IENG/MANE 112

Introduction to Industrial Engineering

lecture 3

Decision Theory

Decision Theory

- A general approach to decision making that is suitable to a wide range of operations management decisions
  - Capacity planning
  - Product and service design
  - Equipment selection
  - Location planning

Characteristics of Suitable Problems

- Characteristics of decisions that are suitable for using decision theory
  - A list of alternatives from which to choose
  - A set of possible future conditions that will have a bearing on the results of the decision
  - A known payoff for each alternative under each possible future condition

Decision Process

- Steps:
  1. Identify the problem
  2. Specify objectives and criteria for a solution
  3. Develop suitable alternatives
  4. Analyze and compare alternatives
  5. Select the best alternative
  6. Implement the solution
  7. Monitor to see that the desired result is achieved
- Errors
  - Failure to recognize the importance of each step
  - Skipping a step
  - Failure to admit mistakes

Decision Environments

- There are three general environment categories:
  - **Certainty**
    - Environment in which relevant parameters have known values
  - **Risk**
    - Environment in which certain future events have probable outcomes
  - **Uncertainty**
    - Environment in which it is impossible to assess the likelihood of various future events

Decision Making Under ***Uncertainty***

- Decisions are sometimes made under complete uncertainty: No information is available on how likely the various states of nature are.
- At the time a decision is made, the decision maker is uncertain which states of nature will occur in the future and has no control over them.
- A ***state of nature*** is an actual event that may occur in the future.

Decision Making Under ***Uncertainty***

- Suppose a distribution company is considering purchasing a computer to increase the number of orders it can process. If economic conditions remain good, the company will realize a large increase in profit; however, if the economy takes a downturn, the company will lose money.
- possible decisions are:
  - to purchase the computer and
  - to not purchase the computer.
- The states of nature are:
  - good economic conditions and
  - bad economic conditions.

Decision Making Under ***Uncertainty***

- Decision Criteria:
  - **Maximin**
    - Choose the alternative with the best of the worst possible payoffs
  - **Maximax**
    - Choose the alternative with the best possible payoff
  - **Laplace**
    - Choose the alternative with the best average payoff
  - **Minimax regret**
    - Choose the alternative that has the least of the worst regrets

Example – Payoff Table

Payoff Table: A table used to show the payoffs that can result from decisions under various states of nature

	Possible Future Demand		
Alternatives	Low	Moderate	High
Small Facility	$10	$10	$10
Medium Facility	7	12	12
Large Facility	-4	2	16

- A decision is being made concerning which size facility should be constructed
- The present value (in millions) for each alternative under each state of nature is expressed in the body of the above payoff table

Example – Maximin Criterion

	Possible Future Demand		
Alternatives	Low	Moderate	High
Small Facility	$10	$10	$10
Medium Facility	7	12	12
Large Facility	-4	2	16

•The worst payoff for each alternative is

Small facility: $10 million

Medium facility $7 million

Large facility -$4 million

•Choose to construct a small facility

Example – Maximax Criterion

Alternatives	Possible Future Demand Low	Moderate	High
Small Facility	$10	$10	$10
Medium Facility	7	12	12
Large Facility	-4	2	16

- The best payoff for each alternative is
  - Small facility: $10 million
  - Medium facility $12 million
  - Large facility $16 million
- **Choose to construct a large facility**

Example – Laplace Criterion

Alternatives	Possible Future Demand Low	Moderate	High
Small Facility	$10	$10	$10
Medium Facility	7	12	12
Large Facility	-4	2	16

- The average payoff for each alternative is
  - Small facility: (10+10+10)/3 = $10 million
  - Medium facility (7+12+12)/3 = $10.33 million
  - Large facility (-4+2+16)/3 = $4.67 million
- **Choose to construct a medium facility**

Example – Minimax Regret

Alternatives	Possible Future Demand: Low	Moderate	High
Small Facility	$10	$10	$10
Medium Facility	7	12	12
Large Facility	-4	2	16

- Construct a **regret** (or **opportunity loss**) table
  - The difference between a given payoff and the **best** payoff for a state of nature

