**EASTERN MEDITERRANEAN UNIVERSITY**



**Department of Industrial Engineering**

**IENG514 Stochastic Processes and Applications**

**HOMEWORK 4 Fall 2019-20**

1. A machine goes out of order whenever a component part fails. The failure of this part is in accordance with a Poisson process with mean rate of 3 per week.
2. What is the probability that three weeks pass after the last failure.
3. Suppose that there are 6 extra parts of the component in an inventory. What is the probability that the next supply for new component is not due in 10 weeks?
4. What is the expected time for running out those 5 extra parts?
5. Divided the interval *[0,1)* into a large number *n* of small intervals of length *h* and suppose that in each small interval, Bernoulli trails with probability of success are held this means that trial with only two outcomes, success with probability  and failure with probability *(1-**)*. Show that the number of successes in an interval of length *t* is a Poisson process with mean. State the assumptions you make.
6. An average of 12 jobs per hour arrive at our departmental printer.
7. Use two different computations (one involving the Poisson and another exponential random variable) to determine the probability that no job will arrive during the next 10 minutes.
8. What is the probability that 3 or fewer jobs will arrive during the next 20 minutes?
9. What is the probability to wait more than 90 minutes to finish the 40th job by this printer?
10. Order arrive in a pizza restaurant according to Poisson process with mean time between order is 10 min. Order are either vegetarian pizza (*p=0.25*) or chicken pizza (*q= 0.75)*. The company makes 3 TL profit on vegetarian pizza and 4.5 TL on chicken pizza. Find the mean & variance of the profit of the company in an 8 hour day.
11. The number of failures *N* (*t*), which occur in a computer network over the time interval [0*, t*), can be described by a homogeneous Poisson process. On an average, there is a failure after every 5 hours, i.e. the intensity of the process is equal = 0*.*2(*h*)-1.

(a) What is the probability of at most 1 failure in [0*;* 9), at least 2 failures in [9*,* 18), and at most 1 failure in [18,27) (time unit: hour)?

(b) What is the probability that the third failure occurs after 9 hours?

1. If *N1(t)*, *N2(t)* are two independent Poisson processes with parameters , respectively, then show that :



1. Suppose that *N1(t)*, *N2(t)* are two independent Poisson processes with parameters, respectively. Compute the expected value and variance of *N(t)=* *N1(t)*- *N2(t).*
2. Show that for large *t* the observation *N(t)/t* is a reasonable estimation for the mean of a Poisson process.
3. Assume that arrival of vehicles to a public transport station (buses and taxis) is according to a Poisson process with rate 20/hour. The station is starting to work from 6:00 AM.
4. Find the probability that there is no vehicle arriving in a 6 minute interval.
5. Given that a vehicle arrived at 10:00; find the probability that the next arrival happen before 10:07.
6. If the probability that a bus arrival be 0.2. What is the probability of seven buses arrived to the station during 2 hours given that 25 vehicles arrived in last hour.
7. Suppose that of each bus carry 16 passengers with variance 0.09 and each taxi carry 3 passengers with variance 0.05. How many passengers can carry from this public transportation station in 3 hours?
8. Given that the number of taxis which arrive until 12:00 is 180, what is the probability that 65 taxis arrive to the station until 9:30?
9. The time between arrivals of buses follows an exponential distribution, with a mean of 20 minutes.
10. That at least two buses will arrive during the next one and half an hours?
11. That no buses will arrive during the next 2 hours? A bus has just arrived.
12. What is the probability that it will be between 40 and 90 minutes before the next bus arrives?
13. What is the probability that the waiting time to receive the fifth bus be 2 hours from now?
14. My home uses two light bulbs. On average, a light bulb lasts for 22 days (exponentially distributed). When a light bulb burns out, it takes an average of 2 days (exponentially distributed) before I replace the bulb.
15. Formulate a three-state birth–death model of this situation.
16. Determine the fraction of the time that both light bulbs are working.
17. Determine the fraction of the time that no light bulbs are working.