
5. Specify the classes of the following Markov chains, and determine whether they are transient or recurrent or aperiodic.

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1. Show that if state *i* in a Markov chain is recurrent and *i* dose not communicate with state *j* then, *pij=0*.
2. Let 0 be an observing state and for *j>0, pjj=p, pjj-1=1-p*. Show that . Find *Fj0*.
3. If *k* is a transient state and *j* is an arbitrary state then is convergence and.
4. A particle starting from the origin moves from *j* to position *(j+1)* with probability *aj* and returns to origin with probability *(1-aj)*. Suppose that the states, after *n* moves are, 0,1,2,…. Show that state 0 is persistent if and only if 
5. A factory has two machines and one repair crew. Assume that probability of any one machine breaking down on a given day is 0.25. Assume that if the repair crew is working on a machine, the probability that they will complete the repairs in one more day is 0.4. For simplicity, ignore the probability of a repair completion or a breakdown taking place expect at the end of the day. Let *Xn* be the number of machines in operation at the end of the *n*th day. Assume that the behaviour of *Xn*can be modelled as a Markov chain.
6. Find one step transition matrix for the chain.
7. If the system starts out with both machine operating, what is the probability that both will be in operation two day later?
8. With which probability this factory can continue to work with two machines without any problem?
9. What is the probability that the factory can always work by two machines?
10. Find *Pn* and the limiting probabilities for the chain having transition probability matrix.



1. Each morning an individual leaves his house and goes for a run. He is equally likely to leave either from his front of back door. Upon leaving the house, he chooses a pair of running shoes (or goes running barefoot if there are not shoes at the door from which he departed). On his return he is equally likely to enter, and leave his running shoes, either by the front or back door. If he owns a total of k pairs of running shoes, what proportion of the time does he run barefooted?
2. Consider a system with two components. We observe the state of the system every hour. A given component operating at time *n* has probability *p* of failing before the next observation at time *n + 1*. A component that was in a failed condition at time *n* has a probability *r* of being repaired by time *n + 1*, independent of how long the component has been in a failed state. The component failures and repairs are mutually independent events. Let *Xn* be the number of components in operation at time *n*. The process *{Xn; n = 0, 1, …}* is a discrete time homogeneous Markov chain with state space *I = 0, 1, 2*.
3. Determine its transition probability matrix, and draw the state diagram.
4. Obtain the steady state probability vector, if it exists.