Alternatives	Regrets: Low	Moderate	High
Small Facility	$0	$2	$6
Medium Facility	3	0	4
Large Facility	14	10	0

Example – Minimax Regret

Alternatives	Low	Moderate	High
Small Facility	$0	$2	$6
Medium Facility	3	0	4
Large Facility	14	10	0

- Identify the worst regret (i.e. maximum value) for each **alternative**
  - Small facility $6 million
  - Medium facility $4 million
  - Large facility $14 million
- Select the alternative with the **minimum of the maximum regrets**
  - **Build a medium facility**

Decision Making Under *Risk*

- Decisions made under the condition that the **probability** of occurrence for each state of nature can be estimated
- A widely applied criterion is **expected monetary value** (**EMV**)
  - **EMV**
    - Determine the expected payoff of each alternative, and choose the alternative that has the best expected payoff
  - This approach is most appropriate when the decision maker is neither risk averse nor risk seeking

Example– EMV

Alternatives	Low (.30)	Moderate (.50)	High (.20)
Small Facility	$10	$10	$10
Medium Facility	7	12	12
Large Facility	-4	2	16

$EMV_{small} = .30(10) + .50(10) + .20(10) = 10$

$EMV_{medium} = .30(7) + .50(12) + .20(12) = 10.5$

$EMV_{large} = .30(-4) + .50(2) + .20(16) = \$3$

Build a medium facility

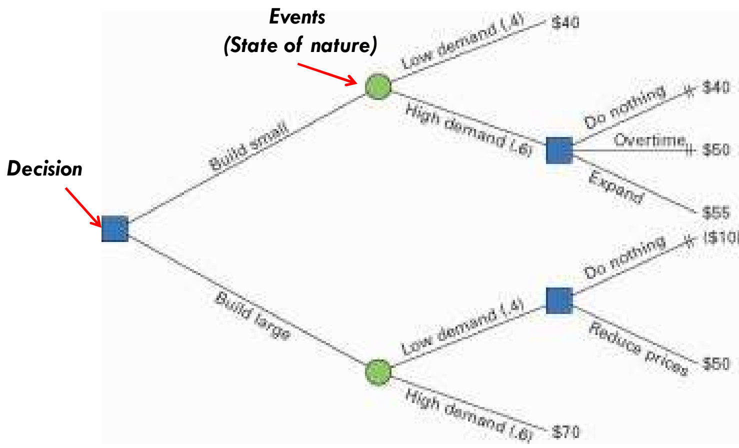
Decision Tree

- **Decision tree**
  - A schematic representation of the available alternatives and their possible consequences
  - Useful for analyzing sequential decisions

Decision Tree

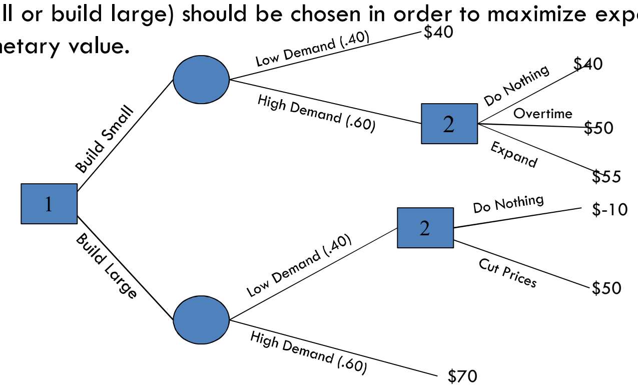
- Composed of
  - Nodes
    - Decisions – represented by square nodes
    - Chance events – represented by circular nodes
  - Branches
    - Alternatives– branches leaving a square node
    - Chance events– branches leaving a circular node
- Analyze from right to left
  - For each decision, choose the alternative that will yield the greatest return
  - If chance events follow a decision, choose the alternative that has the highest expected monetary value (or lowest expected cost)

Format of a Decision Tree

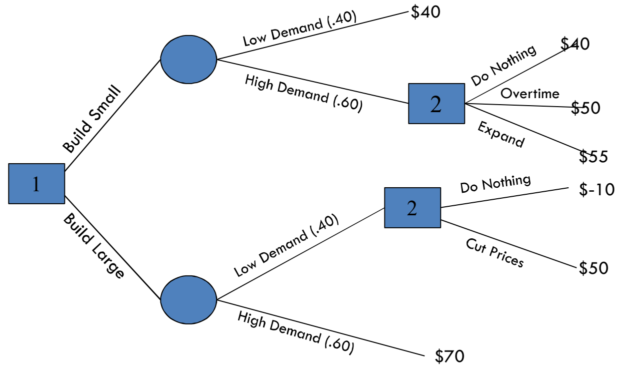


Example– Decision Tree

- A manager must decide on the size of a video arcade to construct. The manager has narrowed the choices to two: large or small. Information has been collected on payoffs, and a decision tree has been constructed. Analyze the decision tree and determine which initial alternative (build small or build large) should be chosen in order to maximize expected monetary value.



Example– Decision Tree



EV_{Small} = .40(40) + .60(55) = \$49
EV_{Large} = .40(50) + .60(70) = \$62

Build the large facility

Expected Value of Perfect Information

- **Expected value of perfect information (EVPI)**
  - The difference between the expected payoff with perfect information and the expected payoff under risk
  - Two methods for calculating EVPI
    - EVPI = expected payoff under certainty – expected payoff under risk
    - EVPI = minimum expected regret

Expected payoff under certainty is calculated by identifying the best payoff under each state of nature, then combining these by weighing each payoff by the probability of that state of nature and adding the amounts.

Example – EVPI

Alternatives	Low (.30)	Moderate (.50)	High (.20)
Small Facility	$10	$10	$10
Medium Facility	7	12	12
Large Facility	-4	2	16

- $EV_{\text{with perfect information}}$ = .30(10) + .50(12) + .20(16) = $12.2
- EMV = $10.5 (see slide no. 19)
- EVPI = $EV_{\text{with perfect information}}$ – EMV
- = $12.2 – 10.5
- = $1.7
- You would be willing to spend up to $1.7 million to obtain perfect information

Example– EVPI (using the expected regret approach)

Regret Table:

Alternatives	Low (.30)	Moderate (.50)	High (.20)
Small Facility	$0	$2	$6
Medium Facility	3	0	4
Large Facility	14	10	0

- Expected Opportunity Loss
  - EOL_{Small} = .30(0) + .50(2) + .20(6) = $2.2
  - EOL_{Medium} = .30(3) + .50(0) + .20(4) = $1.7
  - EOL_{Large} = .30(14) + .50(10) + .20(0) = $9.2
- The minimum EOL is associated with the building the medium size facility. **This is equal to the EVPI, $1.7 million**

Example 1

- Fenton and Farrah Friendly, husband-and-wife car dealers, are soon going to open a new dealership. They have three offers: from a foreign compact car company, from a U.S.-producer of full-sized cars, and from a truck company. The success of each type of dealership will depend on how much gasoline is going to be available during the next few years. The profit from each type of dealership, given the availability of gas, is shown in the following payoff table:

Example 1 (cont.)

Dealership	Shortage (0.6)	Surplus (0.4)
Compact cars	$300,000	$150,000
Full-sized cars	100,000	600,000
Trucks	120,000	170,000

- Determine which type of dealership the couple should purchase

Example 2

- The management of First American Bank was concerned about the potential loss that might occur in the event of a physical catastrophe such as a power failure or a fire. The bank estimated that the loss from one of these incidents could be as much as $100 million, including losses due to interrupted service and customer relations. One project the bank is considering is the installation of an emergency power generator at its operations headquarters.
- The cost of the emergency generator is $800,000, and if it is installed, no losses from this type of incident will be incurred.
- However, if the generator is not installed, there is a 10% chance that a power outage will occur during the next year. If there is an outage, there is a .05 probability that the resulting losses will be very large, or approximately $80 million in lost earnings. Alternatively, it is estimated that there is a .95 probability of only slight losses of around $1 million. Using decision tree analysis, determine whether the bank should install the new power generator.